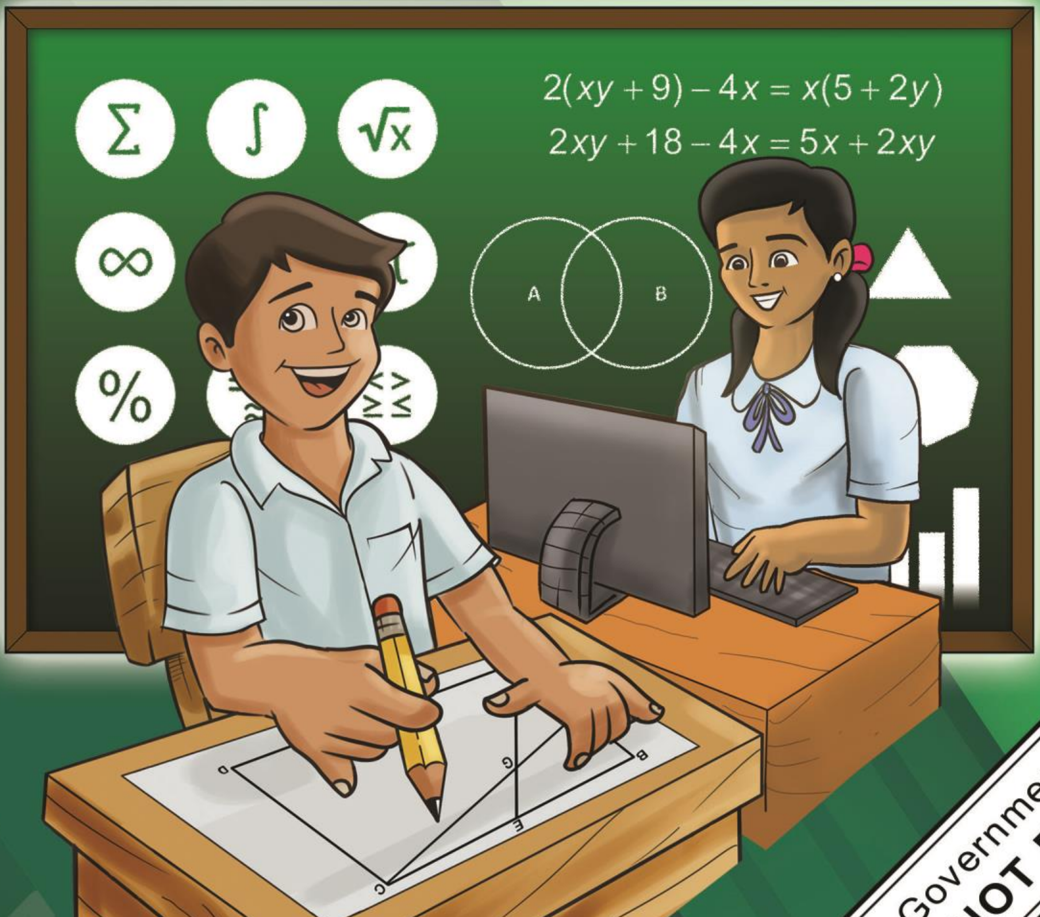
10

NATIONAL LEARNING CAMP

Mathematics

Consolidation Learning Camp

Notes to Teachers



Consolidation Learning Camp
Notes to Teachers

Mathematics Grade 10

Weeks 1 to 3

Contents

Dear Reader,

Every care has been taken to ensure the accuracy of the information provided in this Booklet. Nevertheless, if you identify a mistake, error, or issue, or wish to provide a comment, we would appreciate you informing the **Office of the Director of the Bureau of Learning Delivery** via telephone numbers (02) 8637-4346 and 8637-4347 or by email at bld.od@deped.gov.ph

Thank you for your support.

Notes to Teachers

Part A: Introduction to Mathematics in the 2024 Learning Camp

The Mathematics section of the 2024 Learning Camp for students who have recently completed Grade 10, consists of 15 ‘main’ lessons each focused on a single Key Idea (KI), and 3 ‘deliberate practice’ lessons each week. All 18 lessons are designed for a lesson duration of 45 minutes.

The set of 15 main lessons consists of single lessons addressing each of 15 key ideas of the Grade 10 curriculum content. The Key Ideas are based on selected Most Essential Learning Competencies (MELCs) for the four Quarters of the Grade 10 content.

The three deliberate practice lessons, Lessons 6, 12 and 18, are each delivered at the end of Week 1, 2 and 3 of the 2024 Learning Camp. The lessons are designed to reinforce learning from the Key Idea lessons of the week.

The 15 Key Ideas listed by Quarter are:

Quarter 1:	KI 1	Determine arithmetic means, $n\text{th}$ term of an arithmetic sequence, and sum of the terms of an arithmetic sequence.
	KI 2	Determine geometric means, $n\text{th}$ term of a geometric sequence, and sum of the terms of a finite or infinite geometric sequence.
	KI 3	Solve problems involving sequences.
	KI 4	Solve problems involving polynomials and polynomial equations.
Quarter 2:	KI 5	Solve problems involving polynomial functions.
	KI 6	Solve problems involving circles.
	KI 7	Determine the center and radius of a circle given its equation, and vice versa.
	KI 8	Graph and solve problems involving circles and other geometric figures on the coordinate plane.
Quarter 3:	KI 9	Differentiate permutation from combination of objects taken r at a time.
	KI 10	Solve problems involving permutations and combinations.
	KI 11	Illustrate and find the probability of a union of two events $(A \cup B)$, including for when the events are mutually exclusive.
	KI 12	Solve problems involving probability.
Quarter 4:	KI 13	Calculate and interpret measures of position (quartiles, deciles, and percentiles) of a set of data.
	KI 14	Solve problems involving measures of position.
	KI 15	Use appropriate measures of position and other statistical methods in analyzing and interpreting data.

The Key Ideas are representative of the three content sections of the Grade 10 curriculum as follows:
Patterns and Algebra: KI 1, KI 2, KI 3, KI 4, KI 5
Geometry: KI 6, KI 7, KI 8
Statistics and Probability: KI 9, KI 10, KI 11, KI 12, KI 13, KI 14, KI 15

The 15 main lessons, which each address one (only) of the Key Ideas listed above, are:

