

Republic of the Philippines
Department of Education
NATIONAL CAPITAL REGION
Misamis Street, Bago-Bantay, Quezon City

**UNIFIED SUPPLEMENTARY LEARNING MATERIALS
(USLeM)**



MATHEMATICS

Week 9

EXPECTATIONS:

You will solve problems involving sides and angles of a polygon.
Specifically, this learning material will help you to:

- Identify the given and what is asked in the problem.
- Make a problem solving plan.
- Solve problems using the problem solving plan.

Let us start your journey in learning how to solve problems involving the sides and angles of a polygon. I am sure you are ready and excited to answer the Pretest. Smile and cheer up!

PRE-TEST

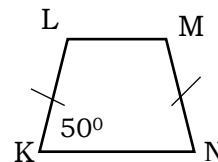
Directions: Read the questions carefully. Encircle the letter of the correct answer.

For items 1-2: The base of a lamp is in the shape of a regular 15-gon.

1. Find the measure of each interior angle.
a. 150 b. 156 c. 2,340 d. 2,700
2. What is the measure of each exterior angle?
a. 150 b. 156 c. 24 d. 18

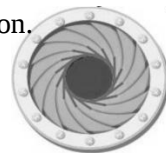
For items 3-5: A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid.

3. Find the measure of angle N.
a. 180° b. 150° c. 130° d. 50°
4. Find the measure of angle M.
a. 180° b. 150° c. 130° d. 50°
5. What is the measure of angle L?
a. 180° b. 150° c. 130° d. 50°



For items 6-8: The aperture on the camera lens shown is a regular 14-sided polygon.

6. What is the sum of the measures of its interior angles?
a. 2160° b. 2520° c. 360° d. 154°
7. What is the measure of each interior angle of the polygon?
a. 150° b. 30° c. 154.3° d. 25.7°
8. Find the measure of an exterior angle of the polygon.
a. 180° b. 360° c. 154.3° d. 25.7°



For items 9-12: A regular polygon is covered by a piece of paper so that only one of its interior angle is visible. You are able to determine that the interior angle is four times as large as the exterior angle.

9. How many sides does the polygon have?
a. 8 b. 9 c. 10 d. 12
10. What type of convex polygon is described in the situation?
a. Octagon b. Nonagon c. Decagon d. Dodecagon
11. What is the sum of the measures of the interior angles?
a. 1080° b. 1260° c. 1440° d. 1800°
12. What is the sum of the measures of the exterior angles?
a. 180° b. 360° c. 540° d. 720°

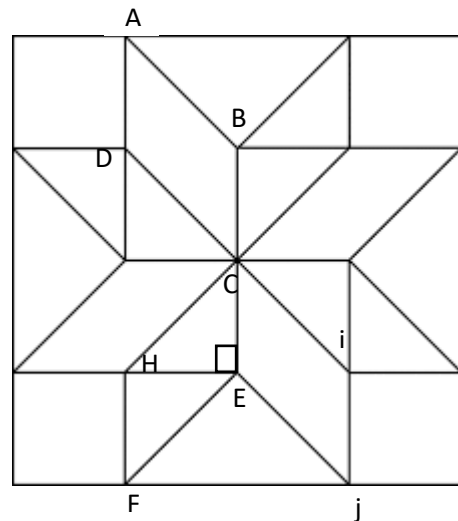
13. Find the number of sides of a polygon if the sum of the measures of its interior angles is twice the sum of the measures of its exterior angles.
 - a. 6
 - b. 2
 - c. 7
 - d. 8
14. The measure of each interior angle of a regular polygon is eight times that of an exterior angle. How many sides does the polygon have?
 - a. 12
 - b. 14
 - c. 16
 - d. 18
15. The measures of the angles of a polygon having four sides are represented as x° , x° , $5x^\circ$ and $(4x - 3)^\circ$. Find the measure of the largest angle.
 - a. 135°
 - b. 145°
 - c. 155°
 - d. 165°

Great, you finished answering the questions. You may request your facilitator to check your work. Congratulations and keep on learning!

