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Republic of the Philippines
Department of Education
NATIONAL CAPITAL REGION
Misamis Street, Bago-Bantay, Quezon City

UNIFIED SUPPLEMENTARY LEARNING MATERIALS (USLeM)

MATHEMATICS

Week 2

EXPECTATIONS:

You will derive relationships of geometric figures using measurements and by inductive reasoning, supplementary angles, complementary angles, congruent angles, vertical angles, adjacent angles, linear pairs, and perpendicular lines.

Specifically, this module will help you to:

- **Define the following terms and use their corresponding concepts to answer some geometric problems:**
  - **Supplementary angle**
  - **Complementary angles**
  - **Congruent Angles**
  - **Vertical Angles**
  - **Adjacent Angles**
  - **Perpendicular Lines**
  - **Linear Pair**
- **Investigate the relationship of vertical angles.**
- **Investigate the relationship of two angles that form a linear pair.**
- **Construct perpendiculars using paper folding.**

Let us start your journey in learning more about angles and lines. I am sure you are ready and excited to answer the Pretest. Smile and cheer up!

PRE-TEST

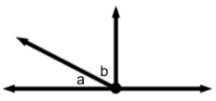
Directions: Read the questions carefully. Encircle the letter of the correct answer.

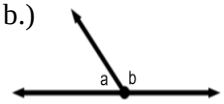
1.) Two angles in the same plane which have a common vertex and a common side but do not have interior points in common are called ___.
   a.) Supplementary angles c.) Adjacent angles
   b.) Congruent angles d.) Vertical angles

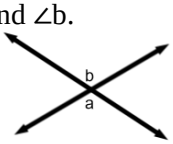
2.) If two angles are both congruent and supplementary, then each is a right angle?
   a.) always true b.) sometimes true c.) never true d.) cannot be determined

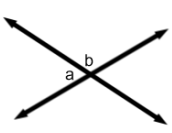
3.) Which of the following conditions would satisfy that the two angles form a linear pair?
   Condition 1: They are adjacent angles
   Condition 2: They are supplementary
   Condition 3: Their uncommon sides are opposite rays
   a.) 1 only b.) 1 and 3 c.) 1, 2 and 3 d.) 1 and 2

4.) Which of the following statements is **not** always true about supplementary angles?
   a.) The sum of supplementary angles is 180.
   b.) If two angles form a linear pair, then they are supplementary.
   c.) Supplementary angles are always adjacent.
   d.) The supplement of an acute angle is an obtuse angle.

5.) Which of the following figures show vertical angles ∠a and ∠b.
   a.) 

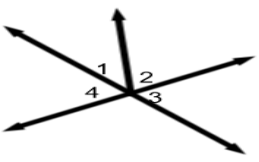
   b.) 

   c.) 

   d.) 

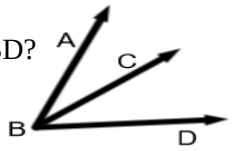
6.) ∠A is the supplement of ∠B. If m ∠A = 80, what is the measure of ∠B?
   a.) 10° b.) 40° c.) 80° d.) 100°

7.) Given the figure on the right, if m∠3 = 70, and m∠1 = 30, what is m∠4?
   a.) 60 b.) 70 c.) 80 d.) 90



8.) The measure of two complementary angles is in the ratio of 2:7. What is the measure of the larger angle?
   a.) 10^0 b.) 20° c.) 70° d.) 80°

9.) Given the figure on the right, if m∠ABC = ½ m∠ABD and m∠ABC = 35, what is m∠CBD?



a.) 17.5 b.) 35 c.) 70 d.) 105

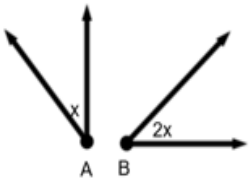
10.) ∠A and ∠B form a linear pair. If m∠A = x and m∠B = ½ x, what is the measure of each angle?

a.) m∠A = 60 ; m∠B = 30 c.) m∠A = 120 ; m∠B = 60
b.) m∠A = 180 ; m∠B = 90 d.) cannot be determined

11.) What is the complement of (x – 8)°?

