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Republic of the Philippines
Department of Education
NATIONAL CAPITAL REGION
Misamis Street, Bago-Bantay, Quezon City

UNIFIED SUPPLEMENTARY LEARNING MATERIALS (USLeM)

MATHEMATICS

Week 8

EXPECTATIONS:

1. Focus on constructing triangles, squares, rectangles, regular polygons, and regular hexagons;
2. Create figures using compass and straightedge.

Let us start your journey in learning more about Union and the Intersection of Events. I am sure you are ready and excited to answer the Pretest. Smile and cheer up!

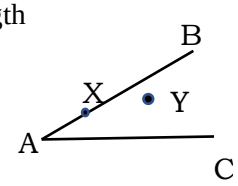
PRE-TEST

Directions: Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

1. Which can be used to measure angles in making a construction?
   a. protractor b. compass c. ruler d. straightedge

2. "Any angle can be bisected by means of a compass and straightedge." – How true is this statement?
   a. always b. sometimes c. never d. often

3. "The midpoint of a segment can be found by constructing the _____ of the segment." - Which word will complete this statement?
   a. angle bisector b. tangent c. perpendicular bisector d. length

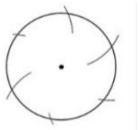


For items 4 – 6, refer to the given figure.

4. To construct a line perpendicular to $\overline{AB}$ at X, the point of the compass would be placed at which point?
   a. A b. B c. C d. X

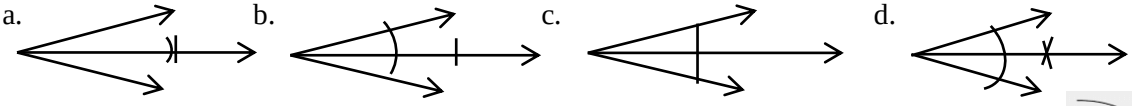
5. "To construct a perpendicular to $\overline{AC}$ from Y, first construct an arc with center at point _____ which intersects _____ in two points." Which values will complete this statement?
   a. Y, $\overline{AC}$ b. X, $\overline{AB}$ c. Y, $\overline{AB}$ d. X, $\overline{AC}$

6. "To construct the perpendicular bisector of $\overline{AB}$, construct intersecting arcs with center at point ___ and _____ ." Which values will complete this statement?
   a. A,C b. A,B c. X,Y d. A,Y

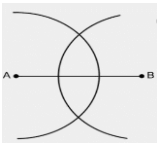


7. Connecting every point where the arcs intersect the circle will finish the construction of which polygon?
   a. square b. rectangle c. equilateral triangle d. hexagon

8. Which diagram shows a correct mathematical construction using only a compass and a straightedge to bisect an angle?
   a. b. c. d.



9. What geometric construction is shown on the right?
   a. an angle bisector c. an angle congruent to a given angle
   b. a line parallel to a given line d. a perpendicular bisector of a segment



10. A triangle with all sides in equal length is what kind of triangle?
    a. an obtuse b. a right c. an isosceles d. equilateral

11. What do you call a tool that only allows you to draw straight lines and not to measure?
    a. protractor b. straightedge c. ruler d. compass

12. If a regular hexagon is divided into six congruent triangles, then which type of triangles is formed?
    a. scalene b. equilateral c. right d. obtuse

13. If all the sides and angles in a parallelogram are equal, then what is it called?
    a. rectangle b. triangle c. square d. rhombus

14. To inscribe a regular polygon what will we first calculate?
    a. all angles b. one angle c. side angle d. center angle

15. What do you call a polygon that is both equilateral and equiangular?
    a. irregular polygon c. segment
    b. regular polygon d. vertices

Great, you finished answering the questions. You may request your facilitator to check your work. Congratulations and keep on learning!

LOOKING BACK TO YOUR LESSON

This activity will give you some ideas on the terms you will encounter in this lesson.

"WORD SEARCH"

i	r	r	e	g	u	l	a	r	l	e	l
n	p	a	n	o	g	a	t	c	o	t	c
s	u	n	r	i	s	e	e	a	s	p	i
s	t	r	a	i	t	l	i	n	e	o	r
c	q	u	s	n	g	l	e	z	k	l	c
b	m	u	s	n	f	h	j	q	s	y	l
s	t	r	a	i	g	h	t	e	d	g	e
d	c	i	p	r	u	v	w	e	r	o	y
e	r	l	m	j	e	s	u	s	i	n	v
t	a	n	o	g	a	x	e	h	n	x	i
a	l	o	c	k	s	x	e	v	n	o	c
n	v	i	c	t	o	r	y	a	m	e	n

Instructions: Find and circle the words listed below that are hidden in the grid. Words may appear in different directions: horizontally, vertically, or diagonally.