LOOKING BACK TO YOUR LESSON

Direction: Answer the following by referring to the fabric design at the right.

1. What is the sum of the interior angles of ABCD?
2. What is the sum of the exterior angles of CHFE?
3. Which sides are parallel to CE?
4. Which side is perpendicular to CE?
5. What is the sum of the measures of $\angle C$ and $\angle H$?
5. What is the measure of each exterior angle of a regular pentagon?



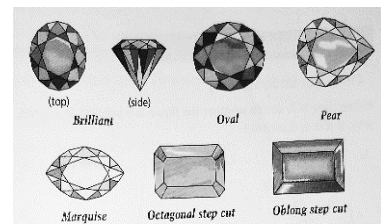
BRIEF INTRODUCTION

THE FACETS OF A GEM

Natural diamond crystals have the shape of an octahedron. To form a gem, a natural crystal is cut and polished in the shape of a more complex polyhedron. The most popular shape for a diamond is called a *brilliant* cut. Each face of a gem is called facet.

A faceted gem is one that is cut so that all its flat surfaces (or facets) are polygons. A gem with many facets tends to reflect light more than one with few facets. The picture above shows some common ways that gems are faceted.

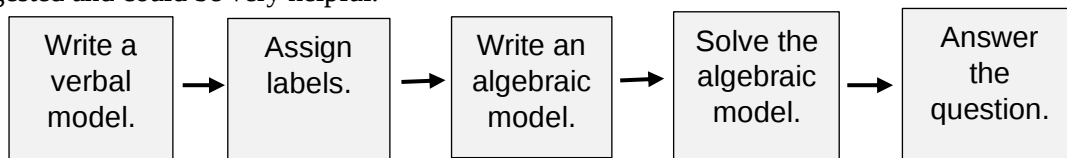
If a brilliant cut gem has 57 facets, how many of the facets do you think are triangles? How many are quadrilaterals? How many are octagons?



WHAT IS PROBLEM SOLVING?

Problem-solving is the process of finding solutions to difficult or complex issues.

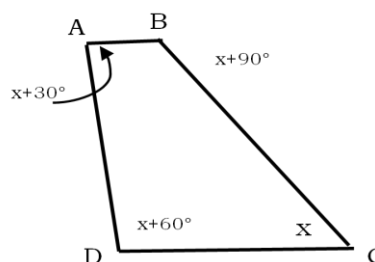
In mathematics, word problems can be intimidating and overwhelming. When solving problems, whether it's a one-step or multi-step word problem, the problem-solving plan below is suggested and could be very helpful.



FINDING ANGLE MEASURES OF A POLYGON

EXAMPLE 1:

In the diagram at the right, you have designed the tailfin of an airplane so that when the four interior angle measures are put in increasing order, each differs from the next large measure by 30° . What are the measures of the four interior angles?



SOLUTION

Verbal Model

From the diagram, order the four angles as follows.

Labels

$$\begin{aligned} m\angle C &= x \\ m\angle A &= x + 30 \\ m\angle D &= x + 60 \\ m\angle B &= x + 90 \end{aligned}$$

Equation

$$x + (x+30^\circ) + (x+60^\circ) + (x+90^\circ) = 360^\circ$$

$$4x + 180^\circ = 360^\circ$$

$$4x = 180^\circ$$

$$x = 45^\circ$$

Therefore, the four angle measures

are: $m\angle C = 45^\circ$, $m\angle A = 75^\circ$,

$m\angle D = 105^\circ$, and $m\angle B = 135^\circ$.

A Problem-Solving Plan

Ask yourself what you need to know to solve the problem. Then write a **verbal model** that will give you what you need to know.



Assign **labels** to each part of your verbal model.



Use the labels to **write an algebraic model** based on your verbal model.



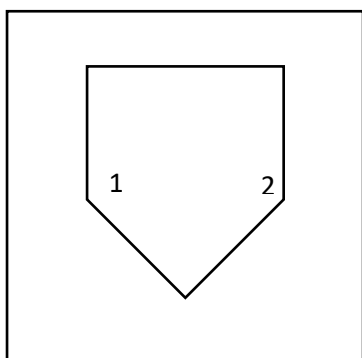
Solve the equation or inequality (the algebraic model).