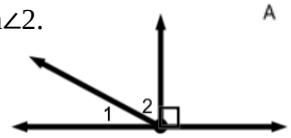
a.) (98 – x)° b.) 98° c.) x° d.) (98 + x)°

12.) Given the figure on the right, if ∠A and ∠B are complementary, find m∠A and m∠B?



a.) m∠A = 60 and m∠B = 30 c.) m∠A = 60 and m∠B =120
b.) m∠A = 30 and m∠B = 60 d.) m∠A = 40 and m∠B = 50

13.) Given the figure on the right, if m∠1 = 3x – 5 and ∠2 = 2x + 20, find m∠2.



a.) 75 b.) 50 c.) 40 d.) 15

14.) Which of the following statements is **NOT** always true?

a.) Vertical angles are congruent.
b.) If two angles form a linear pair, then they are supplementary.
c.) Two lines are perpendicular if and only if they intersect to form right angles.
d.) Adjacent angles are always congruent.

15.) ∠x and ∠y are adjacent angles and the sum of their measures is 113. If the measure of ∠x is 7 less than the measure of ∠y, what is the measure of ∠y?

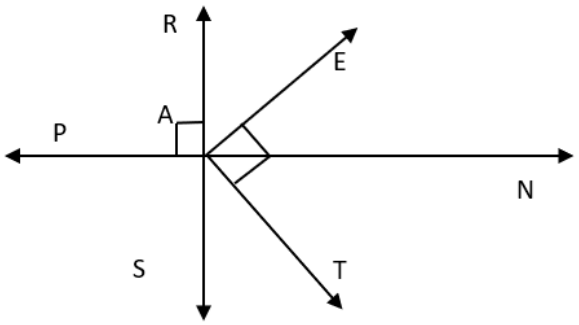
a.) 65 b.) 45 c.) 55 d.) 60

Great, you finished answering the questions. You may request your facilitator to check your work. Congratulations and keep on learning!

LOOKING BACK TO YOUR LESSON

Using the figure, tell whether the following angles are examples of right, acute, or obtuse.

1) ∠NAE
2) ∠PAR
3) ∠PAE
4) ∠RAT
5) ∠EAR
6) ∠SAP
7) ∠EAT
8) ∠SAT
9) ∠EAS
10) ∠TAN



BRIEF INTRODUCTION

Backpackers

Backpacking is a form of low-cost, independent travel, often staying in inexpensive lodging and carrying all necessary possessions in a backpack. Someone who backpacks is called a backpacker. They generally travel for a longer period of time than most other tourists, and they tend to travel in several different countries. Backpacking is perceived not only as a form of tourism, but a means of education.

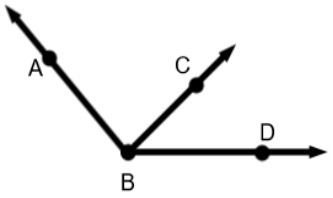
Can you help a backpacker to tour around Manila? How can you use some of the ideas of angle pairs you learned in school to show a backpacker his/her way around Manila?

The relationship that exists between any two angles can be made as to the basis for classifying angles. There are different kinds of angles classified in this manner, and they are called **angle pairs.**

Definition of Adjacent Angles:

Adjacent angles are two angles in the same plane which have a common vertex and a common side but do not have interior points in common.

In the adjacent figure on the right, ∠ABC and ∠CBD are adjacent angles. The common vertex is B and the common side is $\overrightarrow{BC}$. The two angles do not intersect and have no interior points in common.



Example: Which of the figure on the left form an adjacent angle?

Answer: In figure 1, ∠BAC and ∠CAD are adjacent angles but not ∠BAC and ∠BAD. Why? In figure 2. ∠DEK and ∠KHJ are not adjacent angles. Why?

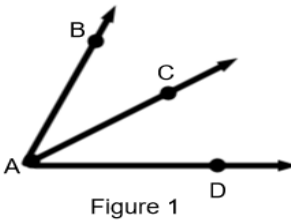


Figure 1

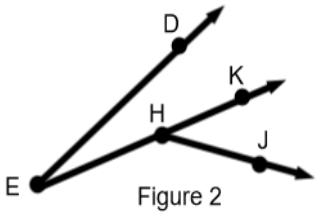


Figure 2

Definition of Congruent Angles

Two angles are **congruent** if and only if their measures are equal.