Compass	Hexagon
Polygon	Straightedge
Triangle	Irregular
Right	Square
Convex	Octagon

BRIEF INTRODUCTION

STAINED GLASS WINDOWS OF MANILA CATHEDRAL

Stained glass, in arts, are colored glasses used for making decorative windows and other objects through which light passes. "Antique" glass remains the basic material used in stained-glass windows to this day.

The artistic designs of the stained-glass windows of the Manila Cathedral are mostly products of the creative genius of Galo Ocampo. Artist Galo Ocampo bathes the church in glorious Marian light.

The image is known as the triple lancet window which features an episode in the life of Mary at the center and flanking figures of Saint Bernard Clairvaux and Pope Innocent XI, both renowned for Marian devotion.

With a closer look at the stained-glass window, the design is a combination of a different polygonal shape. If you were an artist, what designs of the stained-glass window would you make by using the different types of polygons?

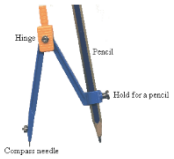
Polygons are plane figures with more than three straight sides. If the sides are not all the same length the figures are said to be irregular polygons, but if they are all the same length the figures are regular polygons.

A polygon is regular if (1) it is convex, (2) all its sides are congruent, and (3) all its angles are congruent. For example, an equilateral triangle is a regular 3-gon, and a square is a regular 4-gon. All regular polygons have **rotation symmetry**. This means that a rotation of less than 360^0 will carry the regular polygon onto itself. A regular *n* – sided polygon has *rotation symmetry* for any multiple of $\frac{360^0}{n}$.

CONSTRUCTIONS

Constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are few tools that you can use:

1. COMPASS



It is a device that allows you to create a circle with a given radius. Not only it can help you to create a circle, but also, they can help you to copy distances.

2. STRAIGHTEDGE

It is a tool that allows you to produce a straight line. A straightedge should not be able to measure distances.

3. PAPER

It is a thin sheet material used to create geometric constructions. When a geometric figure is on a piece of paper, the paper itself can be folded to construct new lines.

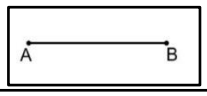
Given the tools for constructions, how do we construct a polygon with ***n*** number of sides? Let's take a look at the examples given.

EXAMPLE1: TRIANGLE

Construct a triangle ABC given $\overline{AB}$.

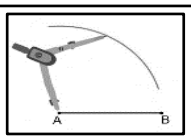
STEPS:

1. Use a straight edge to draw a line segment AB.

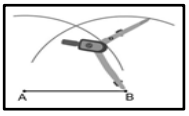


2. Use a compass to measure the length of segment AB.

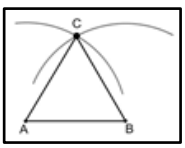
3. Make a partial circle of points/ arcs that is the length of segment AB away from point A.



4. Make another partial circle of points/ arc that is the length of segment AB away from point B.



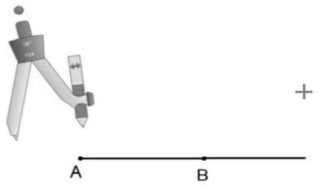
5. The intersection of these two partial circles/ arcs is point C. Then connect the points to form a triangle.



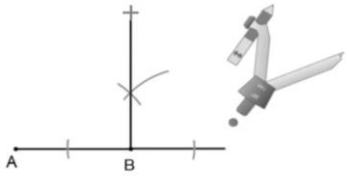
EXAMPLE 2: SQUARE
Construct square ABCD given $\overline{AB}$

STEPS:

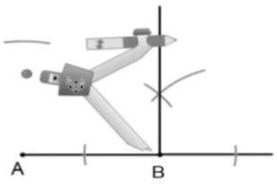
1. Extend line segment AB through B.



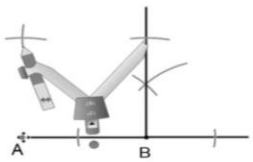
2. Construct a line perpendicular to segment AB through B.



