Republic of the Philippines Department of Education NATIONAL CAPITAL REGION Misamis Street, Bago-Bantay, Quezon City

UNIFIED SUPPLEMENTARY LEARNING MATERIALS (USLeM)



MATHEMATICS Week 3

EXPECTATIONS:

You will derive relationships among angles formed by parallel lines cut by a transversal using measurement and by inductive reasoning.

Specifically, this learning material will help you to:

- Define the following:
 - ➢ Parallel lines
 - Skew Lines \triangleright
 - Intersecting Lines \triangleright
 - \triangleright Transversal
- Define and identify the angles formed by parallel lines cut by a transversal.
 - Alternate Interior Angle
 - Alternate Exterior Angle
 - Corresponding Angle
 - ➢ Interior Angles on the same side of Transversal
 - Exterior Angle on the same side of Transversal
- Investigate the relationship of angles formed by parallel lines cut by a transversal.

Let us start your journey in learning more on Parallel lines. I am sure you are ready and excited to answer the Pretest. Smile and cheer up!

PRE-TEST

Directions: Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E". 1.) What do you call the two lines that are not coplanar and do not intersect? a.) Transversal b.) Skew Lines c.) Intersecting Lines d.) Parallel Lines

- 2.) What do you call a line that intersects two coplanar lines at two different points? a.) Transversal b.) Skew Lines c.) Intersecting Lines d.) Parallel Lines
- 3.) Which does not represent parallel lines in real life? a.) Railroad tracks b.) Guitar Strings c.) Capital T
- d.) Stairs
- 4.) What do you call the two nonadjacent exterior angles on opposite sides formed by two lines cut by a transversal?
 - a.) Alternate Interior Angles
- c.) Interior angles on the same side of transversal
- b.) Corresponding Angles
- d.) Alternate Exterior Angles
- 5.) Given the figure, identify the colored angle pair.
- - a.) Alternate Interior Angles b.) Corresponding Angles

 - c.) Interior Angles on the same side of Transversal
 - d.) Alternate Exterior Angles
- 6.) Given the figure, identify the colored angle pair.
 - a.) Alternate Interior Angles
 - b.) Corresponding Angles
 - c.) Exterior Angles on the same side of Transversal
 - d.) Alternate Exterior Angles
- 7.) Given the figure, identify the colored angle pair.
 - a.) Alternate Interior Angles
 - b.) Corresponding Angles
 - c.) Interior Angles on the same side of Transversal
 - d.) Alternate Exterior Angles
- 8.) Given the figure on the right, if $L_1 \parallel L_2$ and $m \ge 1 = 80$, find $m \ge 2$. a.) 20

b.) 40 c.) 80 d.) 100











LOOKING BACK TO YOUR LESSON

Directions: Fill in the blanks to identify the following definition of terms. Choose the answers from the box on the right.

- angles are nonadjacent angles formed by two intersecting lines. 1.)
- 2.) Two lines are ______ if they lie in the same plane and do not intersect.
- 3.) Two angles are ______ if they have the same measure.
- angles are two coplanar angles that have a common side and a 4.) common vertex but no common interior points.
- if they are adjacent angles and 5.) Two angles form a their uncommon sides are opposite rays.
- 6.) Two angles are ______ if the sum of their measures is 180.7.) Two lines are ______ if they intersect and form right angles.
- 8.) Two angles are _____ if the sum of their measures is 90.
- 9.) Two lines are ______ if they are coplanar and they meet at a common point.
- 10.) If two rays have the same endpoint but do not lie on the same line, then their union is an

BRIEF INTRODUCTION

Trolley Taxis

Every morning, thousands of passengers wait in a train station in Manila. The passengers do this not to ride a train but to ride an alternative mode of transport in the city called trolleys. Aside from allowing commuters to pay cheaper fares, trolleys are used to avoid traffic jams on the road as well.