In symbols: ∠A ≅ ∠B if and only if m∠A = m∠B

A definition is reversible; hence the definition can be written in symbols as follows: m∠A = m∠B if and only if ∠A ≅ ∠B

Definition of Complementary Angles:

If the sum of the measures of two angles is 90, then the angles are called ***complementary,*** and each is called a ***complement*** of the other.

Example:

Which of the figures shown on the right are complementary if,

1.) m ∠1 = 30 and m∠2 = 60

2.) m∠*A* = 40 and m∠B = 50

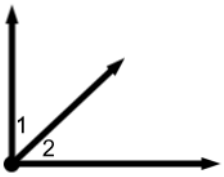


Figure 1

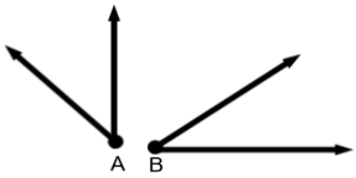


Figure 2

Answer:

1.) m∠1 and m∠2 are complementary because m∠1 + m∠2 = 90

2.) ∠A is a complement of ∠B because m∠A + m∠B = 90. Complementary angles need not be adjacent.

Definition of Supplementary Angles:

If the sum of the measures of two angles is 180, then the angles are called ***supplementary,*** and each is called a ***supplement*** of the other.

Example:

Which of the two angle pairs shown above are supplementary if,

1.) m ∠1 = 130 and m∠2 = 50

2.) m∠3 = 135 and m∠4 = 45

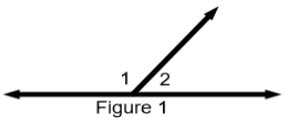


Figure 1

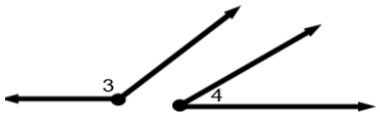


Figure 2

Answer:

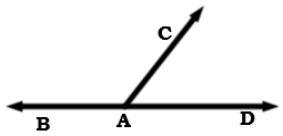
1.) m∠1 and m∠2 are supplementary because m∠1 + m∠2 = 180

2.) ∠3 is a supplement of ∠4 because m∠3 + m∠4 = 180. Supplementary angles need not be adjacent.

Note: The sum of the measures of supplementary angles is 180^0. If one of the angles is acute, then the other is always obtuse. In addition, if one of the angles is right, then the other angle is also right.

Definition of Linear Pairs:

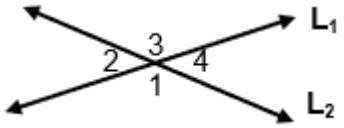
Two angles form a ***linear pair*** if and only if, (1) they are adjacent angles, (2) their uncommon sides are opposite rays.



∠BAC and ∠CAD are adjacent angles and their common side is $\overrightarrow{AC}$. Their uncommon sides $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are opposite rays. Therefore ∠BAC and ∠CAD form a linear pair.

Definition of Vertical Angles:

Two angles are ***vertical angles*** if they are nonadjacent angles, formed by two intersecting lines.



Example:

∠1 and ∠3 are vertical angles because they are nonadjacent angles formed by two intersecting lines.
∠2 and ∠4 are also vertical angles.

ACTIVITY 1: Exploring Angle Pairs Formed by Intersecting Lines

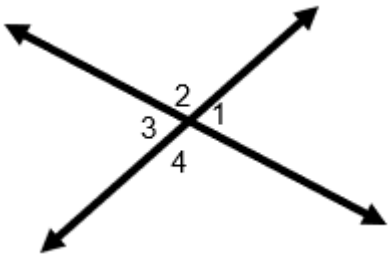
Objective: Find the relationships between the measures of the angles in each pair

Materials Needed: Pencil, Straight edge or ruler, and protractor

Procedure:

A. Using a straight edge, draw at least 5 different pairs of intersecting lines.
B. Label the angles formed as 1, 2, 3, and 4 on counterclockwise movement, so that every set of intersecting lines has the same numbering system.
C. Use a protractor to find each measure.