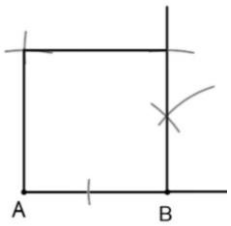
3. Set the compass to length AB and swing arcs above A and B.



4. Move the compass to the new intersection point above B and swing an arc so it intersects the arc above point A.



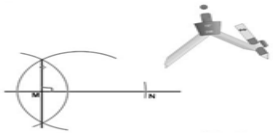
5. Connect the four points of the square and label.



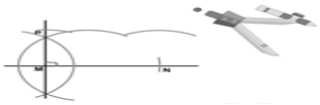
EXAMPLE 3: RECTANGLE
Construct a rectangle given its length and its width.

STEPS:
1. Construct a line perpendicular to segment MN through M.

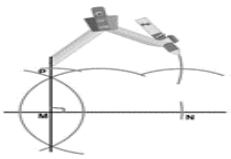
2. Copy the measure of the given length.



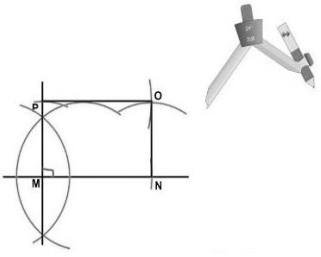
3. Copy the measure of the width and swing arcs above M and N, then mark them as point P and Q respectively.



4. Set the compass to length of MN and move the compass to the new intersection point above M, and swing an arc so it intersects the arc above point N.

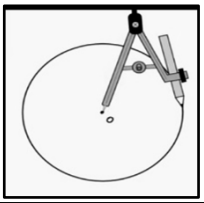


5. Connect the four points of the rectangle and label.

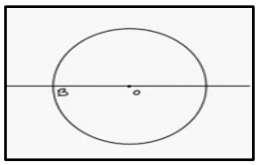


EXAMPLE 4: REGULAR PENTAGON

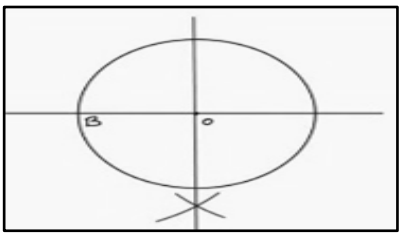
1. Draw a circle and label the center as point O.



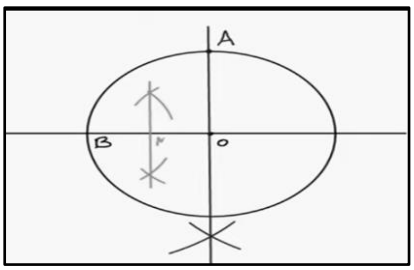
2. Draw a horizontal line through the center of the circle. Label the point of intersection as B.



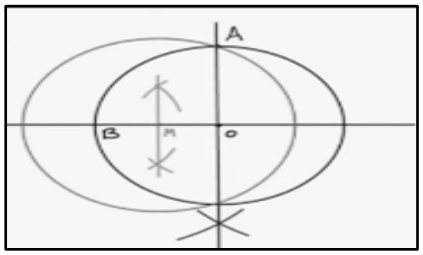
3. Construct a perpendicular bisector of a diameter. Label one of the points of intersection as point A.



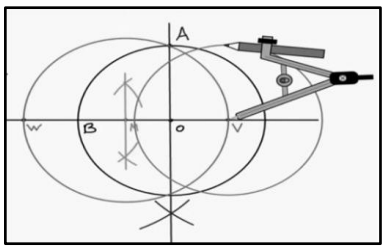
4. Construct the midpoint of O and B. Label the midpoint M.



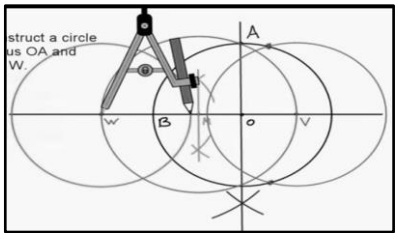
5. Construct a circle centered at M through point A. Label the intersecting points outside the circle as W. Label the intersecting point inside the circle as V.



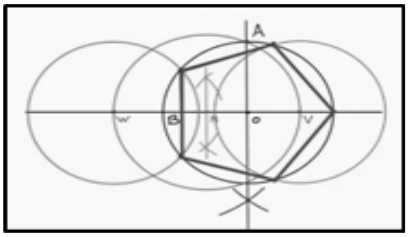
6. Construct a circle with radius OA and center V. The circle intersects the original circle at two of the vertices of the pentagon.