Trolleys are hand-made wooden carts being pushed by trolley boys along the railroad tracks. They push the trolley forward like a skateboard. Such a mode of transport is an illegal way of transporting commuters but is still being done anyway. Trolley boys know the schedule of trains to avoid fatal accidents that will surely kill them and their passengers.

Trolley taxis borrowed the railroad tracks to ply their carts. The carts must fit the tracks to make it work. If you were to make a trolley, what geometrical concept will you use to make it? How can you make it fit the railroad tracks? How can you make sure that it will not fall off the tracks?

Angle Adjacent Complementary Congruent Intersecting Linear pair Parallel Perpendicular Supplementary Vertical

These are the three ways in which lines can be Situated in Space.



Definition of Terms:

Intersecting Lines

- > are two coplanar lines that have a common point which is called the point of intersection.
- **Parallel lines** (symbol: ||)
 - > are coplanar lines that do not intersect.
- **Skew lines**
 - > are non-coplanar lines that do not intersect and are not parallel.

Transversal Line

is a line that intersects two or more coplanar lines at different points. \triangleright

Based on the definition which among the figures has a transversal line?



Answer: Figure 1, 2, and 4 has a transversal line.

Figure 3 has no transversal line because the third line did not intersect the two lines at two different points. Transversal Line



The nonparallel line that intersects the pair of parallel lines is called the transversal line. As you can see in Figure 5.1, angles are formed when the transversal line cuts the parallel lines. They can be grouped and be given a special name.



Angles Formed by Parallel Lines Cut by a Transversal:

- The angles lying between parallel lines L_1 and L_2 are called **INTERIOR ANGLES.** These * angles are 3,4,5, and 6.
- The remaining angles 1,2,7 and 8 are called EXTERIOR ANGLES because they lie on the * outside region of the parallel lines and the transversal line.

With these groups of angles, pairs of congruent angles are determined. Some of them are alternate interior angles and alternate exterior angles.

1. Alternate Interior Angles

are two nonadjacent interior angles \geq on opposite sides of the transversal.





2. Alternate Exterior Angles

➤ are two nonadjacent exterior angles on opposite sides of the transversal.



Can you name the remaining alternate interior angles and alternate exterior angles in Figure 5.1?

3. Corresponding angles

➤ are two nonadjacent angles, <u>one interior</u>, and <u>one exterior</u> on the same side of the transversal.



4. Interior Angles on the Same Side of the transversal are two nonadjacent interior angles on the same side of a transversal.





 $\angle 4 \text{ and } \angle 8 \text{ ; } \angle 1 \text{ and } \angle 5$ are examples of Corresponding Angles.

Can you name the remaining Corresponding Angles in the Figure 5.1?

5. Exterior Angles on the Same Side of the transversal are <u>two nonadjacent **exterior**</u> angles on the **same side** of a transversal.



Can you name the remaining Interior Angles on the Same Side and Exterior Angles on the Same Side in Figure 5.1?

Let us Do this!!!

Materials needed: Lined paper, ruler, and protractor.

- Draw at least 5 sets of parallel lines (different sizes) using lined paper or the two edges of a ruler to make vertical parallel lines. Then draw a transversal that intersects the two parallel lines. Transversals may tilt in any direction.
- ➤ Use a protractor to measure each of the angles formed. Record the measures on your drawing.
- Draw another 5 sets of lines that are not parallel. Then draw a transversal that intersects the two non-parallel lines. Transversals may tilt in any direction.
- ▶ Use a protractor to measure each of the angles formed. Record the measures on your drawing.
- Make a conclusion about the measures of alternate interior angles, alternate exterior angles, corresponding angles, interior angles on the same side of the transversal, and exterior angles on the same side of the transversal.
- > Compare your conclusions on each set of parallel lines.
- > Also, compare your conclusion on each set of non parallel lines.