Angle	Measure of Angle (Possible Answer may vary)
m∠1	*60*
m∠2	*120*
m∠3	*60*
m∠4	*120*
m∠1 + m∠2	*180*
m∠2 + m∠3	*180*
m∠3 + m∠4	*180*
m∠1 + m∠4	*180*



D. Make a conclusion based on your observation.

Guide Questions:

Note: Italic words inside *[square bracket]* are the possible answers.

1.) Name a pair of vertical angles and a linear pair of angles in your diagram in step A.
[*Vertical Angles: ∠1 & ∠3 or ∠2 & ∠4 ;*
*Linear Pair: ∠1 & ∠2, ∠2 & ∠3, ∠4 & ∠3, and ∠1 & ∠4****]***

2.) What do you notice about the measures of a pair of vertical angles?
. ***[****Vertical angles have the same measure and so are congruent.****]***

3.) What do you notice about the sum of the measures of angles that form a linear pair?
[*The angles in a linear pair add up to 180, so they are supplementary.****]***

4.) Compare all your answer. Make a conclusion based on your observation about a pair of vertical angles and a pair of angles that form a linear pair.
[*Vertical angles are congruent. If two angles form a linear pair, then they are supplementary****]***

To **wrap up** activity 1, answer the following questions:

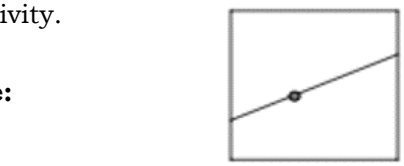
1.) How would you describe a pair of supplementary angles in the drawing of intersecting lines? ***[****They are adjacent angles.****]***

2.) Which pair of angles, if any, are congruent in a drawing of two intersecting lines?
[*Opposite angles (vertical angles) are congruent****]***

Summary of Conclusion for the activity

1.) Vertical angles are congruent.
2.) If two angles form a linear pair, then they are supplementary.

ACTIVITY 2: How to Construct Perpendicular Lines Using Paper Folding
Materials needed: pencil, ruler, and onion skin paper or tracing paper or any paper that are thin and translucent
Procedure: For each activity, construct at least 3 of each activity.



Activity 2.1:
To Construct a Perpendicular to a given point on the line:
1.) Fold a line segment on your paper from one side of the paper to the other. Use a ruler and pencil to draw in the line segment. Put a point on your line segment.
2.) Fold the line segment on top of itself so that the fold contains the given point. Unfold the paper and draw a line through the crease with a ruler and pencil.

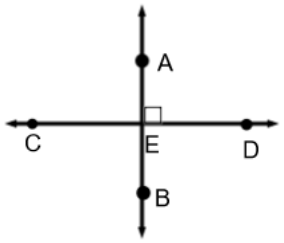
Activity 2.2:
To construct the Perpendicular bisector of a given segment:
1.) Use a ruler to draw a segment on your paper and put the endpoints on the end as shown.
2.) Fold the paper so that one endpoint lies on top of the other point. Unfold the paper and draw in the line segment on the crease with a ruler and pencil.

Guide Question:
1.) What do you notice about the angle formed by the perpendicular lines? Use your protractor to measure the angles formed by perpendicular lines. ***[****The four angles formed measures 90, so the lines formed four right angles.****]***

2.) Compare all your answers. Are they all the same? ***[****yes****]***

Perpendiculars

When two lines intersect, four angles are formed. If each of these angles is a right angle, then the intersecting lines are **perpendicular.**
In the figure, ∠AED, ∠AEC, ∠CEB, and ∠BED are all right angles. Line $\overleftrightarrow{AB}$ is perpendicular to line $\overleftrightarrow{CD}$. " ⊥ " is the symbol for perpendicular. In symbol, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ (read as line AB is perpendicular to line CD). In the figure, the following are also perpendicular: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, $\overleftrightarrow{EA} \perp \overleftrightarrow{CD}$, $\overleftrightarrow{EA} \perp \overleftrightarrow{EC}$, $\overleftrightarrow{EA} \perp \overleftrightarrow{ED}$, and so on.

