Republic of the Philippines Department of Education NATIONAL CAPITAL REGION Misamis Street, Bago-Bantay, Quezon City

# UNIFIED SUPPLEMENTARY LEARNING MATERIALS (USLeM)



# MATHEMATICS

### **EXPECTATIONS:**

You will **represent a point, line, and plane using concrete and pictorial models, illustrate subsets of a line** and **classify different kinds of angles and measure an angle.** Specifically, this learning material will help you to:

- Describe and name a point, a line, and a plane
- Illustrate the relationships of points, lines, and planes
- Define, identify, and name the subsets of a line
- Illustrate and define an angle
- Name and identify the parts of an angle
- Illustrate the different kinds of angles: acute, right, and obtuse

Let us start your journey in learning more about Points, Lines, and Planes, Subsets of Line and Angles. I am sure you are ready and excited to answer the Pretest. Smile and cheer up!

# PRE-TEST

**Directions:** Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

1) Which of the followi	ng is NOT consi	dered as u	ındefin	ed term?					
a) Point	b) Line	C	c) Plane	2	d) Angle				
2) It is a subset of a line consisting of two endpoints and all the points in between.									
a) Ray	b) Line Segmen	nt c	c) Oppo	osite Rays	d) Congr	uent Seg	ments		
3) Given the three states	ments below, ide	ntify the s	stateme	ent/s that is alwa	ys true abo	out point	s?		
I.	Coplanar points	s lie on the	e same	line.					
II.	A point denotes a position in space and has no dimension.								
a) Statement II only	A line has two e	endpoints.	2) Stata	ments L and III	only				
b) Statements II and	III only	(	1) State	ments I II and	III.				
4) Which of the followi	ng is an angle th	at maasur	as hatu	$r_{\text{ren}} = 0^{\circ}$ and $00^{\circ}$	) )				
a) Right Angle	b) Obtuse Angle	e (	c) Acut	e Angle	d) Reflex	Angle			
5) What is the vertex of	F / TOY?		,	8	.,	0			
a) T	b) O	C	c) Y		d) cannot	t be deter	rmined		
6) How many distinct li	ine segments can	be named	in the	figure? A	B	С	D		
a) 3 b) 4	c) 5	d) 6			•	<b></b>	<b></b>	m	
7) Using the same figur	e in #6, how mai	ny rays ca	n be na	med in the figur	re?				
a) 3	b) 4	C	c) 5		d) 6				
8) Using the same figur	e in #6, which is	not the co	orrect r	name for the figu	ure?				
a) AC	b) DA	C	c) Am		d) <i>BC</i>				
9) What is formed by tw	wo non-collinear	rays with	a com	mon endpoint ca	alled vertex	<b>c</b> ?			
a) line	b) opposite rays	s (	c) angle	es	d) plane				
10) Which statement is	TRUE?								
a) The intersection	of a line and a pl	ane 1s a li	ne.	c) The intersec	tion of two	planes 1	s a poir	it.	
b) I wo lines will al	ways intersect at	a point.		d) I wo copiana	called ske	ew lines.	ntersect	are	
For items $11 - 13$ , refer	to the figure at t	he right.			T.				
11) Which of the follow	ving cannot be us	sed to nam	ne the g	given angle?	•	A			
a)∠D	b)∠ADI	c) ∠IDA		d)∠AID	N				
2) Which point lies in th	he interior of the	angle?				• N	1		
a) 1	0) E	C) M		d) D	D	_			
							•		
						<b>E</b> ●	1		



## **BRIEF INTRODUCTION**

Did you ever play billiards? Billiards is a game played on a table with pockets in it, and with the use of a cue stick to strike small balls into the pocket. Many Filipinos considered Efren 'Bata' Reyes as one of the greatest pool players of all time. Pool is a game like billiard. What comes into your mind when you see the picture at the right? What lessons in geometry is it related?



## **Undefined Terms: Points, Lines and Planes**

Undefined terms are basic terms that cannot be defined but can be described. Some of these undefined terms in geometry are point, line, and plane. These three terms are used as the basis for defining other terms in geometry.