Answer the original question.
Check that your answer is reasonable.

EXAMPLE 2:

Home plate on a baseball field has three right angles and two congruent angles as shown as $\angle 1$ and $\angle 2$ in the figure below. Find the measures of $\angle 1$ and $\angle 2$.



SOLUTION

Verbal Model

Homeplate's shape is pentagon with three right angles and two congruent angles. The sum of interior angles of pentagon is 540° .

Labels

$$\begin{aligned} x &= m\angle 1 = m\angle 2 \\ 3(90^\circ) &= 270^\circ \end{aligned}$$

Equation

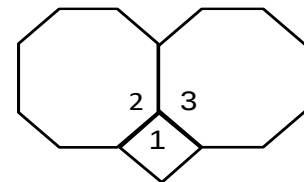
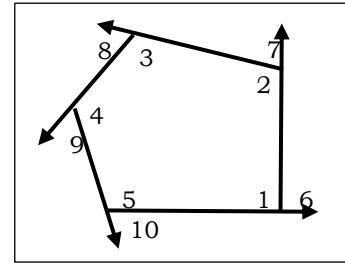
$$\begin{aligned} 2x + 3(90^\circ) &= 540^\circ \\ 2x &= 270^\circ \\ x &= 135^\circ \end{aligned}$$

Therefore, the measure of $m\angle 1 = 135^\circ$ and $m\angle 2 = 135^\circ$.

ACTIVITIES

ACTIVITY 1:

1. In the diagram at the right, you are given $m\angle 2=100^\circ$, $m\angle 8=40^\circ$, $m\angle 4 = m\angle 5 = 110^\circ$. Find the measures of the other labeled angles.
2. The sum of the measures of seven angles of an octagon is 1000. Find the measure of the eighth angle.
3. How many sides does a regular polygon have if each exterior angle has a measure of 15?
4. How many sides does a regular polygon have if each interior angle has a measure of 108?
5. The tiling pattern to the right consists of regular octagons and a square. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ and verify that their sum is 360.



You may explore more to check your understanding!

REMEMBER

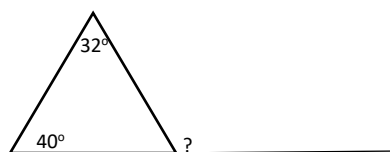
- Problem-solving is the process of finding solutions to a difficult or complex issue.
- Problem-Solving Plan
 - **Write a verbal model** that will give you what you need to know.
 - **Assign labels** to each part of your verbal model.
 - Use the labels to **write an algebraic model** based on your verbal model.
 - **Solve** the equation or inequality (the algebraic model).
 - Answer the original question. Check that your answer is reasonable.

CHECKING YOUR UNDERSTANDING

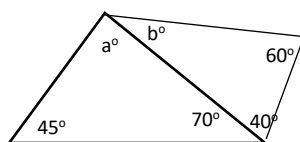
THINK AND REFLECT

Analyze and solve the following problems.

1. Josh says that the missing angle is 72° . Do you agree with Josh? Explain why?



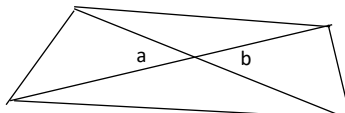
2. Find the angles marked with letter in this quadrilateral.



3. An isosceles triangle has an angle of 44° . What measures could the other two angles be?

4. Is the following statement always true or never true? Explain your answer.

$$a + b = 180^\circ$$



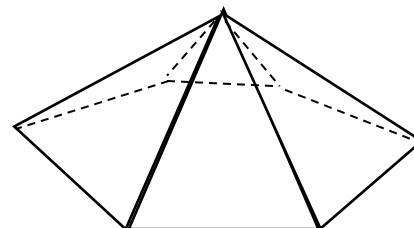
CREATING A CARDBOARD MODEL

Use cardboard and scissors to cut three groups of six congruent, isosceles triangles. In the first group, make $m\angle A < 60^\circ$; in the second group, make $m\angle A = 60^\circ$; and in the third group, make $m\angle A > 60^\circ$.