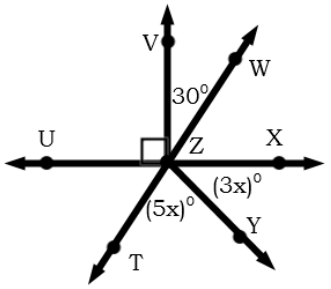


Definition of Perpendicular Lines:

Two lines are **perpendicular** if and only if they intersect to form right angles.

Segments or rays are perpendicular if and only if they are contained in perpendicular lines.

Example: Use the given figure to answer the following questions. $\overleftrightarrow{UX}$ and $\overleftrightarrow{TW}$ intersect at point Z. Find the value of x and the measures of ∠WZX, ∠XZY, ∠YZT, and ∠UZT. Explain your answer.



Answer:
∠UZV and ∠XZV are a linear pair formed by an intersecting line and ray $\overleftrightarrow{UX}$ and $\overrightarrow{ZV}$, so they are supplementary, and the sum of their measure is 180. Since ∠UZV is a right angle, m∠UZV = 90, so m∠VZX = 180 – 90 = 90. m∠VZW + m∠WZX = m∠VZX = 90, so they are complementary, therefore, ***m∠WZX*** *= 90 – 30 =* ***60****.*

∠WZX and ∠XZT are supplementary because they form a linear pair, so m∠XZT = 180 – 60 = 120.

m∠XZY + m∠YZT = m∠XZT, by substitution, 3x + 5x = 120, 8x = 120, and ***x = 15.***

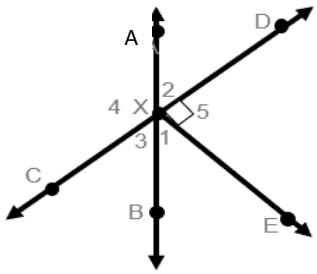
*Given that ∠****XZY*** *= (3x)° = (3)(15) =* ***45****° and ∠****YZT*** *= (5x)° = 5(15) =* ***75****°.*

*∠UZT and ∠WZX are congruent because they are vertical angles, therefore, ∠****UZT = 60****° also.*

ACTIVITIES

A. Answer the following:

Given the adjacent figure in which $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at X and ∠5 = 90°.



1.) Are ∠ 1 and ∠ 2 adjacent angles?
2.) Name an angle supplement to $\angle DXE$.
3.) Name a right angle other than ∠5 .
4.) Are ∠1 and ∠2 complementary angles?
5.) Are ∠1 and ∠4 vertical angles?
6.) Name an angle vertical to ∠2.
7.) If ∠1 = 50°, what is the measure of ∠2?
8.) Name an angle complement to ∠1.
9.) Can you say that ∠1, ∠5, and ∠2 are supplementary?
10.) What is the measure of ∠3?

B. Angle Pair Relationship Activity:

Given the figure set below with angle pairs ∠1 and ∠2, categorize each of the angle pairs according to their angle pair relationship. Sort the given angles by writing the letter of the angle pairs in their correct categories in the table below. Angle pair may belong to more than 1 category.