Lesson 1: Determining Arithmetic Means, $n\text{th}$ term of an Arithmetic Sequence, and Sum of the Terms of an Arithmetic Sequence.
Lesson 2: Determining Geometric Means, $n\text{th}$ term of a Geometric Sequence, and Sum of the Terms of a Finite or Infinite Geometric Sequence.
Lesson 3: Solving Problems involving Sequences.
Lesson 4: Solving Problems involving Polynomials and Polynomial Equations.
Lesson 5: Solving Problems involving Polynomial Functions.
Lesson 6: Deliberate Practice: Solving Problems involving Sequences; Solving Problems involving Polynomials and Polynomial Equations; Solving Problems involving Polynomial Functions.
Lesson 7: Solving Problems involving Circles.
Lesson 8: Determining the Center and Radius of a Circle given its Equation, and vice versa.
Lesson 9: Graphing and Solving Problems involving Circles and other Geometric Figures on the Coordinate Plane.
Lesson 10: Differentiating Permutation from Combination of Objects taken r at a time.
Lesson 11: Solving Problems involving Permutations and Combinations.
Lesson 12: Deliberate Practice: Solving Problems involving Circles; Graphing and Solving Problems involving Circles and Other Geometric Figures on the Coordinate Plane; Solving Problems involving Permutations and Combinations.
Lesson 13: Illustrating and finding the Probability of a Union of Two Events $(A \cup B)$, including for when the events are mutually exclusive.
Lesson 14: Solving Problems involving Probability.
Lesson 15: Calculating and Interpreting Measures of Position (quartiles, deciles, and percentiles) of a Set of Data.
Lesson 16: Solving Problems involving Measures of Position.
Lesson 17: Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data.
Lesson 18: Deliberate Practice: Solving Problems involving Probability; Solving Problems involving Measures of Position; Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data.

Note: The three deliberate practice lessons each address and correspond to multiple Key Ideas. The Deliberate Practice lessons, with the associated Content Section/s and Key Ideas are:
Lesson 6: Content Section: Patterns and Algebra; Key Ideas: KI 3, KI 4, KI 5
Lesson 12: Content Sections: Geometry, Statistics and Probability; Key Ideas: KI 6, KI 8, KI 10
Lesson 18: Content Section: Statistics and Probability; Key Ideas: KI 12, KI 14, KI 15

Each of the 15 main lessons and three deliberate practice lessons is written in a standard format, made up of five sequential lesson components.

The components are:
Lesson Component 1: Lesson Short Review
Lesson Component 2: Lesson Purpose/Intention
Lesson Component 3: Lesson Language Practice
Lesson Component 4: Lesson Activity
Lesson Component 5: Lesson Conclusion – Reflection/Metacognition on Student Goals

The nature of these lesson components for all 18 Mathematics KI lessons is described and discussed in Part B.

Part B: Commentary on Lesson Components in All Lessons

Overview

The NLC lessons emphasizes consolidating and, where possible, extending student knowledge in previously covered topics. Lesson sets are designed to strengthen students' current foundational knowledge ready for future learning. The review lessons have been designed to be interactive among teachers and their students, and with students and their peers.

At the same time, the expectation is that teachers will enhance their own pedagogical practices and subject knowledge as well as refine further their teaching methods. The thinking behind the Camp lessons is grounded in the 'Science of Learning' framework, creating a dynamic, learning environment employing the findings of cognitive research and evidence-informed approaches.

Lesson Component 1 (Lesson Short Review)

Component 1 offers teachers the chance to:

- settle the class quickly;
- review or preview previously encountered information;
- address previous content in the form of a few targeted questions that are *relevant to the current lesson;*
- note what students already know;
- elicit answers from the class to reinforce the important content needed for the lesson; and
- briefly address issues that may arise.

Overall, Component 1 acts as a partial advance organizer designed to remind students of previous work that has relevance to activities to be undertaken in the current lesson. When done carefully, this replay of previous information directed at what is to come in the lesson helps students prepare for future memory recall and decision-making.

Reminding students of *relevant* information at the beginning of a lesson, prior to encountering the main lesson learning focus, can enhance the brain's ability to:

- access information to be used in problems/questions/information to come;
- prepare, and have some oversight, for the direction of future learning; and
- further consolidate ideas in long-term memory.

The teacher should note any issues that may arise in student answers. This may be addressed later in the lesson or in later lessons, if relevant.

Lesson Component 2 (Lesson Intention)

This component offers teachers the opportunity to explain to the class the intention or purpose of the lesson. The explanation should link with student prior knowledge or experience. This may mean connecting the purpose to the responses and levels of understandings in Component 1. The words and phrases used by the teacher should be familiar to, and understandable by, students. Information could include ideas personal to students that could facilitate student engagement in the lesson such as:

- the provision of a relevant context;
- asking a question that sounds interesting to that age group; and/or
- addressing an aspect that has a special interest to the class.

In addition, this component is an appropriate time to address what students might expect/aim to achieve, i.e., the lesson goal(s). Teachers should clarify, in clear language, the learning intention and what success looks like. (Note**:** Evaluation of the degree of success or partial success of student learning intention should occur as part of Component 5.)

Lesson Component 2 is about activating, in the student brain, ideas already relevant to the students. The purpose is to help students contextualize their new learning experiences and to help them make sense of any new information.

Design considerations in statements of the lesson intention are about promoting student engagement and enthusiasm. This is best done by stating things in ways that make sense to as many students as possible in the class. In terms of timing, this component is relatively brief. Its presence, as one-of-five components, lies in *its importance* to the student brain and learning. Finally, it is important *not to* overwhelm students with excessive and unnecessary detail that could disengage them at this early point in the lesson.

Lesson Component 3 (Lesson Language Practice)

Component 3 concerns language use – speaking, hearing, listening, and comprehending. The focus is on words or phrases that are important to this lesson. It maybe language that has the potential to cause difficulties for the students through speech, interpretation, or understanding, or simply a reminder that these words are important to, or will be used in, this lesson. Typically, the language identified is restricted to about 6 words/phrases so that there is enough time for students to use them in practice.

Deliberate practice concerns repeating aspects of learning that the teacher has deliberately identified/selected because it is where students are making an error that needs to be corrected, or because of its important role in learning. In the case of unfamiliar or unknown textual or symbolic language, deliberate practice can help students reduce cognitive load (reduce working memory) by making some aspect more familiar, enabling students to re-allocate resources to a problem solution, comprehending a passage, answering a question, explaining a concept, or describing some event or story, etc.

Overall, Component 3 can help achieve language familiarity by saying the word/phrase, being able to spell it, or using it in a specific context. This may also involve helping students to understand or unpack a visual text, diagram or graph, e.g., for a graph, the teacher may need to point out such things as the graph heading, the axes, units, data points, or trend lines.

Lesson Component 4 (Lesson Activity)

Addressing the key idea for the lesson is the focus of Component 4. It involves students applying known content to solve non-routine problems or interpreting new texts. This requires students to interpret/understand the meaning of the stem of the problem correctly and then answer a few questions of varying degrees of complexity related to the stem. The stem holds the needed information that will be the basis for the questions. Following the stem is a small number of questions that can be answered by utilizing students' background content knowledge and understanding, together with information in the stem.

From a learning perspective, the lessons are intended to help students consolidate their understanding at different levels of difficulty, e.g., the early questions are at an elementary level allowing the students to get started, then the next level is directed at the majority of students and usually requires a number of steps to reach a conclusion, and, finally, the third question attempts to offer all students the opportunity to be challenged and experience enhancements of their learning through seeing how ideas are connected or applied.

(Note: The level of difficulty of the questions should not stop any student from being given the opportunity to experience, with support, questions at higher levels, including the more challenging questions, and to hear about, and be involved in, discussions about the answers.