7. Construct a circle of radius OA and center W. This circle intersects the original circle at two of the vertices of the pentagon.

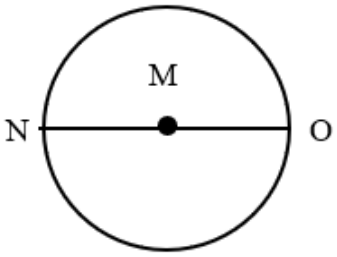


8. The 5th vertex is the intersection of the horizontal line with the original circle. Connect the vertices using red markings to form a pentagon.

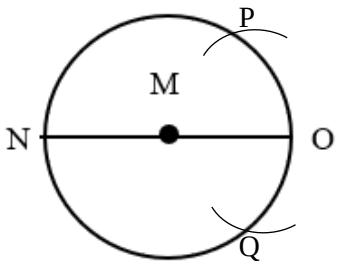


EXAMPLE 5: REGULAR HEXAGON

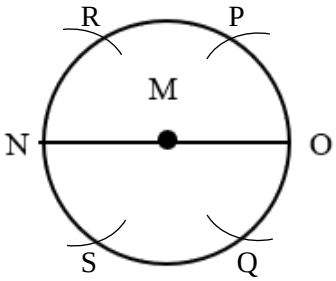
1. Draw circle M with diameter $\overline{NO}$



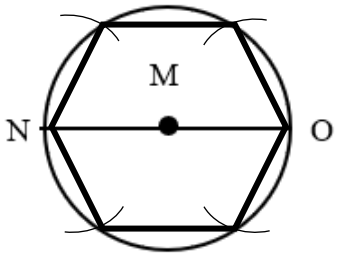
2. With the same opening of the compass place the tip of the compass on O, and then draw arcs that intersect the circle into two. Name the intersections as P and Q.



3. Repeat step 2, but this time, place the tip of the compass on N. Name the intersection as R and S.



4. Connect the points of intersection and endpoints of the diameter consecutively to form a hexagon.



ACTIVITIES

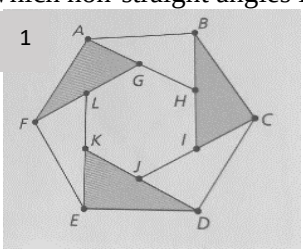
ACTIVITY 1: DRAWING DESIGNS

A. Use a straightedge and compass to draw each design. Begin by marking off six successive arcs around a circle. On each completed design, label all endpoints of segments.

(a.) Which segments in the design are bisected?

(b.) Which non-straight angles in the design are bisected?

1



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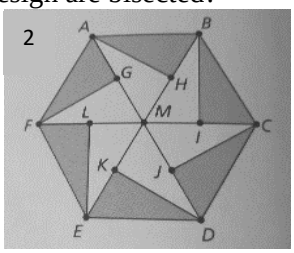
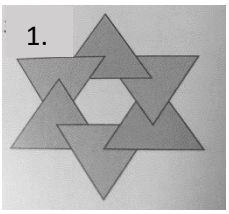
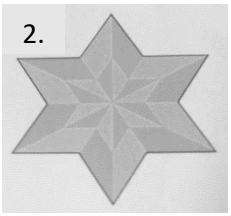


B. You can construct geometric designs using a straight edge and a compass. The following are some examples.





- To create your own design, you may use a "computer paint" program.

ACTIVITY 2: PROBLEM SET

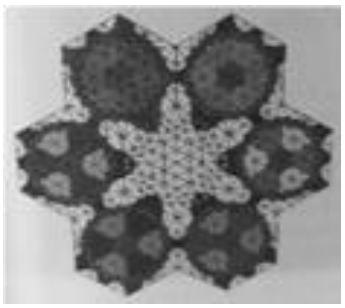
1. What quadrilateral, if any, is equilateral but not regular? Equiangular but not regular?
2. Determine the measure of each angle of a regular polygon of 5 sides: 9 sides; 12 sides;15 sides; and 24 sides.
3. Sketch a polygon that has congruent sides and all right angles.
4. How would you construct a square inscribed in the circle below?