Undefined Terms	Description	Representation	Naming	Other Information	Concrete and Pictorial Model
Point	a position in space	A	A point is named using a capital letter. The representation can be named as <b>point A.</b>	Has no dimen- sion, no length, no width, no thickness, and does not occupy an area	tip of a needle, mole,
Line	set of continuous points that extends indefinitely in both directions	m A B	A line is named using any two points on the line with a line over the two letters or by using a single lower- case script letter. The representation can be named as: $\overrightarrow{AB}$ , $\overrightarrow{BA}$ , or $\overrightarrow{m}$	It has no width and no thickness. It is straight and has an infinite length	edge of a ruler, string
Plane	set of points contained in a flat surface and extends indefinitely in all directions		A plane is named by a single script capital letter or by any three points in the plane which are not on the same line. The representation can be named <b>plane N</b> , <b>plane ABD</b> , <b>plane</b> <b>ABCD</b>	Has infinite length and width but no thickness. A plane is com- monly denoted as a closed slanted four- sided figure	a sheet of paper, the surface of a table

## **Relationship between Points, Lines, and Planes**

A *definition* is a statement that gives clear meaning to a word or phrase. Using the undefined terms, we can define other terms in geometry.

1) *A space* is the set of all points.

2) A *figure* is any set of points.

3) Collinear Points are points that lie on the same line.

4) Non-collinear Points are points that do not lie on the same line

5) *Coplanar Points* are points that lie in the same plane

6) Non-coplanar Points are points that do not lie in the same plane.



The intersection of geometric figures is the set of all points that are common to the given figures.



*Figure 1* represents the intersection of two lines  $\overrightarrow{\text{DF}}$  and  $\overleftarrow{\text{EG}}$ . Notice that both lines contain point F. *Therefore, the intersection of two lines is a point and that point is called the point of intersection.* 

*Figure 2* represents the intersection of two planes  $\mathcal{D}$  and  $\mathcal{C}$ . Notice that common to both planes  $\mathcal{D}$  and  $\mathcal{C}$  is  $\overrightarrow{AB}$ . *Therefore, the intersection of two planes is a line.* 

*Note:* The broken line symbolizes that it is in the back part of the figure.

*Figure 3* represents the intersection  $\overleftarrow{EF}$  and Plane *G*. The line passes through the plane. Notice that common to both line and the plane is point F. *Therefore, the intersection of a line and a plane is a point*. (Note: The intersection of a line and plane can be an empty set if the line is parallel to the plane. The intersection of a line and a plane can also be a line if the plane contains that line.)

*Figure 4* represents plane  $\mathcal{I}$  containing two lines  $\overrightarrow{HJ}$  and  $\overrightarrow{LM}$ . The two lines do not have any point in common and even if you extend the two lines, there will be no intersection. These are called parallel lines. *Therefore, two coplanar lines that do not intersect are called parallel lines*.

#### Subsets of Line

A line segment is a subset of a line consisting of two endpoints and all the points in between.

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Figure 1 may be called  $\overline{SO}$ , read as "line segment SO".  $\overline{SO}$  is just the same with  $\overline{OS}$ .

Given this line with points A, B, C, and D:



In figure 2, we can form 6 different line segments which are as follows:  $\overline{AB}$ ,  $\overline{AC}$ ,  $\overline{AD}$ ,  $\overline{BC}$ ,  $\overline{BD}$  and  $\overline{CD}$ . Note that naming different line segments is based on the two endpoints. Without listing the different line segments, we can know the *number of line segments* formed in a *given line* using this formula:  $\frac{n(n-1)}{2}$ , where n is the number of points in the given line.

**Two line segments** are said to be **congruent segments** if they have the same measure or length. To know the measure of a line segment in a given number line, we get the absolute value of the difference of the coordinates of the two given points.

A ray is a subset of a line with only one endpoint and extends infinitely in only one direction.

Figure 3 may be called MU, read as "ray MU". We name a ray with its endpoint first followed by another point on the ray.