- A. For each group, tape the legs of the six triangles together as shown at the right. When taped together, one of the groups forms a rigid peak. Which is it?



- B. Describe the shapes formed by the other two taped groups.



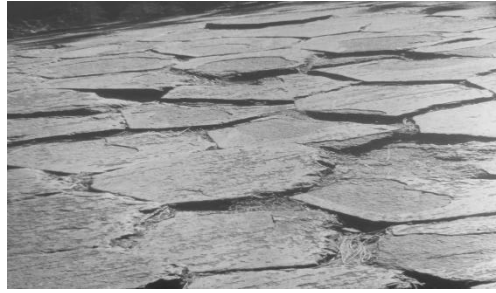
POST-TEST

Directions: Read the questions carefully. Encircle the letter of the correct answer.

For items 1-3: During the halftime performance for a football game, the color guard is planning a new formation in which seven members stand around a central point and stretch their flag to the person immediately to their left.

- What is the measure of each exterior angle of the formation?
 - 25.7°
 - 128.6°
 - 51.4°
 - 360°
- What type of polygon is formed?
 - Pentagon
 - Hexagon
 - Heptagon
 - Octagon
- What is the measure of the sum of the interior angles of a polygon?
 - 720°
 - 900°
 - 1080°
 - 1260°

For items 4 – 6: Many rock formations have faces that are polygons. For instance, the lava columns in the photo tend to have hexagonal cross sections.



4. If four angles of a hexagon have the following measures: 72° , 157° , 85° , and 124° , what is the measure of the remaining angles if they are congruent?
 - a. 141°
 - b. 282°
 - c. 85°
 - d. 124°
5. If the hexagon is a regular polygon, what is the sum of the interior angles?
 - a. 360°
 - b. 900°
 - c. 540°
 - d. 720°
6. How many sides do the polygon formed?
 - a. 5
 - b. 6
 - c. 7
 - d. 8

For items 7- 11: In the image at the right, a two-bedroom house plan was designed. The side sections of the plan is divided into congruent quadrilaterals.



7. If the shape of the entire house is a polygon, how many sides is the house made of?
 - a. 5
 - b. 6
 - c. 7
 - d. 8
8. What is the measure of one of the exterior angles of the verandah that is adjacent to one of the interior angles which measures 105° ?
 - a. 85°
 - b. 75°
 - c. 65°
 - d. 15°
9. What is the sum of the interior angles of all the verandahs?
 - a. 1080°
 - b. 720°
 - c. 360°
 - d. 900°
10. What is the sum of the interior angles of the house plan?
 - a. 1080°
 - b. 900°
 - c. 720°
 - d. 540°
11. What is the shape of the inner polygon of the house plan?
 - a. Nonagon
 - b. Dodecagon
 - c. Undecagon
 - d. Octagon
12. The measure of the second angle of a triangle is 20 more than the measure of the first, and the measure of the third angle is 10 less than three times the first. What is the measure of each of the three angles of a triangle?
 - a. 50, 30, 100
 - b. 45, 50, 85
 - c. 34, 54, 92
 - d. 42, 58, 80
13. The measure of one angle of a parallelogram is 20 more than three times the next consecutive angle. What are the measures of the angles of the parallelogram?
 - a. 40, 120, 100, 50
 - b. 60, 120, 60, 120
 - c. 70, 90, 100, 80
 - d. 40, 140, 40, 140

14. In a conservationist's heptagonal sanctuary, when interior angles are put in increasing order, each differs from the next by 25° . Find the measure of the smallest interior angle of the field to the nearest tenth of a degree.
- a. 53.6° b. 125.0° c. 75.0° d. 128.6°
15. An after- school club was doing a craft with paper cut out in the shape of a convex regular polygon. If one of the interior angles has a measure of 135° , what is the name of the polygon?
- a. Tridecagon b. Pentakaidecagon c. Heptagon d. Octagon