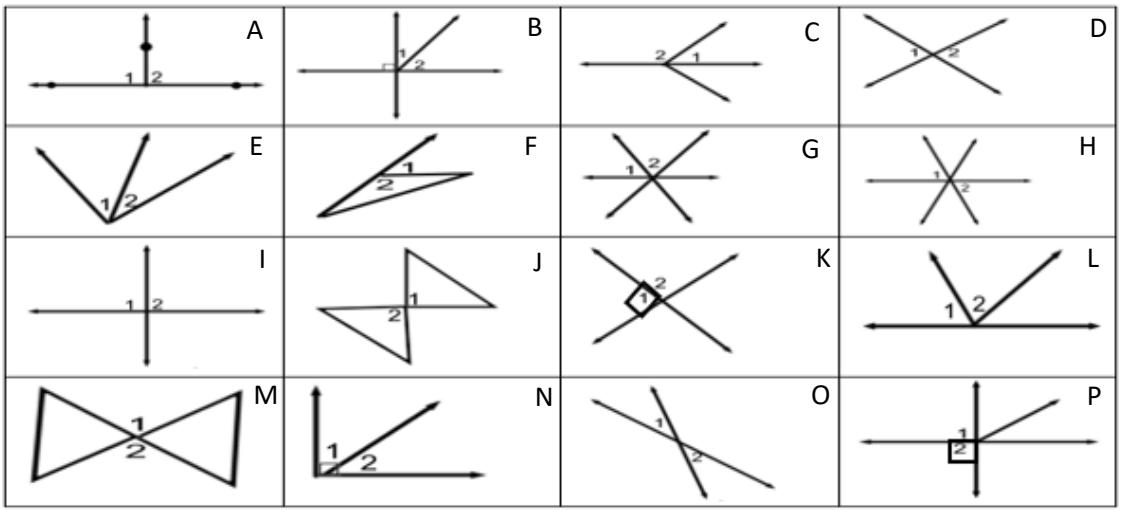


Figure Set:
Table of Categories:

Vertical	Complementary	Supplementary	Adjacent	Congruent

You may explore more to check your understanding!

REMEMBER

Definition of Terms:

Adjacent angles are two angles in the same plane which have a common vertex and a common side but do not have interior points in common.

If the sum of the measures of two angles is 90, then the angles are called ***complementary,*** and each is called a ***complement*** of the other.

If the sum of the measures of two angles is 180, then the angles are called ***supplementary,*** and each is called a ***supplement*** of the other.

Two angles form a ***linear pair*** if and only if, (1) they are adjacent angles, (2) their uncommon sides are opposite rays.

Two angles are ***vertical angles*** if they are nonadjacent angles formed by two intersecting lines.

Two lines are ***perpendicular*** if and only if they intersect to form right angles. Segments or rays are perpendicular if and only if they are contained in perpendicular lines.

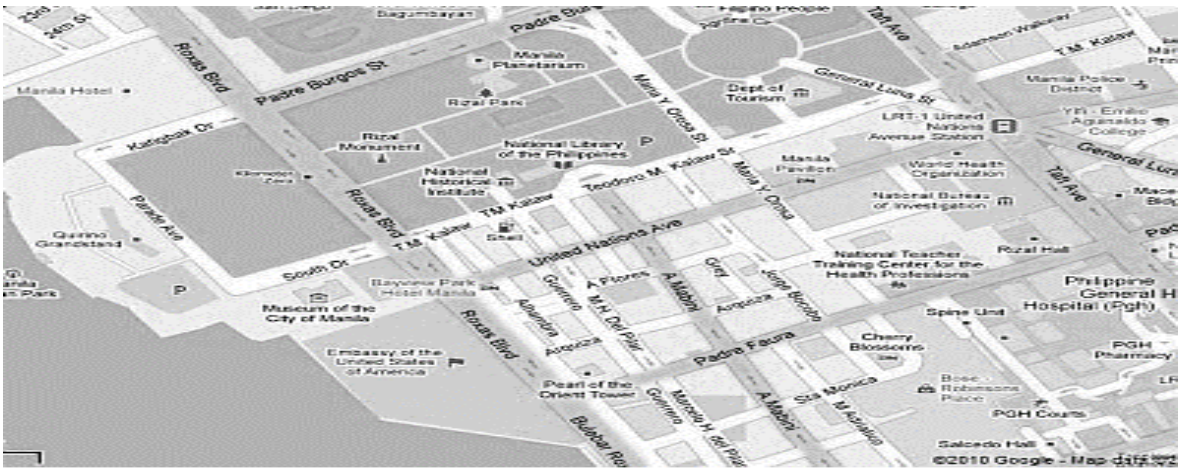
Angle Pair Relationship:

1.) The sum of complementary angles is 90.
2.) The sum of supplementary angles is 180.
3.) Vertical angles are congruent.
4.) If two angles form a linear pair, then they are supplementary.
5.) Perpendicular lines formed four right angles.

CHECKING YOUR UNDERSTANDING

Application in Real Life

The photo shown below is the map of Manila.



