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Republic of the Philippines
Department of Education
NATIONAL CAPITAL REGION
Misamis Street, Bago-Bantay, Quezon City

UNIFIED SUPPLEMENTARY LEARNING MATERIALS
(USLeM)

MATHEMATICS

Week 6

EXPECTATIONS:

You will derive inductively the **relationship of exterior and interior angles of a convex polygon.**

Specifically, this learning material will help you to:

- determine how to find the sum of the interior angles of any convex polygons;
- explore the relationship of the number of sides of a regular polygon and the interior angles;
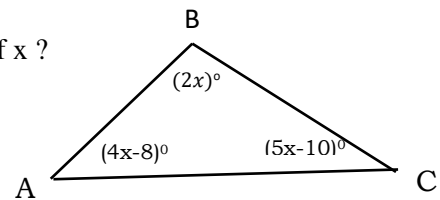- find the sum of the exterior angles of a convex polygon.

Let us start your journey in learning more about angle measures of polygons. I am sure you are ready and excited to answer the Pretest. Smile and cheer up!

PRE-TEST — — — — — — — — — — — — — — —

Directions: Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

1) For items 1 – 2, use the figure on the right. What is the value of x ?
       a. 64   b. 80
       c. 36   d. 18

2) Find the measure of angle A.
       a. 64^0 b. 80^0 c. 36^0 d. 18^0

3) What is the sum of exterior angles for a decagon?
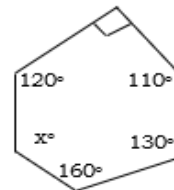   a. 180°   b. 360°   c. 1440°   d. 1800°

4) Which set of angle measurements could be from a convex polygon?
       a. 90, 90, 90,90   b. 60, 60, 20, 270, 20
       c. 180, 30, 30, 120   d. 90, 90, 30, 270, 30

5) What is the measure of x in the given hexagon on the right?
       a. 160^0 b. 120^0 c. 113^0 d. 110^0
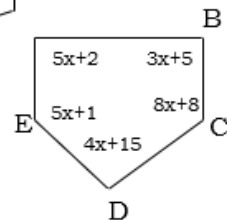
6) How many angles does a dodecagon have?
       a. 12   b. 10   c. 15   d. 20

7) Find the measure of angle E in the figure on the right.
       a. 20.36   b. 102.8   c. 101   d. 110

8) Three angles in a quadrilateral are 90, 145, and 80. What is the measure of the fourth angle?
       a. 25   b. 45   c. 115   d. 205

9) A triangle has the following known angles: 45, 45, and x. What is the value of x?
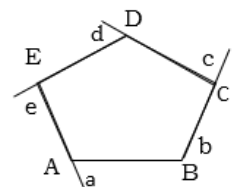   a. 45   b. 30   c. 60   d. 90

10) What is the formula used to find the sum of interior angles in a polygon?
       a. 180(n-2)/2   b. 180(n- 2)   c. 360/n   d. 360 –n/180

11) What is the interior angle of a regular polygon with an exterior angle of 65 degrees?
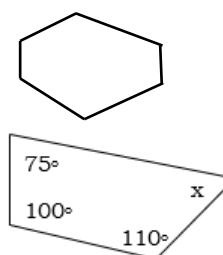   Hint: An interior angle and its exterior angle form a linear pair.
       a. 180   b. 115   c. 360   d. 195

12) If the polygon is regular, what is the measure of angle d?
     a. 72^0 b. 360^0 c. 540^0 d. 108^0

13) Find the angle sum of the interior angles of the polygon.
     a. 180^0 b. 540^0 c. 720^0 d. 1080^0

14) Find the missing angle.
     a. 75^0 b. 85^0 c. 100^0 d. 45^0

15) If each interior angle of a regular polygon is 135 degrees, what is the name of the polygon?
   a. pentagon   b. hexagon   c. heptagon   d. octagon

Great, you finished answering the questions. You may request your facilitator to check your work. Congratulations and keep on learning!

LOOKING BACK TO YOUR LESSON

2 – D SHAPE ROBOT BUNDLE

Directions: Complete the table below by identifying the name of the shapes in the robot's body also write the number of sides of each figure.

Number	Name of the Shapes	Number of Sides
1		
2		
3		
4		
5		
6		
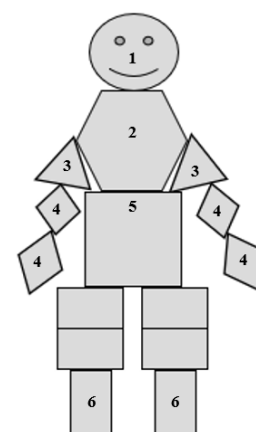
BRIEF INTRODUCTION

SCALE MODEL

A scale model is a representation or copy of an object that is larger or smaller than the actual size of the object being represented.