Guide Questions:

- 1. What can you say about the measures of any pairs of the following angles formed by parallel lines and a transversal:
- a.) alternate interior angles b.) alternate exterior angles c.) corresponding angles
- 2. What do you notice about the sum of the measures of any pairs of angles on (a) interior angles on the same side of the transversal and (b) exterior angles on the same side of transversal formed by parallel lines?
- 3. What can you say about the measures of any pair of the following angles formed by non-parallel lines and transversal:
 - a.) alternate interior angles b.) alternate exterior angles c.) corresponding angles

Possible Answer:

They are congruent for a, b, and c. [The sum of their measure is 180, so they are supplementary] The angles are not congruent for a, b, and c.

Summary of the Conclusion:

If the lines are parallel, then:

- > Any pairs of alternate interior angles are congruent.
- > Any pairs of alternate exterior angles are congruent.

- > Any pairs of corresponding angles are congruent.
- Any pairs of interior angles on the same side of the transversal are supplementary because the sum of their angle measure is 180.
- > Any pairs of exterior angles on the same side of the transversal are supplementary.
- The conclusion for parallel lines is not true for non parallel lines, therefore it is applicable only to parallel lines.

Example: Given the figure with lines $l_1 \parallel l_2$ and $m \ge 1 = 110$, find the measure of the other angles. Which angles are congruent, and which angles are supplementary?



- $m \angle 8 = 110$ because $\angle 8 \& \angle 1$ are alternate exterior angles
- $m \angle 5 = 110$ because $\angle 1 \& \angle 5$ are corresponding angles
- $m \angle 7 = 70$ because $\angle 1 \& \angle 7$ are exterior angles on the same side of the transversal therefore they are supplementary
- $m \angle 2 = 70$ because $\angle 2 \& \angle 7$ are alternate exterior angles
- $m \angle 6 = 70$ because $\angle 6 \& \angle 2$ are corresponding angles
- $m \angle 4 = 110$ because $\angle 4 \& \angle 5$ are alternate interior angles
- $m \angle 3 = 70$ because $\angle 3 \& \angle 6$ are alternate interior angles.

Congruent angles: $\angle 1$, $\angle 4$, $\angle 5$ & $\angle 8$ and $\angle 2$, $\angle 3$, $\angle 6$, & $\angle 7$

Supplementary Angles: $\angle 5 \& \angle 3; \angle 4 \& \angle 6; \angle 1 \& \angle 7; \angle 2 \& \angle 8$



Relationships between Pairs of Angles Formed by Parallel Lines Cut by a Transversal:

Remember: If two parallel lines are cut by a transversal, then:

- Corresponding angles are congruent.
- Alternate-Interior angles are congruent.
- > Alternate-Exterior angles are congruent.
- > Interior angles on the same side of the transversal are supplementary.
- > Exterior angles on the same side of the transversal are supplementary.

To show these relationships, let us consider the figure on the right and apply the relationships between pairs of angles formed by parallel lines cut by a transversal.

Note: symbol for congruent \cong

Pairs of	$\angle 1$ and $\angle 3$	$\angle 1 \cong \angle 3$
Corresponding Angles	∠2 and ∠4	$\angle 2 \cong \angle 4$
	∠5 and ∠7	$\angle 5 \cong \angle 7$
	∠6 and ∠8	$\angle 6 \cong \angle 8$
Pairs of Alternate	∠2 and ∠7	$\angle 2 \cong \angle 7$
Interior Angles	∠3 and ∠6	∠3 ≅∠6
Pairs of Alternate	∠1 and ∠8	$\angle 1 \cong \angle 8$
Exterior Angles	∠4 and ∠5	$\angle 4 \cong \angle 5$



Notes:

Examples:

Given the figure above (Figure 5.2) $\ell_1 \| \ell_2$ cut by a transversal. $m \ge 1 = 110^0$ Find the measures of the numbered angles.