Most students should be able to make some progress and be acknowledged for that. The point of question levels is to at least have students experience these more demanding questions and their answers as the start of the process for their learning journey. It is also designed to offer teachers a more realistic view of potential expectations of students in their class.)

Component 4 has three aspects, 4A, 4B, and 4C. Students are first presented in 4A with the stem. This can be a stimulus or passage/text or diagram or … and are given the time/opportunity to understand the stem.

Then, in 4B and 4C, two separate set of questions related to the same stem are presented. This process involves a set of three questions based on the same stem, which is then repeated, resulting in one set of questions in each of 4B and another set of questions in 4C.

Note: The early components, Components 1, 2 and 3, can be seen as bringing together the pre-requisite information that will place the student in the best possible position to be successful in Component 4. Component 4 begins with 4A.

4A Reading and Understanding the Stem

4A involves understanding the language of the stem. The purposes here are for the teacher:

- to model fluent reading of the stem (first);
- to identify any unfamiliar language the student (possibly addressed in Component 3);
- to read the passage or describe the figure; etc
- to hear and experience fluency in reading the stem.

Other activities here could include students:

- reading to each other;
- reading silently to themselves; and
- exploring the meaning of the vocabulary.

4B Solving the First Set of Questions

4B involves students answering questions associated with the stem. The students will recognize that they have a stem (previously met in **4A**) and that this is followed by a small set of questions. Students find their own way to a response for each question in the set. The students write down responses or attempts at each question. It is important that every student in the class is expected to have a response. To achieve this desired result, it is important for teachers to ensure all students start on time at the same time.

When the students are finished, or sufficient time has been allocated, students provide answers to the questions and the teacher marks the questions. Discussion takes place about:

- the quality of the answers;
- the implications of errors; and
- what this information tells the class about the content.

The time allocated for 4B provides teachers with an opportunity to observe the quality and levels of student response, which they can build on as a base of what the student knows.

Note: It is important that students start the questions promptly. This involves student self-regulation concerning focus and attitude to work, and may need to be consistently encouraged or reinforced by the teacher.

Teachers can seek out different responses or approaches or thinking exhibited. Errors made by students should be *acknowledged and valued* for their contribution to the class discussion and student learning. Those who achieve correct answers on different questions should also be acknowledged. **Note:** The questions are usually arranged in increasing difficulty from basic to more challenging.

4C Solving the Second Set of Questions

4C uses the same Stem as **4B** and repeats the same process as **4B** but offers students a second (different) batch of questions, again in order of increasing difficulty. When all questions are completed, as was the case in **4B**, students provide answers to all questions, i.e., the students write down responses to, or attempts at, each question. When they are finished, the questions are marked (either using teacher or student answers) and discussion takes place about the quality of correct answers and the implications of errors and what this tells the class about the content.

Note: 4C offers a new start for students regardless of how they performed in **4B**. It allows all students to see **4C** as a new starting point and the class focus for all students should now be around the content and answers in **4C**.

For teachers this approach serves two purposes. *First,* it is a practical way to ensure all students have experiences and are able to contribute perspectives with all questions asked. *Second,* the teacher will have the opportunity to practice further problem-solving questions where different sets of questions can be used with a familiar Stem. This approach is efficient as students obtain more problem-solving practice on the same underlying content.

Reducing cognitive load (working memory demands) is important in writing a stem. Stems in the lessons are designed to facilitate students reading and interpretation. This is achieved by restricting materials to several sentences and a few paragraphs in length, with no more than one diagram for each item. The teacher could have students read the stems together or individually to assist the development of their fluency with the language used.

In Component 4 students are expected to provide answers using:

- factual knowledge
- application of skills and procedures (fluency)
- understanding
- communicating skills
- reasoning and justification.

Clear feedback to students is very important. Teachers should assist students at a level that they can understand in addressing issues, misconceptions or errors that have arisen.

Lesson Component 5 (Lesson Conclusion)

Component 5 offers a student-focused summary of the lesson intention. **Students** reflect on their progress, achievement, or partial achievement of goals (lesson intention) and their performance and understandings. It takes up comments from Component 2 about teacher expectations. Here teachers can confirm student progress. Honesty is needed, as positive as circumstances permit, including the long-term impact of student effort and persistence.

Component 5 has a high metacognitive aspect for students – thinking about their own thinking – which can be further enhanced by teacher modelling.

Part C: Syllabus References, Matters for Students to Observe, and Worked Answers for the Individual Lessons

Information provided in Part C includes matters that are important for students to observe in the 15 main lessons and three deliberate practice lessons. This information, together with the 'Syllabus Codes' (if applicable) from the **K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016**, is listed, as well as worked answers to the Component 1 and Component 4 questions. This is provided for assistance, as needed, in working through the respective lessons.

Lesson 1: Determining Arithmetic Means, $n\text{th}$ term of an Arithmetic Sequence, and Sum of the Terms of an Arithmetic Sequence

Syllabus Code/s: —

K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016

Quarter: Grade 10 – First Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of sequences, polynomials, and polynomial equations.

Performance Standard

The learner is able to formulate and solve problems involving sequences, polynomials, and polynomial equations in different disciplines through appropriate and accurate representations.

Most Essential Learning Competency

The learner determines arithmetic means, $n\text{th}$ term of an arithmetic sequence, and sum of the terms of a given arithmetic sequence.

Key Idea

Determine arithmetic means, $n\text{th}$ term of an arithmetic sequence, and sum of the terms of a given arithmetic sequence.

Matters for Students to Observe

Students need to:

- know well that an *arithmetic sequence* (also called an *arithmetic progression* or *AP*) is a sequence of numbers in which each number (called a *term*) is generated by adding a fixed number d (called the *common difference*) to the previous number.
- be able to readily determine whether a sequence is arithmetic by understanding that for an arithmetic sequence $(a_1, a_2, a_3, a_4, \ldots)$ that $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$.
- know well that the *general* term (or $n\text{th}$ term) of an arithmetic sequence is given by $a_n = a_1 + (n-1)d$, where a_1 is the first term and d is the common difference.
- know well that if three numbers a, b, c are in arithmetic sequence, then the middle term, b, is called the *arithmetic mean* of a and c, and that $b = \frac{a+c}{2}$.
- know well that the *sum to n terms* of an arithmetic sequence is given by $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where a_1 is the first term and d is the common difference. If the first and last terms are known, the *sum to n terms* of an arithmetic sequence may be found using the formula $S_n = \frac{n}{2}(a_1 + l)$, where a_1 is the first term and l is the last term.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) Arithmetic mean of -7 and $9 = \frac{-7+9}{2}$

$$= 1$$

(ii) Arithmetic mean of $2\sqrt{3} - 3$ and $2\sqrt{3} + 3 = \frac{2\sqrt{3}-3+2\sqrt{3}+3}{2}$

$$= \frac{4\sqrt{3}}{2}$$

$$= 2\sqrt{3}$$

2. (i) Common difference is -5. $\therefore$ next term is $-8 - 5 = -13$

(ii) Using $a_n = a_1 + (n-1)d$, where $a_1 = 2$ and $d = -5$,

$$a_n = 2 + (n-1)(-5)$$

$$= 7 - 5n$$

$$a_{15} = 7 - 5(15)$$

$$= -68$$

3. (i) Using $a_n = 7 - 5n$,

$$-93 = 7 - 5n$$

$$5n = 100$$

$$n = 20$$

$\therefore -93$ is the 20th term.