Very often the scale model is smaller than the original and used as a guide to make the object full size.

Scale models are built and collected for many reasons including, in engineering, for testing the likely performance of a design object at an early stage without the expense of building the full-sized prototype, in remote control vehicles, and on TV and in the movie industry, for constructing objects or sets that cannot be built in full size.

The photo above shows a scale model of a residential house made by Galvan Brothers, Inc.

Suppose you are to draw a floor plan of your own house, what design will you make for the different parts of your house?

In the previous chapter, you have learned that the sum of the angle measures of a triangle is 180^0. You can use this property to find the sum of the interior angles of a convex polygon.
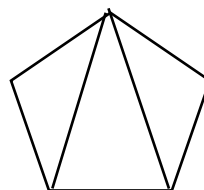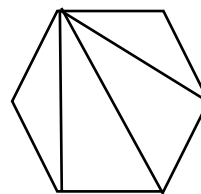
Let us look at the given figures below.
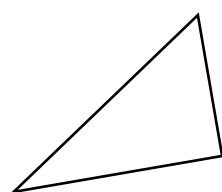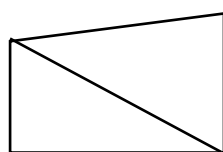
| Triangle | Quadrilateral | Pentagon | Hexagon |

In each polygon, diagonals are drawn from one vertex. Notice that this divides each polygon into triangular regions.

From the figures above, you can see that the sum of the angles of the quadrilateral is equal to the sum of the angles of two triangles (180^0). Similarly, the sum of the angles of a pentagon is 3 (180^0) and of a hexagon is 4 (180^0). Continuing this procedure, you can compile the table below.

The Sum of the Interior Angles of the different Polygons

Polygon	Number of Sides, n	Number of Triangles, n-2	Sum of Measures of Interior Angles
Triangle	3	1	$1(180^0) = 180^0$
Quadrilateral	4	2	$2(180^0) = 360^0$
Pentagon	5	3	$3(180^0) = 540^0$
Hexagon	6	4	$4(180^0) = 720^0$
Heptagon	7	5	$5(180^0) = 900^0$
Octagon	8	6	$6(180^0) = 1080^0$
Nonagon	9	7	$7(180^0) = 1260^0$
Decagon	10	8	$8(180^0) = 1440^0$

From the table, it appears that **the Sum of the measures of the Interior Angles of a convex n–gon is given by** $\boxed{\mathbf{S = 180^0(n-2)}}$

Examples:

a. Find the sum of the measures of the interior angles of a convex 13-sided polygon.

SOLUTION: n = 13
Formula: $\mathbf{Sum = 180^0(n-2)}$
$= 180(13-2)$
$= 180(11)$
$= \mathbf{1,980^0}$

b. The sum of the measures of the interior angles of a polygon is 1620. How many sides does the polygon have?

SOLUTION: S = 1620

Formula: $180^0(n-2) = 1620$
$180n - 360 = 1620$
$180n = 1620 + 360$
$\frac{180n}{180} = \frac{1980}{180}$
$\mathbf{n = 11}$

Interior Angles of a Regular Polygon

But how do we find the interior angles of a regular polygon?

Recalling the definition of a <u>regular polygon</u> which states that a polygon is regular when *all its sides and all of its angles are congruent*. In short, it must be equilateral and equiangular. If the sum of the angles of a regular polygon is divided by the number of its sides, the resulting quotient is the measure of each angle.

Why? Because all the angles of a regular polygon are congruent. This implies also that the measure of **each interior angle** of a regular polygon is $\dfrac{180^0(n-2)}{n}$

Examples:

a. Find the measure of each interior angle of a regular decagon.

SOLUTION: decagon = 10 sides (n)

Measure of each interior angle $= \dfrac{180\,(n-2)}{n}$

$= \dfrac{180\,(10-2)}{10}$

$= \dfrac{180\,(8)}{10} = \dfrac{1440}{10}$

$= \boxed{\mathbf{144^0}}$

b. Each angle of a polygon has a measure of 140^0. How many sides does the polygon have?