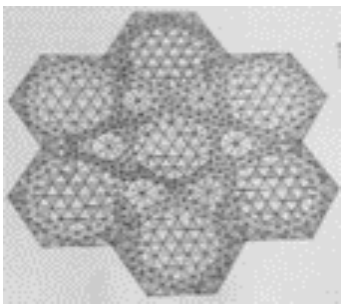
REMEMBER

- A polygon is regular if (1) it is convex, (2) all of its sides are congruent, and (3) all of its angles are congruent. For example, an equilateral triangle is a regular 3-gon, and a square is a regular 4-gon.
- Basic tools for constructions:
  - Straightedge - It is a tool (ruler), with no markings, that allows you to produce a straight line.
  - Compass - It is a device that allows you to create a circle with a given radius. Not only it can help you to create circles, but it can also help you copy distances.
  - Paper - It is a thin sheet material used to create geometric constructions.
- We can construct regular n-gons with any number of sides by the following methods:
  (1) Begin with a circle, with center Q and radius r.
  (2) Divide the circle into n congruent arcs, end to end. Each arc has a measure of 360/n.
  (3) For each little arc, draw the corresponding chord.

CHECKING YOUR UNDERSTANDING

Just like the stained-glass windows of Manila Cathedral which was designed creatively and caught the eyes of many visiting tourists in the country, the designs below are very attractive also and are taken from the book Images 2 by Roger Burrows, published by Running Press. Each design consists of seven congruent hexagons that are filled with triangles. To create the patterns, Burrows colored the triangles to form a ring of turtles, a pattern, and a bowl of fruit. Create your own "seven-hexagon" design. (If you have access to a "computer paint" program, try using it to create the design.)

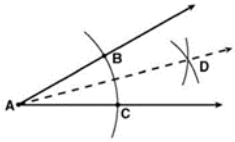




POST-TEST

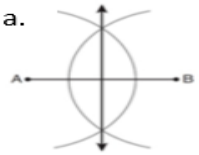
Directions: Read the questions carefully. Encircle the letter of the correct answer. If the answer is not found in the choices, write "E".

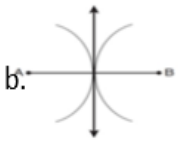
1. Given angle A. What is the first step in constructing the angle bisector of angle A?



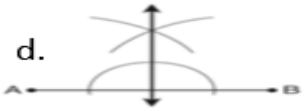
   a. Draw ray AD.
   b. Draw a line segment connecting points B and C.
   c. From points B and C, draw equal arcs that intersect at D.
   d. From point A, draw an arc that intersects the sides of the angle at points B and C.

2. Which is the correct construction for a perpendicular bisector?

a. 

b. 

c. 

d. 

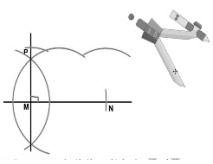
3. To inscribe a square inside a circle, first you must draw a chord anywhere across the circle. What should your next step be?
   a. Construct a perpendicular bisector.
   b. Draw a second diameter to the circle.
   c. Construct a line tangent to the circle.
   d. Set your compass the length of the radius.

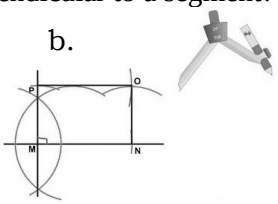
4. What do you call the intersection of the sides of a polygon?
   a. vertices of the polygon
   b. sides of the polygon
   c. angles of the polygon
   d. diagonals of the polygon

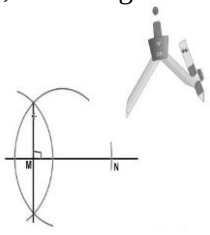
5. "Diagonals divide the parallelogram into two congruent _____." - Which word will best complete this statement?
   a. lines b. triangles c. circles d. squares

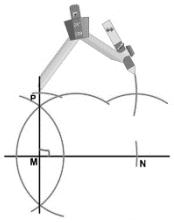
6. To construct a square ABCD given $\overline{AB}$, you need to construct a line perpendicular to $\overline{AB}$ through B. What should your next step be?
   a. Connect the four points of the square and label.
   b. Move the compass to the new intersection point above B and swing an arc to intersect another arc above A.
   c. Set the compass to the length AB and swing arc above A and B.
   d. Extend line segment AB through B.

7. To construct a rectangle given its length and its width, which figure should be the next step after constructing a line perpendicular to a segment?

a. 