Using Figure 2, we can form 6 different rays which are as follows:  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ ,  $\overrightarrow{CD}$ ,  $\overrightarrow{DC}$ ,  $\overrightarrow{CB}$  and  $\overrightarrow{BA}$ . Without listing the different rays, we can know the *number of rays* formed in a *given line* using this formula: 2(n - 1), where n is the number of points in the given line.

To know the *number of rays* formed in a *given ray*, we use this formula: n - 1, where n is the number of points in the given ray.

**Opposite rays** are rays with a common endpoint but extending in opposite directions. Examples:

1) Name the different line segments, rays and, opposite rays formed by the figure below:

U Ε **Line segments:**  $\overline{CU}$ ,  $\overline{CT}$ ,  $\overline{CE}$ ,  $\overline{UT}$ ,  $\overline{UE}$  and  $\overline{TE}$ Solutions: **Rays:**  $\overrightarrow{CU}$ ,  $\overrightarrow{UT}$ ,  $\overrightarrow{TE}$ ,  $\overrightarrow{ET}$ ,  $\overrightarrow{TU}$  and  $\overrightarrow{UC}$ **Opposite Rays:**  $\overrightarrow{UC}$  and  $\overrightarrow{UT}$ ,  $\overrightarrow{TU}$  and  $\overrightarrow{TE}$ 

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2) Find the number of line segments and rays formed in a given line with: a) 7 points b) 13 points Solutions:

a) Line segments:  $\frac{n(n-1)}{2} = \frac{7(7-1)}{2} = \frac{7(6)}{2} = \frac{42}{2} = 21$ **Rays:** 2(n-1) = 2(7-1) = 2(6) = 12b) Line segments:  $\frac{n(n-1)}{2} = \frac{13(13-1)}{2} = \frac{13(12)}{2} = \frac{156}{2} = 78$  Rays: 2(n-1) = 2(13-1) = 2(12) = 24

3) Determine the measure of the following line segment using the figure below and identify which are congruent segments.



Solutions:

a) The coordinate of I is -9 and L is -5. Therefore, IL = |-9 - (-5)| = |-9 + 5| = |-4| = 4

b) The coordinate of L is -5 and O is -2. Therefore, LO = |-5 - (-2)| = |-5 + 2| = |-3| = 3

c) The coordinate of V is 2 and E is 6. Therefore, VE = |2 - 6| = |-4| = 4

d) The coordinate I is -9 and E is 6. Therefore, IE = |-9 - 6| = |-15| = 15

e) The coordinate L is -5 and U is 10. Therefore, LU = |-5 - 10| = |-15| = 15

Notice that the symbol above the letters was removed because we are referring to its measurement. IL = VE therefore they are *congruent segments*. ( $\overline{\text{IL}} \cong \overline{\text{VE}}$ )

IE = LU therefore they are also *congruent segments*. ( $\overline{IE} \cong \overline{LU}$ )

#### Naming and Parts of an Angle

An **angle** is formed by two non-collinear rays with a common endpoint. The two rays are called the sides of the angle and the common endpoint is called the vertex of the angle.

An angle is named using a number, the vertex of the angle or the vertex with a point on each side of the angle.

An angle separates a plane into three parts: the interior of the angle, the exterior of the angle, and the points on the side of the angle

#### Examples:

1) a) Name the angle formed in Figure 1. Identify its vertex and sides



vertex in the middle of the three letters) b) Different angles formed:  $\angle DEF$  or  $\angle FED$ ,  $\angle FEG$  or  $\angle GEF$  or  $\angle 1$ , and  $\angle DEG$  or  $\angle GED$ 

(Note: We cannot name figure 2 as  $\angle E$  only because there is more than 1 angle whose vertex is E)

2) Using Figure 3, give an example to each of the following:

a) interior of $\angle$ HIJ	d) interior of ∠HIK
b) exterior of ∠JIK	e) exterior of ∠HIK
c) points on the side of $\angle$ HIK	

c) H, I and K



## Angle Measurement and Kinds of Angles

The **measure of an angle** is the smallest amount of rotation about the vertex from one ray to the other. **Degree** is the unit of measure of an angle and the tool used to measure a given angle is **protractor**.

d) L and J

e) None

*Example:* The measure of  $\angle ABC$  is 40 can be written as "m $\angle ABC = 40$ "

In this example, the measure of an angle must be limited to greater than  $0^{\circ}$  but less than  $180^{\circ}$  only. To measure an angle or to find the number of degrees in an angle, we have to follow these steps:

1) Place the center of the protractor at the vertex of the angle

2) Place the straight edge of the protractor along one side of the angle.