Solution:

- 1.) $\angle 1$ and $\angle 8$ are alternate exterior angles so their measures are congruent.
- 2.) ∠1 *and* ∠3 are corresponding angles so their measures are congruent.
- 3.) ∠1 *and* ∠4 are exterior angles on the same side of the transversal so they are supplementary.
- 4.) $\angle 5$ and $\angle 4$ are alternate exterior angles so their measures are congruent.
- 5.) \angle **5** *and* \angle **7** *are corresponding angles* so their measures are congruent.



■ || symbol for parallel

 \blacktriangleright symbol for angle





Note: The relationships between pairs of angles formed by parallel lines cut by a transversal are for parallel lines only; it is not valid for non –parallel lines.

ACTIVITIES

Activity 1: Identify the kinds of line shown in each picture. Write Parallel, Intersecting, or Skew.



x

Activity 2: Use the figure below to determine the angles being asked.

Interior angles: Ex	terior angles:	1
1)	5)	1 .7
2)	6) Z	_ / /*
3)	7)	\sim / /
4)	8)	
Corresponding Angles		5/7
9) and	11) and	8/6
10) and	12) and	4/2
Interior Angles on the Same Side	Alternate Interior Angles	
Interior Angles on the Same Side	Alternate Interior Angles	
13) and	15) and	- /
14) and	16) and	4
Exterior Angles on the Same Side	Alternate Exterior Angles	
17) and	19) and	
18) and	20) and	

Activity 3: Use the figure at the right to find the measure of the angles below. Given: line m || line n cut by a transversal.



REMEMBER

Definition of Terms

- Intersecting Lines are two coplanar lines that have a common point which is called the point of intersection.
- > Parallel lines (symbol: ||) are coplanar lines that do not intersect.
- Skew lines are non-coplanar lines that do not intersect and are not parallel.
- > A **Transversal Line** is a line that intersects two or more coplanar lines at different points.

Angles Formed by Parallel Lines Cut by a Transversal.

- Alternate interior angles are two nonadjacent interior angles on opposite sides of the transversal.
- Alternate exterior angles are two nonadjacent exterior angles on opposite sides of a transversal.
- Corresponding angles are two nonadjacent angles, one interior, and one exterior on the same side of the transversal.
- Interior angles on the same side of the transversal are two nonadjacent interior angles on the same side of a transversal.
- Exterior angles on the same side of the transversal are two nonadjacent exterior angles on the same side of a transversal.

***** If two parallel lines are cut by a transversal, then:

- ✓ Corresponding angles are congruent.
- ✓ Alternate-Interior angles are congruent.
- ✓ Alternate-Exterior angles are congruent.
- \checkmark Interior angles on the same side of the transversal are supplementary.
- \checkmark Exterior angles on the same side of the transversal are supplementary.

CHECKING YOUR UNDERSTANDING

A. Directions: Consider the situation below and determine which establishments are on each corner of the city blocks based on Bimbo's given information. Assume North and South Street are parallel to each other. The dashed lines are the railroad track where the trolley taxi passes.

Bimbo rode a trolley along Sta. Mesa, Manila. While on this journey, he saw many establishments and wanted to remember them according to their street location.

- a. The **Bakery** and **Hospital** corners have a corresponding angle relationship.
- b. **Hospital** and **Barangay Hall** corners have an alternate interior angle relationship.
- c. The **Hair Salon** and **Gas Station** corners have an alternate exterior angle relationship.
- d. The **Fire Station** and **Hospital** corners have an interior angle on the same side of the transversal relationship.
- e. The **Milk Tea House** and **Hair Salon** corners have an exterior angle on the same side of the transversal relationship.
- f. The **Hair Salon** and **Coffee Shop** corners have a corresponding angle relationship.

B. Going back to the Brief Introduction discussed about the Trolley Taxi. If you were to make a trolley taxi, what geometrical concept will you use to make it? How can you make it fit the railroad tracks? How can you make sure that it will not fall off the tracks?