(ii) Using $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where $a_1 = 2$, $d = -5$, $n = 10$,

$$S_{10} = \frac{10}{2}(2(2) + (10-1)(-5))$$

$$= 5(4 + 9(-5))$$

$$= -205$$

Component 4

Part 4B Item 1

1. Arithmetic mean of 2 000 000 and 2 600 000 $= \frac{2\,000\,000 + 2\,600\,000}{2}$

$$= 2\,300\,000$$

$\therefore$ Marcella was paid ₱2 300 000 in her second year with ABC Enterprises.

2. (i) Using $a_n = a_1 + (n-1)d$, where $a_1 = 2\,000\,000$, $d = 300\,000$, $n = 10$,

$$a_{10} = 2\,000\,000 + (10-1)(300\,000)$$

$$= 4\,700\,000$$

$\therefore$ Marcella would have been paid ₱4 700 000 in her tenth year with ABC Enterprises.

(ii) Using $a_n = a_1 + (n-1)d$, where $a_n = 7\,100\,000$, $a_1 = 2\,000\,000$, $d = 300\,000$,

$$7\,100\,000 = 2\,000\,000 + (n-1)(300\,000)$$

$$\frac{5\,100\,000}{300\,000} = n - 1$$

$$n - 1 = 17 \Rightarrow n = 18$$

$\therefore$ Marcella would have received a salary of ₱7 100 000 in her 18th year with ABC Enterprises.

3. Using $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where $a_1 = 2\ 000\ 000$, $d = 300\ 000,\ n = 10$,

   $$S_{10} = \frac{10}{2}(2(2\ 000\ 000) + (10-1)(300\ 000))$$
   $$= 5(4\ 000\ 000 + 9(300\ 000))$$
   $$= 33\ 500\ 000$$

   ∴ Marcella would have earned a total of ₱33 500 000 if she had remained with ABC Enterprises for ten years.

Part 4C Item 2

1. If 4 arithmetic means are inserted between 2 500 000 and 2 900 000, then 2 500 000, the 4 arithmetic means, and 2 900 000 form an arithmetic sequence of 6 terms.

   For this sequence, $a_1 = 2\ 500\ 000$, $a_1 + 5d = 2\ 900\ 000, \Rightarrow 5d = 400\ 000 \Rightarrow d = 80\ 000$.

   ∴ The 4 arithmetic means between 2 500 000 and 2 900 000 (the salaries (in PHP) that Marcella would have been paid each half year between the first half-year and sixth half-year) are
   $2\ 580\ 000, 2\ 660\ 000, 2\ 740\ 000, 2\ 820\ 000$.

2. (i) Using $a_n = a_1 + (n-1)d$, where $a_1 = 2\ 500\ 000$, $d = 200\ 000,\ n = 15$,

   $$a_{15} = 2\ 500\ 000 + (15-1)(200\ 000)$$
   $$= 5\ 300\ 000$$

   ∴ Marcella will be paid ₱5 300 000 in her 15th year with XYZ Enterprises.

   (ii) Need to find smallest integer value of n for which $a_n > 4\ 000\ 000$.

   i.e., for which $a_1 + (n-1)d > 4\ 000\ 000$, where $a_1 = 2\ 500\ 000$, $d = 200\ 000$,

   Solving $2\ 500\ 000 + (n-1)(200\ 000) > 4\ 000\ 000$,

   $$n - 1 > \frac{4\ 000\ 000 - 2\ 500\ 000}{200\ 000}$$

   $n - 1 > 7.5 \Rightarrow n > 8.5 \Rightarrow n = 9$, since n must be smallest integer greater than 8.5.

   ∴ Marcella will first receive a salary of over ₱4 000 000 In her 9th year with XYZ Enterprises.

3. Using $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where $a_1 = 2\ 000\ 000$, $d = 300\ 000,\ n = 20$,

   Total amount Marcella would have earned after 20 years at ABC Enterprises:

   $$S_{20} = \frac{20}{2}(2(2\ 000\ 000) + (20-1)(300\ 000)$$
   $$= 97\ 000\ 000$$

   ∴ Marcella would have earned ₱97 000 000.

   Total amount Marcella would have earned after 20 years at XYZ Enterprises:

   $$S_{20} = \frac{20}{2}(2(2\ 500\ 000) + (20-1)(200\ 000)$$
   $$= 88\ 000\ 000$$

   ∴ Marcella would have earned ₱88 000 000.

   ∴ Greater amount earned after 20 years at ABC Enterprises= ₱97 000 000 − ₱88 000 000
   = ₱9 000 000

Lesson 2: Determining Geometric Means, $n\text{th}$ term of a Geometric Sequence, and Sum of the Terms of a Finite or Infinite Geometric Sequence

Syllabus Code/s: —

Quarter: Grade 10 – First Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of sequences, polynomials, and polynomial equations.

Performance Standard

The learner is able to formulate and solve problems involving sequences, polynomials, and polynomial equations in different disciplines through appropriate and accurate representations.

Most Essential Learning Competency

The learner determines geometric means, $n\text{th}$ term of a geometric sequence, and sum of the terms of a finite or infinite geometric sequence.

Key Idea

Determine geometric means, $n\text{th}$ term of a geometric sequence, and sum of the terms of a finite or infinite geometric sequence.

Matters for Students to Observe

Students need to:

- know well that a *geometric sequence* (also called a *geometric progression* or *GP*) is a sequence of numbers in which each number (called a *term*) is generated by multiplying a constant number r (called the *common ratio, which is a non-zero number*) by the previous number.