SOLUTION: $140 = \dfrac{180\,(n-2)}{n}$

$140 = \dfrac{180n - 360}{n}$

$140 = \dfrac{180n - 360}{n}$

$140n = 180n - 360$

$360 = 180n - 140n$

$\dfrac{360}{40} = \dfrac{40n}{40}$

$\boxed{9 = n}$

Exterior Angles of a Polygon

A property of the exterior angles of a polygon can be developed by the following method. Consider the figure below. At each vertex of pentagon ABCDE there is a linear pair of angles.
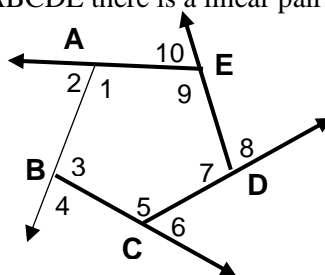
Use the Linear Pair relationship

$$m\angle 1 + m\angle 2 = 180°$$
$$m\angle 3 + m\angle 4 = 180°$$
$$m\angle 5 + m\angle 6 = 180°$$
$$m\angle 7 + m\angle 8 = 180°$$
$$m\angle 9 + m\angle 10 = 180°$$

Add the left members and the right members of the above equations to obtain.

(exterior sum) + (interior sum) = 5 (180)
(exterior $\angle$ sum) + (5 – 2) 180 = 5 (180)
            (exterior $\angle$ sum) = 5 (180) – 3 (180)
            (exterior $\angle$ sum) = 900 - 540
            (exterior $\angle$ sum) = 360°

If 5 sides are replaced by n sides, the Sum of the Exterior Angles of polygons will be 360°.
The measure of each **Exterior Angle** of a regular n- gon is $\boxed{\dfrac{360}{n}}$

Note: Only convex polygons are being discussed here.

The sum of the exterior angles of any polygon is **360 degrees.**

This is a result of the interior angles summing 180(n-2) degrees and each exterior angle being, by definition, supplementary to its interior angle.

Examples:

a. Find the measure of an exterior angle of an octagon.	**b.** The measure of an exterior angle of a regular polygon is 24°. How many sides does the polygon have?
SOLUTION: Let n = the number of sides **n= 8 (octagon)** $= \dfrac{360}{n}$ $= \dfrac{360}{8}$ $\boxed{= 45^0}$	SOLUTION: Let n = the number of sides $24 = \dfrac{360}{n}$ $\dfrac{24n}{24} = \dfrac{360}{24}$ $\boxed{\textbf{n = 15}}$

ACTIVITIES
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Activity 1: Find the sum of the interior angles of a polygon with the following number of sides.

_____ 1.) 8   _____ 4.) 15   _____ 7.) 18

_____ 2.) 11   _____ 5.) 19   _____ 8.) 14

_____ 3.) 12   _____ 6.) 21   _____ 9.) 6

Activity 2: Find the number of sides of a polygon given the sum of the measures of the interior angles.

_____ 1.) 540^0 _____ 3.) 1260^0 _____ 5.) 1440^0

_____ 2.) 720^0 _____ 4.) 1800^0 _____ 6.) 1080^0

Activity 3: Determine the measure of one interior angle given the number of sides of the following regular polygons.

_____ 1.) 3   _____ 3.) 18   _____ 5.) 9

_____ 2.) 6   _____ 4.) 12

Activity 4: Determine the measure of an exterior angle given the number of sides of the following regular polygon.

_____ 1.) 15   _____ 3.) 4   _____ 5.) 12

_____ 2.) 9   _____ 4.) 8

Activity 5: Determine the number of sides of a regular polygon given the measure of an exterior angle of the regular polygon.

_____ 1.) 15   _____ 3.) 4   _____ 5.) 12

_____ 2.) 9   _____ 4.) 8

REMEMBER
- - - - - - - - - - - - - - - -

- ❖ A polygon is regular if it is both equilateral and equiangular.
- ❖ The sum of the measures of the interior angles of a convex n – gon is 180^0 (n-2)
- ❖ The measure of each interior angle of a regular polygon is given by $\dfrac{180\,(n-2)}{n}$
- ❖ The sum of the exterior angles of any polygon is 360°.
- ❖ All exterior angles of a regular polygon are congruent.
- ❖ A polygon is regular if it is both equilateral and equiangular.
- ❖ The measure of each exterior angle of a regular n-gon is $\dfrac{360}{n}$.