POST-TEST

Direction: Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

1) Which shows a corresponding angle?



2) If two parallel lines are cut by a transversal, which pair/s of angles could be supplementary?

- Alternate interior angles I.
- II. Interior angles on the same side of the transversal
- III. Corresponding angles b.) II only

c.) III only d.) I and III only

3) Two lines that are coplanar but do not intersect. a.) Transversal b.) Skew Lines

a.) I. II and III

c.) Intersecting Lines d.) Parallel Lines 4) Which condition/s would always be true if the two parallel lines are cut by a transversal?

Condition 1: Corresponding angles are congruent.

Condition 2: Alternate interior angles are congruent.

Condition 3: Interior angles on the same side of transversal are congruent. b.) Condition 1 and 2 c. Condition 1, 2 and 3 d) Condition 1 and 3 a.) Condition 1 only

5) Which statement is NOT always true?

a) If two lines are cut by a transversal, then the alternate interior angles are congruent.

b) A transversal is a line which intersects two coplanar lines at two different points.

c) Skew lines are lines that do not intersect and not coplanar.

d) Two lines that are coplanar but do not intersect are called parallel lines.

6.) If two different lines are cut by a transversal, what do you call the two nonadjacent interior angles on opposite side of the transversal?

a.) Corresponding Angles

c.) Alternate Interior Angles

d.) Interior Angles on the same side of transversal

b.) Alternate Exterior Angles 7) Which figure does not have a transversal?



_ and a pair of alternate exterior angles are 8.) If two lines are cut by a congruent, then the lines are parallel. What should be placed on the blank? a.) Transversal b.) Skew Lines c.) Intersecting Lines d.) Parallel Lines For numbers 9 - 15, refer to the given figure on the right. Given: lines x and y are cut by transversal t, and $x \parallel y$. 9) m $\angle 1 = 115$, what is m $\angle 6$? a) 35 b.) 65 c.) 75 d.) 115 10) $\angle 3 = (2y)^\circ$, and $\angle 6 = (y + 12)^\circ$, what is the value of y? a) 3 b.) 6 c.) 12 d.) 24 11) x || y. If $\angle 7 = 36^{\circ}$ and $\angle 4 = (5a + 4)^{\circ}$, what is the value of a? a) 28 b.) 36 c.) 40 d.) 144 12) Which among the choices have all the interior angles in the given figure? a) $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$ b) ∠3, ∠4, ∠5, ∠6 c.) $\angle 1$, $\angle 2$, $\angle 7$ d.) $\angle 3$, $\angle 4$, $\angle 5$, 13) If $m \angle 5 = 150$ What is the measure of $m \angle 4$? a) 30 b.) 40 d.) 150 c.) 50 _. What should be 14) ∠7 and ∠8 are Linear pairs therefore their measurement is equal to_ d.) 170⁰ placed on the blank? a) 90° b.) 100⁰ c.) 180° 15) Which pairs of angles below considered alternate interior angles? a) $\angle 3$ and $\angle 5$ b.) ∠4*a nd* ∠6 c.) $\angle 3$ and $\angle 6$ d.) ∠4 and ∠7

E-SITES

To further explore the concept learned today, you may visit the following links to enhance your knowledge.

https://learnzillion.com/lesson_plans/255-recognize-congruent-angles-formed-by-parallel-lines-cutby-a-transversal-by-identifying-and-justifying-which-angles-are-congruent/?card=5666 https://www.mathword.co/math-words-starting-with-letter-s/skew-lines/

https://www.khanacademy.org/math/basic-geo/basic-geo-angle/angles-between-lines/v/angles-formed-by-parallel-lines-and-transversals

https://www.mathplanet.com/education/geometry/perpendicular-and-parallel/angles-parallel-lines-and-transversals

https://www.mathsisfun.com/geometry/parallel-lines.html

REFERENCES

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