- be able to readily determine whether a sequence is geometric by understanding that for a geometric sequence $(a_1, a_2, a_3, a_4, \ldots)$ that $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} \ldots = \frac{a_n}{a_{n-1}}$.
- know well that the *general* term (or $n\text{th}$ term) of a geometric sequence is given by $a_n = a_1 r^{n-1}$, where a_1 is the first term and r is the common ratio and is not equal to one.
- have a sound understanding that they can solve an equation where the unknown is in the index on one side of the equation by expressing both sides of the equation as powers of the same number.
- know well that if three numbers a, b, c are in geometric sequence, then the middle term, b, is called the *geometric mean* of a and c, and that $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac \Rightarrow b = \pm\sqrt{ac}$.
- know well that the *sum to* n *terms* of a geometric sequence is given by $S_n = \frac{a_1(r^n - 1)}{r-1}$ or $S_n = \frac{a_1(1-r^n)}{1-r}$, where a_1 is the first term and r is the common ratio.
- have clear knowledge that a sequence is *finite* if the number of terms is able to be counted, or *infinite* if the number of terms increases indefinitely.
- know well that the *sum to* $infinity$ (the limiting value of the sum of the terms) of a geometric sequence, where $|r| < 1$, is given by $S_\infty = \frac{a_1}{1-r}$.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. Geometric mean of -4 and $-16 = \pm\sqrt{(-4 \times -16)}$

$$= \pm\sqrt{64}$$

$$= \pm\sqrt{8}$$

2. (i) Common ratio is $\frac{1}{3}$. ∴ next term is $5 \times \frac{1}{3} = \frac{5}{3} (= 1\frac{2}{3})$

(ii) Using $a_n = a_1 r^{n-1}$, where $a_1 = 45$ and $r = \frac{1}{3}$,

$$a_n = 45(\tfrac{1}{3})^{n-1}$$

$$a_6 = 45(\tfrac{1}{3})^{6-1}$$

$$= \frac{5}{27}$$

3. (i) Using $a_n = 45(\frac{1}{3})^{n-1}$,

$$\frac{5}{9} = 45(\tfrac{1}{3})^{n-1}$$

$$\frac{1}{81} = (\tfrac{1}{3})^{n-1}$$

$$(\frac{1}{3})^{n-1} = (\frac{1}{3})^4 \Rightarrow n - 1 = 4 \Rightarrow n = 5$$

∴ $\frac{5}{9}$ is the 5th term.

(ii) Using $S_n = \frac{a_1(1-r^n)}{1-r}$ where $a_1 = 45, r = \frac{1}{3}, n = 5$

$$S_5 = \frac{45(1-(\frac{1}{3})^5)}{1-\frac{1}{3}}$$

$$= \frac{3}{2} \times 45 \times \frac{242}{243}$$

$$= 67\frac{2}{9}$$

Using $S_\infty = \frac{a_1}{1-r}$ where $a_1 = 45$ and $r = \frac{1}{3}$,

$$S_\infty = \frac{45}{1-\frac{1}{3}}$$

$$= \frac{3}{2} \times 45$$

$$= 67\frac{1}{2}$$

Component 4

Part 4B Item 1

1. If 3 geometric means are inserted between 60 and $\frac{15}{4}$, then 60, the 3 geometric means, and $\frac{15}{4}$ represent 5 terms in geometric sequence.

   For this sequence of terms, $a_1 = 60$, $a_1 r^{5-1} = \frac{15}{4}, \Rightarrow r^4 = \frac{1}{16} \Rightarrow r = \frac{1}{2}$.

   ∴ The 3 geometric means between 60 and $\frac{15}{4}$ are 30, 15 and $\frac{15}{2}$ (or $7\frac{1}{2}$).

2. (i) Using $a_n = a_1 r^{n-1}$, where $a_1 = 60$ and $r = \frac{1}{2}$,

   $a_n = 60(\frac{1}{2})^{n-1}$

   (ii) $a_7 = 60(\frac{1}{2})^{7-1}$

   $= 60 \times (\frac{1}{2})^6$

   $= \frac{15}{16}$

3. (i) Using $S_n = \frac{a_1(1-r^n)}{1-r}$ where $a_1 = 60, r = \frac{1}{2}, n = 5$,

   $S_5 = \frac{60(1-(\frac{1}{2})^5)}{1-\frac{1}{2}}$

   $= 2 \times 60 \times \frac{31}{32}$

   $= \frac{465}{4}$ (or $116\frac{1}{4}$)

   (ii) Using $S_\infty = \frac{a_1}{1-r}$ where $a_1 = 60$ and $r = \frac{1}{2}$,

   $S_\infty = \frac{60}{1-\frac{1}{2}}$

   $= 2 \times 60$

   $= 120$

Part 4C Item 2

1. Positive geometric mean of 60 and $\frac{80}{3} = \sqrt{(60 \times \frac{80}{3})}$

$$= \sqrt{1600}$$

$$= 40$$

2. (i) Using $a_n = a_1 r^{n-1}$, where $a_1 = 60$ and $r = \frac{2}{3}$,

$a_n = 60(\frac{2}{3})^{n-1}$

$\therefore$ Formula for the sequence of heights described for Cher's ball (Sequence T) is $a_n = 60(\frac{2}{3})^{n-1}$.

(ii) Using $a_n = 60(\frac{2}{3})^{n-1}$,

$$\frac{320}{27} = 60(\frac{2}{3})^{n-1}$$

$$\frac{16}{81} = (\frac{2}{3})^{n-1}$$

$$(\frac{2}{3})^{n-1} = (\frac{2}{3})^4 \Rightarrow n - 1 = 4 \Rightarrow n = 5$$

$\therefore \frac{320}{27}$ is the 5th term. This means that Cher's ball attains a height of $\frac{320}{27}$ meters on its fourth bounce.

3. (i) Using $S_n = \frac{a_1(1-r^n)}{1-r}$ where $a_1 = 60$ and $r = \frac{2}{3}$,

$$S_5 = \frac{60(1-(\frac{2}{3})^5)}{1-\frac{2}{3}}$$

$$= 3 \times 60 \times \frac{211}{243}$$

$$= \frac{4220}{27} \text{ (or } 156\frac{8}{27})$$

(ii) For Sequence S:

from Part 4B Question 3. (ii), $S_\infty = 120$.

For Sequence T:

using $S_\infty = \frac{a_1}{1-r}$ where $a_1 = 60$ and $r = \frac{2}{3}$,

$$S_\infty = \frac{60}{1-\frac{2}{3}}$$

$$= 3 \times 60$$

$$= 180$$

$$\frac{\text{Limiting sum of Sequence } T}{\text{Limiting sum of Sequence } S} = \frac{180}{120} = 1\frac{1}{2}$$

$\therefore$ Limiting sum of Sequence T is $1\frac{1}{2}$ times greater than the limiting sum of Sequence S.

Lesson 3: Solving Problems involving Sequences

Syllabus Code: M10AL-If-2

Quarter: Grade 10 – First Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of sequences, polynomials, and polynomial equations.

Performance Standard

The learner is able to formulate and solve problems involving sequences, polynomials, and polynomial equations in different disciplines through appropriate and accurate representations.

Most Essential Learning Competency

The learner solves problems involving sequences.

Key Idea

Solve problems involving sequences.