3) The other side of the angle needs to cross the numbered part of the protractor.

4) The size of the angle can be read off the scale.

Other information:  $1^{\circ} = 60$  minutes written as "60"

b) H and L

Solutions:

a) L

1 minute = 60 seconds written as "60" So therefore,  $1^\circ = 59' 60"$ 

Angle Addition Postulate: If D is in the interior of  $\angle BAC$ , then  $m \angle BAD + m \angle DAC = m \angle BAC$ **Kinds of Angles** 

A **right angle** is an angle that measures 90°.

An **acute angle** is an angle whose measure is greater than 0° but less than 90°.

An obtuse angle is an angle whose measure is greater than 90° but less than 180°.





Note that right angles are marked using a small square on the vertex.

## ACTIVITIES

Activity 1: Tell whether each of the following represents a point, a line, or a plane.

 81	r , , , , , , , , , , , , , , , , , , ,
 1) star in the sky as seen from the Earth	6) a clothesline
 _ 2) curtain rod	7) the crease in a folded paper
 _ 3) edge of a ruler	8) corner of a room
 _ 4) cartolina	9) a page of a book
 5) a knot on a piece of thread	10) a magic wand

Activity 2: Use the figure at the right to determine whether each of the following is True or False.

Figure:

- 1) A, N and D are collinear. 2) A, R, Z, K are coplanar.
- \_\_\_\_\_\_ 3) C, R, Z and T are non-coplanar.
- \_\_\_\_\_ 4) M, K and T are coplanar.
- \_\_\_\_\_ 5) M, R, A and I are coplanar
- \_\_\_\_\_\_ 6) R, Z and T are non-collinear.
- \_\_\_\_\_7) E, C and M are collinear.
- \_\_\_\_\_ 8) M, K, C and I are coplanar.
- \_\_\_\_\_9) M, R, Z, T and K are coplanar.
- \_\_\_\_\_ 10) A, I, R and N are non-collinear.



Activity 3: Use the figure at the right to answer the following questions.
1) Name all the distinct line segments in PS
2) How many line segments were formed? \_\_\_\_\_\_\_\_
3) Name all the rays in MN \_\_\_\_\_\_\_\_
4) How many rays were formed? \_\_\_\_\_\_\_\_
5) Name the opposite rays \_\_\_\_\_\_\_

Activity 4: Determine the number of line segments and rays formed in a line with the given number of points.

1) 6 points	line segment:	rays:
2) 10 points	line segment:	rays:
3) 12 points	line segment:	rays:
4) 9 points	line segment:	rays:
5) 15 points	line segment:	rays:

Activity 5: Determine the measure of the following segments using the figure below.

Α	R			Ε			Y		0		U			W	Ι	L			D		
-11 -10	0-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
	1	) AR				_ 2) Ī	RY	_		3)	EU			4	) W	D			5) E	Ĺ	

Activity 6: Use the figure at the right to complete the table below.

Name of the different Angles	Sides of the Angle	Points in the Interior	Points in the Exterior	Points on the side		P. <sup>4</sup>	E 🐙
∠PKS	$\overrightarrow{\mathrm{KP}}$ and $\overrightarrow{\mathrm{KS}}$	NONE	E and A	P, K and S			
					S	K	A

Activity 7: Find the measure of the angles using the protractor below.



Activity 8: Tell whether the given measure will form a right angle, an acute angle or an obtuse angle.