A. Answer the following:

1. What is the sum of the measures of the interior angles of a convex 12 – sided polygon?
2. What is the measure of each interior angle of a regular 20 – sided polygon?
3. A convex pentagon has interior angles that measure 90^0, 100^0, 110^0, and 120^0.
   What is the measure of the fifth interior angle?
4. A convex hexagon has exterior angles that measure 32^0, 54^0, 67^0, 72^0, and 100^0.
   What is the measure of the sixth exterior angle?
5. What is the measure of each exterior angle of a regular nonagon?
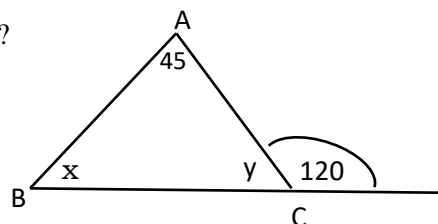
B. Find the **measure of ∠ A.**
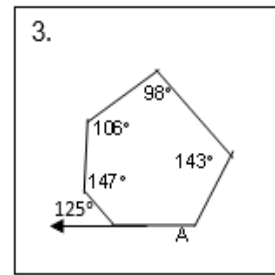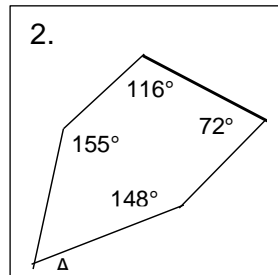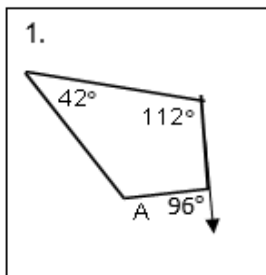
1. 42° 112° A 96°

2. 116° 72° 155° 148° A

3. 98° 106° 143° 147° 125° A

Directions: Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

1) Find the sum of the interior angles of a polygon with 12 sides.
   a. 1440   b. 1980   c. 1800   d. 3420

2) If the sum of the measures of four exterior angles of a pentagon is 270^0, what is the measure of the fifth exterior angle?
   a. 70^0 b. 90^0 c. 45^0 d. 30^0

3) How many sides does a polygon have if the sum of the interior angles is 900?
   a. 6 sides   b. 8 sides   c. 7 sides   d. 9 sides

4) What is the measure of one exterior angle of a regular polygon with 8 sides?
   a. 135^0 b. 45^0 c. 1440^0 d. 360^0

5) How many sides does a regular polygon have if the measure of one exterior angle is 72^0?
   a. 5   b. 6   c. 7   d. 9

6) Find the number of sides in a regular polygon in which the measure of one interior angle is 108^0.
   a. 5   b. 6   c. 7   d. 9

7) What is the formula used to find the sum of interior angles in a polygon?
   a. 180(n – 2)/2   b. 180(n – n)   c. 180(n -5)   d. 180(n -2)

8) What are the measurements of the missing angles?

   a. x = 45, y = 45
   b. x = 75, y = 60
   c. x = 32, y = 90
   d. x = 34, y = 55

   A 45 B x y 120 C

9) Six angles of a convex octagon are congruent. Each of the two remaining angles is 20 degrees \
   more than one of the other six angles. Find the measure of each angle.
      a.   1200   b. 1500   c. 1300, 1500   d.1200,1300

10) What is the formula to find an exterior angle of a polygon?
      a.   180(n-2)   b. 360/n   c. 360 –n/180   d. 360

11) Three of the exterior angles of a hexagon have a sum of 2400. The remaining exterior angles are
   congruent to each other. Determine the measure of the remaining angles.
      a.   30   b. 36   c. 40   d. 45

12) What is the sum of the measures of the angles of a right triangle?
         a. 90   b. 180   c. 60   d. 360

13) What is the measure of each angle of a regular octagon?
         a. 125   b. 135   c. 145   d. 155

14) What is the sum of the measures of the interior angles of a nonagon?
         a. 1260   b. 1620   c. 1980   d. 900

15) The measure of each angle of an equiangular triangle is _____.
         a. 180   b. 90   c. 70   d. 60

E-SITES

To further explore the concept learned today and if it possible to connect the internet, you may visit the following links:

https://www.youtube.com/watch?v=qG3HnRccrQU&feature=share
https://www.youtube.com/watch?v=BG1HpadfiKw&feature=share
https://www.google.com/search?q=skeletal+scale+model+house&tbm=isch&hl=en- GB&client=ms-android-samsung-gj-rev1&prmd=isvn&hl=en-GB&ved=2ahUKEwi1DKnbbpAhXHAKYKHQVkCzsQgowBegQIARAD&biw=360
https://www.sciencedaily.com/terms/scale_model.htm

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Clemens, et al,(1994). Geometry.Addison-Wesley Publishing Company, Inc.,USA

Jurgensen, Ray C., et al,(1965). Modern Geometry Structure and Method. Houghton Mifflin Company, USAReuters/Erik De Castro. (February 22, 2017) Wooden trolleys provide alternative means of transportation in Manila. GMA News Online February 22,2017.

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Malate Jose S., Amid et al (2014) Understanding Mathematics Grade 7. Vicarish Publication and Trading, Inc.