Matters for Students to Observe

Students need to:

- have knowledge of the fact that the arithmetic mean of two positive numbers is greater than or equal to their positive geometric mean.
- be able to readily distinguish arithmetic sequences from geometric sequences by understanding that an arithmetic sequence $(a_1, a_2, a_3, a_4, \ldots)$ has a common difference $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$,whereas a geometric sequence $(a_1, a_2, a_3, a_4, \ldots)$ has a common ratio $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} \ldots = \frac{a_n}{a_{n-1}}$.
- know that the symbols $\cong$ and $\sim$ mean 'is approximately equal to'.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. Arithmetic mean of -3 and $-27 = \frac{-3+(-27)}{2}$

   $= -15$

   Geometric mean of -3 and $-27 = \pm\sqrt{(-3 \times -27)}$

   $= \pm\sqrt{81}$

   $= \pm 9$

2. (i) For Sequence A: $-2, 1, 4, \ldots,$

   using $a_n = a_1 + (n-1)d$, where $a_1 = -2$, $d = 3$, $n = 8$,

   $a_8 = -2 + (8-1)(3)$

   $= 19$

For Sequence B: $2, -1, \frac{1}{2}, \ldots,$

using $a_n = a_1 r^{n-1}$, where $a_1 = 2, r = -\frac{1}{2}, n = 8,$

$a_n = 2(-\frac{1}{2})^{n-1}$

$a_8 = 2(-\frac{1}{2})^{8-1}$

$= 2 \times (-\frac{1}{2})^7$

$= -\frac{1}{64}$

(ii) For Sequence A: $-2, 1, 4, \ldots,$

using $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where $a_1 = -2, d = 3,\ n = 8,$

$S_8 = \frac{8}{2}(2(-2) + (8-1)(3))$

$= 4(-4 + 7(3))$

$= 68$

For Sequence B: $2, -1, \frac{1}{2}, \ldots,$

using $S_n = \frac{a_1(1-r^n)}{1-r}$ where $a_1 = 2, r = -\frac{1}{2}, n = 8,$

$S_8 = \frac{2(1-(-\frac{1}{2})^8)}{1-(-\frac{1}{2})}$

$= 2 \times \frac{255}{256} \times \frac{2}{3}$

$= \frac{85}{64}$ (or $1\frac{21}{64}$)

3. (i) Using $a_n = 2(-\frac{1}{2})^{n-1}$,

$-\frac{1}{16} = 2(-\frac{1}{2})^{n-1}$

$-\frac{1}{32} = (-\frac{1}{2})^{n-1}$

$(-\frac{1}{2})^{n-1} = (-\frac{1}{2})^5 \Rightarrow n - 1 = 5 \Rightarrow n = 6$

$\therefore -\frac{1}{16}$ is the 6th term.

(ii) Using $S_\infty = \frac{a_1}{1-r}$ where $a_1 = 2$ and $r = -\frac{1}{2}$,

$S_\infty = \frac{2}{1-(-\frac{1}{2})}$

$= 2 \times \frac{2}{3}$

$= \frac{4}{3}$

Component 4

Part 4B Item 1

1. (i) Since the number of trees on the plantation increased by a constant number of trees (2500) each year over the first 15 years, the number of trees on the plantation each year formed an arithmetic sequence.

Using $a_n = a_1 + (n-1)d$, where $a_1 = 45\ 000$, $d = 2500$,

$a_n = 45\ 000 + (n-1)2500$

$= 2500n + 42\ 500$

$a_5 = 2500(5) + 42\ 500$

$= 55\ 000$

∴ there were 55 000 trees on the plantation at the beginning of the 5th year of the plantation.

(ii) Using $a_n = 2500n + 42\ 500$,

$65\ 000 = 2500n + 42\ 500$

$2500n = 22\ 500$

$n = 9$

∴ at the beginning of the 9th year of the plantation there were 65 000 trees on the plantation.

2. (i) Using $a_n = 2500n + 42\ 500$

$a_{16} = 2500(16) + 42\ 500$

$= 82\ 500$

∴ there were 82 500 trees on the plantation at the beginning of the 16th year of the plantation.

(ii) Arithmetic mean $x = \frac{a+b}{2}$, where $a = 45\ 000, b = 82\ 500$

$$= \frac{45\ 000 + 82\ 500}{2}$$

$$= 63\ 750$$

3. Using $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where $a_1 = 45\ 000$, $d = 2500,\ n = 16$,

$$S_{16} = \frac{16}{2}(2(45\ 000) + (16-1)(2500))$$

$$= 1\ 020\ 000$$

∴ Total subsidy received= $1\ 020\ 000 \times$ ₱100

$=$ ₱$102\ 000\ 000$

Part 4C Item 2

1. At the beginning of the 16th year of the plantation, it was decided to sell-off each year $40\%(=\frac{2}{5})$ of the trees remaining on the plantation. The number of trees that remained on the plantation at the beginning of the 17th year of the plantation (i.e., one year after beginning to sell-off trees) was equal to $\left(82\,500-\frac{2}{5}(82\,500\right)=\frac{3}{5}(82\,500)=49\,500$.

2. (i) The number of trees remaining on the plantation at the beginning of the 16th year of the plantation and then at the beginning of each subsequent year ($100\%-40\%=60\%=\frac{3}{5}$ of the trees on the plantation at the beginning of each previous year) formed a geometric sequence, with first term $49\,500$ and common ratio $\frac{3}{5}$.

   Using $a_n=a_1r^{n-1}$, where $a_1=49\,500, r=\frac{3}{5}, n=5$,

   $a_5=49\,500(\frac{3}{5})^{5-1}$

   $a_5\cong 6415$

   ∴ The number of trees that remained on the plantation at the beginning of the 21st year of the plantation was approximately 6415.

   (ii) Using $a_n=a_1(r)^{n-1}$, where $a_1=49\,500, r=\frac{3}{5}, n=10$,

   $a_9=49\,500(\frac{3}{5})^{10-1}$

   $\cong 499\ (<500)$

   ∴ there were less than 500 trees left on the plantation at the beginning of the 10th year after beginning to sell-off trees.

3. Limiting amount (S_∞) of subsidy $=\frac{49\,500}{1-0.6}\times 100$ PHP (using $S_\infty=\frac{a_1}{1-r}$, where $a_1=49\,500, r=0.6$).

   $=123\,750\ \times 100$ PHP
   $=12\,375\,000$ PHP.

∴ Limiting amount of government subsidy that the plantation could receive after beginning to sell off trees is $12\,375\,000$ PHP.

Lesson 4: Solving Problems involving Polynomials and Polynomial Equations

Syllabus Code: M10AL-Ij-2

Quarter: Grade 10 – First Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of sequences, polynomials, and polynomial equations.

Performance Standard

The learner is able to formulate and solve problems involving sequences, polynomials and polynomial equations in different disciplines through appropriate and accurate representations.

Most Essential Learning Competency

The learner solves problems involving polynomials and polynomial equations.

Key Idea

Solve problems involving polynomials and polynomial equations.