1) 45°	6) 90°	11) 90° 1′ 13″
2) 75°	7) 89° 59′ 60″	12) 179°
3) 111°	8) 90.001°	13) 45° + 35°
4) 89.99°	9) 32.5°	14) 75° + 15°
5) 178°	10) 178° 13′ 40″	15) 128° - 48°

## REMEMBER

- The three **undefined** terms are point, line, and plane
- A *point* denotes a position in space and has no dimension.
- A *line* is a series of points that extends in opposite directions without end.
- ✤ A *plane* is a flat surface.
- Points that lie on the same line are collinear while points that do not lie on the same line are non-collinear.
- Points and lines on the same plane are coplanar while points that do not lie on the same plane are non-coplanar.
- The intersection of two lines is a *point*.
- The intersection of a line and plane is a *point*. It can be an empty set if the line is parallel to the plane. It can also be a line if the plane contains the line.
- The intersection of two planes is a *line*.
- Subsets of a Line:
  - A line segment is subset of line consisting of two endpoints and all the points in between.
  - $\blacktriangleright$  A ray is a subset of a line with only one endpoint and extends infinitely in only one direction.
- **Opposite rays** are rays with a common endpoint but extending in opposite directions.
- To know the *number of line segments* formed in a *given line*, use this formula:  $\frac{n(n-1)}{2}$ , where n is the number of points in the given line.
- ★ To know the *number of rays* formed in a *given line*, use this formula: 2(n 1), where n is the number of points in the given line.
- An angle is formed by two non-collinear rays with a common endpoint. The two rays are called the sides of the angle and the common endpoint is called the vertex of the angle.
- ✤ An angle separates a plane into three parts: the interior of the angle, the exterior of the angle, and the points on the side of the angle
- The measure of an angle is the smallest amount of rotation about the vertex from one ray to the other.
- Degree is the unit of measure of an angle and the tool used to measure a given angle is a protractor.
- Kinds of Angles
  - > A right angle is an angle that measures 90°.
  - > An **acute angle** is an angle whose measure is greater than  $0^{\circ}$  but less than  $90^{\circ}$
  - $\blacktriangleright$  An **obtuse angle** is an angle whose measure is greater than 90° but less than 180°.

# CHECKING YOUR UNDERSTANDING

A. Fill in the blanks using *always*, *sometimes*, or *never*.

- 1) A point \_\_\_\_\_\_ occupies an area.
- 2) Two points are \_\_\_\_\_ collinear.
- 3) Three lines that intersect at exactly one point are \_\_\_\_\_\_ contained in one plane.
- 4) Given plane M and plane N, then their intersection is \_\_\_\_\_\_ a line.
- 5) If line p and line a intersect, then their intersection is \_\_\_\_\_\_ a point.
- 6) Two distinct lines \_\_\_\_\_\_ intersect in more than one point.
- 7) Three points are \_\_\_\_\_ collinear.
- 8) A line and a point on the line are \_\_\_\_\_\_ contained in one plane only.
- 9) Three points \_\_\_\_\_\_ determine a plane.
- 10) Two lines that intersect are \_\_\_\_\_ contained in two planes.

B. Tell whether the following statement is always true (AT), sometimes true (ST), or never true (NT).

\_\_\_\_\_1) The measure between two points A and B with coordinates -2 and -6 respectively can be negative.

\_\_\_\_\_2) Given two rays with the same endpoint, then they are opposite rays.

\_\_\_\_\_3) Given two opposite rays, then they have the same endpoint.

\_\_\_\_\_4) A line has two endpoints.

- \_\_\_\_\_5)  $\overrightarrow{AB}$  is the same as  $\overrightarrow{BA}$ .
- \_\_\_\_\_6) If A is between C and T, then  $\overrightarrow{AC}$  and  $\overrightarrow{AT}$  are opposite rays.

\_\_\_\_7)  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$  are opposite rays, then B is between A and C.

8 D, O and T are collinear points. O is between D and T and DT = 2DO, therefore DO = OT

- \_\_\_\_\_9) A ray consists of one point.
  - 10)  $\overrightarrow{XY}$  and  $\overrightarrow{XZ}$  are opposite rays, then X, Y and Z are collinear points.

## **POST-TEST**

**Directions:** Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