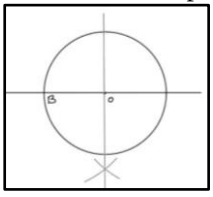
b. 

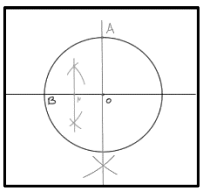
c. 

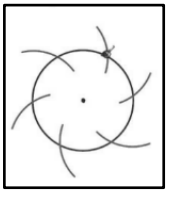
d. 

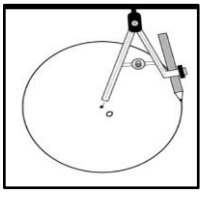
8. In constructing regular n-gons with any number of sides, how do you divide the circle into congruent arcs?
   a. Draw a corresponding chord
   b. Divide 360 by the number of sides
   c. Draw the radii from the center to the vertices
   d. Divide the line into equal parts

9. What is the first step in constructing a pentagon?

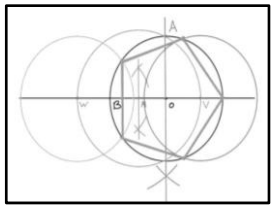
a. 

b. 

c. 

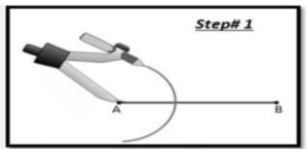
d. 

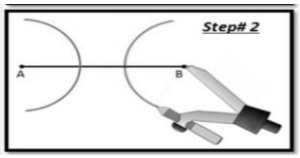
10. What kind of construction is displayed in this picture?
   a. a circle
   b. a circle inscribed in a hexagon
   c. a hexagon inscribed in a circle
   d. a hexagon



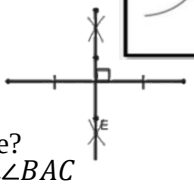
11. Which will you do when you're constructing a line parallel to a given line?
    a. constructing a perpendicular
    b. copying a segment
    c. copying an angle
    d. bisecting a segment

12. Marionne was attempting to construct a perpendicular bisector to $\overline{AB}$ with a compass and a straightedge. Which statement explains what Marionne may have done wrong?
    a. Marionne just needed to open the compass more to create arcs that have a radius of more than half the length of $\overline{AB}$.
    b. On the the 2nd step, Marionne should have placed the compass needle point where the first arc intersected $\overline{AB}$.
    c. Marionne didn't do anything wrong she just needs to connect the opposite endpoints of each arc to finish the construction.
    d. Marionne should have started by putting the compass needle point at the midpoint of $\overline{AB}$.

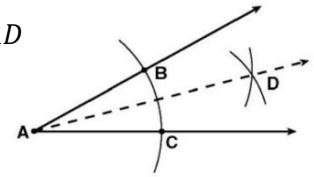




13. What type of construction do you see?
    a. altitude
    b. perpendicular bisector
    c. midpoint
    d. angle bisector



14. Based on the construction below, which statement must be true?
    a. $m\angle BAD = \frac{1}{2}\, m\angle CAD$
    b. $m\angle BAD = m\angle CAD$
    c. $m\angle BAD = m\angle BAC$
    d. $m\angle CAD = \frac{1}{2}\, m\angle BAD$



15. What does the word BISECT mean?
    a. A shape that has three sides.
    b. It is a plane with two sets of wings.
    c. To cut something into more than five pieces.
    d. To cut something into two congruent pieces or in half.

E-SITES

To further explore the concept learned today and if it possible to connect the internet, you may visit the following links:
https://www.youtube.com/watch?v=fx1RTMyzsLQ&feature=share
https://www.youtube.com/watch?v=HfKwW32wH5c&feature=share
https://www.youtube.com/watch?v=ELNDgbRQi4&feature=share
https://www.mathopenref.com/constcirclecenter.html

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Clemens, et al,(1994). Geometry.Addison-Wesley Publishing Company, Inc.,USA
Jurgensen, Ray C., et al,(1965). Modern Geometry Structure and Method. Houghton Mifflin Company, USA
Garcia Anna Khares G., Amid et al (2015) Mathematics for the 21st Century. Diwa Learning System Inc.
https://quizizz.com/admin/quiz/5cb09734352086001a8edee5/inscribed-polygon-construction
https://quizizz.com/admin/quiz/5bfd45f94f7011001b6ecbd3/geometry-constructions

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