Matters for Students to Observe

Students need to:

- have ready recall of the formula $V = l \times b \times h$, where $l = \text{length}, b = \text{breadth}, h = \text{height}$, for determining the volume of a rectangular prism.
- know well how to express the relevant units for length, area, and volume.
- know well, and be able to explain, the meaning of the terms polynomial, polynomial equation, binomial, trinomial, quadratic, expand, factorize, substitute into, long division.
- develop fluency in obtaining products of algebraic expressions.
- develop fluency in substituting values into algebraic expressions.
- develop fluency in factorizing quadratic trinomials.
- have a good understanding and associated skill in performing long division with polynomials.
- develop a sound ability to assess the validity, or otherwise, of particular algebraic solutions in practical contexts.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. Volume of rectangular prism $= (x + 5)(x + 2)(x)$

$$= x(x^2 + 7x + 10)$$

$$= (x^3 + 7x^2 + 10x) \text{ cubic meters}$$

2. (i) Height of boxes$= (x + 6)$ meters and Base area of boxes$= (2x^2 + x - 1)$ square meters

When $x = 2$,

Height of boxes$= (2 + 6)$ meters and Base area of boxes $= (2(2)^2 + 2 - 1)$ square meters

$= 8$ meters $= 9$ square meters.

(ii) Volume of box=height of box × base area of box

$$= (x + 6) \times (2x^2 + x - 1)$$

Volume of box for which $x = -4$

$$= (-4 + 6)(2(-4)^2 + (-4) - 1) \text{ cubic meters.}$$

$$= (2)(27)$$

$$= 54 \text{ cubic meters.}$$

3. Solving,

$2x^2 + x - 1 = 5$

$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$x = -2$ or $1\frac{1}{2}$

Component 4

Part 4B Item 1

1. (i) Base area $= \frac{\text{Volume of storage spaces}}{\text{Height of storage spaces}}$

$$= \frac{6x^3 - 13x^2 + x + 2}{2x - 1}$$

$$2x - 1\overset{3x^2 - 5x - 2}{\overline{)6x^3 - 13x^2 + x + 2}}$$

(by 'long division' of polynomials)

Base area $= (3x^2 - 5x - 2)$ square meters

(ii) From Question 1. (i) Volume in cubic meters $= 6x^3 - 13x^2 + x + 2$

$$= (2x - 1)(3x^2 - 5x - 2)$$

$= (2x - 1)(3x + 1)(x - 2)$ on factorizing $3x^2 - 5x - 2$.

2. (i) It is given that the Set A storage spaces have a height of $(2x - 1)$ meters and a volume of $(6x^3 - 13x^2 + x + 2)$ cubic meters. From Question 1. (i), the base area of the storage spaces is $(3x^2 - 5x - 2)$ square meters.

$\therefore$ When $x = 3$, storage space in Set A has:

Height $= 2(3) - 1 = 5$ meters; Base area $= 3(3)^2 - 5(3) - 2 = 10$ square meters;

Volume = height × base area $= 5 \times 10 = 50$ cubic meters

(ii) The height (h) of the storage spaces in Set A is given by $h = 2x - 1$. On solving $2x - 1 \leq 0$, we obtain $x \leq \frac{1}{2}$. Therefore, if $x \leq \frac{1}{2}$ we would obtain a zero or negative height for the storage space.

3. (i) Since $3x^2 - 7x - 6 = 14$

$3x^2 - 7x - 20 = 0$

$(3x + 5)(x - 4) = 0$

$(3x + 5) = 0$ or $(x - 4) = 0$

$x = -\frac{5}{3}$ or $x = 4$

However, $x = -\frac{5}{3}$ is not a valid solution, since for $x = -\frac{5}{3}$, we would obtain a negative height for the storage space. $\therefore x = 4$ is only solution.

(ii) Height $= 2x - 1 = 2(4) - 1 = 7$ meters

Base area$=$ 14 square meters

$\therefore$ Volume $= 7 \times 14 = 98$ square meters

Part 4C Item 2

1. Volume $=$ height $\times$ base area

$$= (3x - 2)(2x^2 + x - 3)$$

$$= (6x^3 - x^2 - 11x + 6) \text{ cubic meters}$$

2. (i) Base area $= 2x^2 + x - 3$

$$= (2x + 3)(x - 1)$$

(ii) Height $=$ 16

$\therefore 3x - 2 = 16$

$3x = 18$

$x = 6$

Base area $= (2x^2 + x - 3) = 2(6)^2 + 6 - 3) = 75$ square meters

Volume $=$ height $\times$ base area $= 16 \times 75 = 1200$ cubic meters

3. (i) Polynomial equation:

$$(2x^2 + x - 3) + 6 = 3x^2 - 7x - 6$$

$$0 = x^2 - 8x - 9$$

$x + 1 = 0$ or $x - 9 = 0$

$\therefore$ $x = -1$ or $x = 9$

However, $x = -1$ is not a valid solution, since for $x = -1$, we would obtain a negative height for the storage space. $\therefore$ $x = 9$ is only solution.

(ii) Height $= 2x - 1 = 2(9) - 1 = 17$ meters

Base area$=$ $3x^2 - 7x - 6 = 3(9)^2 - 7(9) - 6) = 174$ square meters

$\therefore$ Volume $= 17 \times 174 = 2958$ cubic meters

Lesson 5: Solving Problems involving Polynomial Functions

Syllabus Code: M10AL-IIb-2

Quarter: Grade 10 – Second Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of polynomial functions.

Performance Standard

The learner is able to conduct systematically a mathematical investigation involving polynomial functions in different fields.

Most Essential Learning Competency

The learner solves problems involving polynomial functions.

Key Idea

Solve problems involving polynomial functions.

Matters for Students to Observe

Students need to:

- develop fluency in how to solve quadratic equations.
- have a sound understanding of the term 'axis of symmetry' and how to find the equation of the axis of symmetry of a parabola.
- know the importance of the relationship between the equation of the axis of symmetry and the minimum or maximum value of a quadratic function, and how to find the minimum or maximum value.
- have a sound understanding of how to find the coordinates of the points where parabolas meet/cut the x-axis and the y-axis.
- know, and be able to explain the meanings of, the terms 'profit', 'revenue', and 'break-even'.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) $W(\mathrm{x}) = U(\mathrm{x}) - V(\mathrm{x})$

$$= -10 + 7x - \frac{1}{3}x^2 - (4x + 25)$$

$$= -10 + 7x - \frac{1}{3}x^2 - 4x - 25$$

$$= -35 + 3x - \frac{1}{3}x^2$$

(ii) $-8 + 5x - \frac{1}{2}x^2 = 0$

i.e., $x^2 - 10x + 16 = 0$

$(x-2)(x-8) = 0$

$x = 2$ or $x = 8$

2. Equation of axis of symmetry is given by $x = -\frac{b}{2a}$.

   For $A(x) = -12 + 4x - \frac{1}{4}x^2$, $a = -\frac{1}{4}, b = 4$.

   ∴ Axis of symmetry: $x = -\frac{4}{2\times-\frac{1}{4}} = 8$

3. (i) The polynomial function $Q(x) = -10 + 3x - \frac{1}{5}x^2$ is negative (i.e., $Q(x) < 0$) for the x-values of all points on its graph below the x-axis. For an inverted parabola, these x-values lie on the x-axis 'outside' of the parabola or 'outside' of where the parabola cuts the x-axis (at $(5, 0)$ and $(10, 0)$).

   ∴ The required inequalities are $x < 5$ or $x > 10$.

   (ii) Equation of axis of symmetry is given by $x = -\frac{b}{2a}$.