Use the map to show the streets that show an angle pair. What angle is shown by the intersection of Padre Faura and Mabini St.? What angle pair is formed by Taft Avenue, General Luna, and United Nations Avenue? If the backpacker is in a hostel in the corner of Jorge Bocobo St. and Padre Faura St., using the map shows him the way going to U.S. Embassy. What angle pair is formed? What street intersections will he pass by if you will tour him around Luneta?

POST-TEST

Directions: Read the questions carefully. Encircle the letter of the correct answer.

1.) Which of the following figures show perpendicular lines?

a.) 

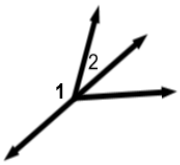
b.) 

c.) 

d.) 

2.) Which of the following conditions would always be true if the two angles are vertical?
Condition 1: The two angles are congruent.
Condition 2: The sum of the two angles is 180.
Condition 3: The two angles are formed by two intersecting lines but not adjacent.

a.) Condition 1 only
b.) Condition 1 and 2
c.) Condition 2 only
d.) Condition 1 and 3



3.) From the given figure, what is the relationship of ∠1 and ∠2?

a.) complementary
b.) congruent
c.) supplementary
d.) vertical

4.) If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, then they form ___

a.) Four right angles
b.) Four obtuse angles
c.) two acute angles and two obtuse angles
d.) Two right angles and two obtuse angles

5.) Which statement is NOT true?

a.) Vertical angles are congruent.
b.) If two angles form a linear pair, then they are supplementary.
c.) If two rays are perpendicular , then they form a right angle.
d.) If two angles are complementary, then their sum is 180.

6.) What do you call the two angles that are nonadjacent and formed by two intersecting lines?

a.) Supplementary angles
b.) Congruent angles
c.) Adjacent angles
d.) Vertical angles

7.) Which statement/s would always be true about perpendicular lines?
Statement 1: Two lines are perpendicular if and only if they intersect to form right angles.
Statement 2: Segments or rays are perpendicular if and only if they are contained in perpendicular lines.
Statement 3: Two lines that are perpendicular form four right angles.

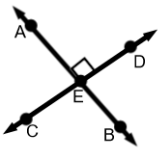
a.) Statement 1 only
b.) Statement 1 and 2
c.) Statements 1, 2, and 3
d.) Statement 1 and 3

8.) How true is the statement: "If two angles are complementary, then they are adjacent." ?

a.) always true
b.) sometimes true
c.) never true
d.) cannot be determined

9.) Which is true about the given figure on the right?

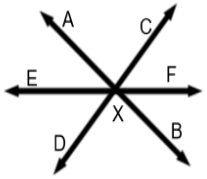
a.) ∠AED and ∠BED are vertical angles
b.) Two right angles are formed by $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.
c.) $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at E
d.) $\overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$



10.) What is the supplement of (2x – 50)°?

a.) (230 + 2x) °
b.) (130 – 2x) °
c.) (230 – 2x) °
d.) (130 + 2x) °

For #**11 – 15**, refer to the given figure. $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at X.



11.) $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at X. If ∠CXF = (2y – 3)° and ∠EXD = 45, what is the value of y?

a.) 21
b.) 24
c.) 45
d.) 90

12.) $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at X. If ∠AXE = 35°, what is the measure of ∠EXB?

a.) 35
b.) 55
c.) 115
d.) 145

13.) $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at X. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, what is m∠AXC?

a.) 90
b.) 80
c.) 45
d.) 30

14.) $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at X. If ∠AXC is a right angle, m∠CXF = y – 12 and m∠AXE = y, what is m∠CXF?

a.) 39
b.) 51
c.) 78
d.) 102

15.) $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ intersect at X. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and m∠DXE = 36, what is m∠AXE?

a.) 36
b.) 45
c.) 54
d.) 90

E-SITES

To further explore the concept learned today, you may visit the following links to further enhance your knowledge.
https://www.khanacademy.org/math/geometry-home/geometry-angles/geometry-vert-comp-supp/v/vertical-adjacent-and-linearly-paired-angles
https://www.youtube.com/watch?v=tPjJ4DI2yO8
https://www.youtube.com/watch?v=QjwbvNdUSTk
https://www.onlinemathlearning.com/pairs-of-angles.html

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Acknowledgements

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