   For $Q(x) = -10 + 3x - \frac{1}{5}x^2$, $a = -\frac{1}{5}, b = 3$.

   ∴ Axis of symmetry of $Q(x)$: $x = -\frac{3}{2\times-\frac{1}{5}} = \frac{15}{2} = 7\frac{1}{2}$

   Maximum value of $Q(x)$ occurs on the axis of symmetry i.e., where $x = 7\frac{1}{2}$.

   ∴ Maximum value of $Q(x) = -10 + 3\left(\frac{15}{2}\right) - \frac{1}{5}\left(\frac{15}{2}\right)^2 = -10 + \frac{45}{2} - \frac{45}{4} = 1\frac{1}{4}$

Component 4
Part 4B Item 1

1. (i) $P_1(x) = R_1(x) - C_1(x)$

$$= 6x - \frac{1}{10}x^2 - (2x + 30)$$

$$= 6x - \frac{1}{10}x^2 - 2x - 30$$

$$= -30 + 4x - \frac{1}{10}x^2$$

   (ii) If $P_1(x) = 0$, then from Part 4B Question 1. (i),

$$-30 + 4x - \frac{1}{10}x^2 = 0$$

   i.e., $x^2 - 40x + 300 = 0$

   $(x - 10)(x - 30) = 0$

   $x = 10$ or $x = 30$

   ∴ 'Break-even' production for GizmoCo in 2022 would have been 10 000 or 30 000 gizmos.

2. (i) Maximum possible profit each month in 2022 occurs when $x = 20$.

   $P_1(20) = -30 + 4(20) - \frac{1}{10}(20)^2$

   $= 10$

   i.e., 10 000 U.S. dollars each month

(ii) The polynomial function $P_1(x) = -30 + 4x - \frac{1}{10}x^2$ is negative (i.e., $P_1(x) < 0$) for the x-values of all points on its graph below the x-axis. For an inverted parabola, these x-values lie on the x-axis 'outside' of the parabola or 'outside' of where the parabola cuts the x-axis (at $(10, 0)$ and $(30, 0)$). (Note solutions of $P_1(x) = 0$ in answer for Part 4B Question 1. (ii).)

∴ The required inequalities are $x < 10$ or $x > 30$.

3. (i) $P_2(x) = R_2(x) - C_2(x)$

$$= 6x - \frac{1}{10}x^2 - (x + 60)$$

$$= 6x - \frac{1}{10}x^2 - x - 60$$

$$= -60 + 5x - \frac{1}{10}x^2$$

(ii) From Part 4B Question 2. (i), the maximum possible profit each month in 2022 occurred when $x = 20$.

Now $P_2(20) = -60 + 5(20) - \frac{1}{10}(20)^2$

$$= -60 + 100 - 40$$

$$= 0$$

∴ GizmoCo would have only broken even each month in 2023.

Part 4C Item 2

1. (i) GizmoCo's 2022 monthly profit is given by $P_1(x) = -30 + 4x - \frac{1}{10}x^2$.

If producing 5000 gizmos, $x = 5$.

$P_1(5) = -30 + 4(5) - \frac{1}{10}(5)^2$

$= -12.5$ i.e., a loss of 12 500 U.S. dollars

GizmoCo's 2023 monthly profit is given by $P_2(x) = -60 + 5x - \frac{1}{10}x^2$

If producing 5000 gizmos, $x = 5$.

$P_2(5) = -60 + 5(5) - \frac{1}{10}(5)^2$

$= -37.5$ i.e., a loss of 37 500 U.S. dollars

∴ GizmoCo would have made the greater loss by producing 5000 gizmos only in 2023.

(ii) From Part 4B Question 1. (ii), 'break-even' production for GizmoCo in 2022 would have been 10 000 (occurring when $x = 10$) or 30 000 (occurring when $x = 30$) gizmos.

From the symmetry of the polynomial function $P_1(x)$, the equivalent loss to that occurring when $x = 10 - 5 = 5$, occurs when $x = 30 + 5$ i.e., when $x = 35$.

∴The production of 35 000 gizmos would have produced the same loss as for the production of 5000 gizmos.

2. (i) If $P_2(x) = 0$, then from Part 4B Question 3. (i),

$$-60 + 5x - \frac{1}{10}x^2 = 0$$

i.e., $x^2 - 50x + 600 = 0$

$(x - 20)(x - 30) = 0$

$x = 20$ or $x = 30$

∴ 'Break-even' production for GizmoCo in 2023 would have been 20 000 or 30 000 gizmos.

(ii) The polynomial function $P_2(x) = -60 + 5x - \frac{1}{10}x^2$ is positive (i.e., $P_2(x) > 0$) for the x-values of all points on its graph above the x-axis. For an inverted parabola, these x-values lie on the x-axis 'inside' the parabola or 'inside' where the parabola cuts the x-axis (at $(20, 0)$ and $(30, 0)$). (Note solutions of $P_2(x) = 0$ in answer for Part 4C Question 2. (i)}.

∴ The required inequality is $20 < x < 30$, which represents the values of x for which the number of gizmos would have resulted in a profit for GizmoCo in 2023.

3. (i) The graph of the polynomial function $P_2(x)$ is an inverted parabola with axis of symmetry given by $x = -\frac{\text{coefficient of } x}{2\times\text{coefficient of } x^2} = -\frac{5}{2\times-\frac{1}{10}} = 25.$

∴ $x = 25$ is the value of x that would have maximised GizmoCo's profit in 2023.

(ii) From Part 4B Question 2.(i), GizmoCo's maximum possible profit each month in 2022 was 10 000 U.S. dollars.

From Part 4C Question 3. (i), GizmoCo's maximum possible profit each month in 2023 occurred when $x = 25$.

$$P_2(25) = -60 + 5(25) - \frac{1}{10}(25)^2$$

$$= 2.5$$

i.e., 2500 U.S. dollars each month.

∴ Maximum possible profit in 2023 is $(10\,000 - 2500) = 7500$ U.S. dollars less than in 2022.

Lesson 6 Deliberate Practice: Solving Problems involving Sequences, Solving Problems involving Polynomials and Polynomial Equations, Solving Problems involving Polynomial Functions

Syllabus Codes: M10AL-If-2, M10AL-Ij-2, M10AL-IIb-2

Quarter: Grade 10 – First Quarter; Second Quarter

Content Section: Patterns and Algebra

Content Standards

The learner demonstrates understanding of key concepts of sequences, polynomials, and polynomial equations.

The learner demonstrates understanding of key concepts of polynomial functions.

Performance Standard

The learner is able to formulate and solve problems involving sequences, polynomials, and polynomial equations in different disciplines through appropriate and accurate representations.

The learner is able to conduct systematically a mathematical investigation involving polynomial functions in different fields.

Most Essential Learning Competencies

The learner solves problems involving sequences.

The learner solves problems involving polynomials and polynomial equations.

The learner solves problems involving polynomial functions.

Key Ideas

Solve problems involving sequences.

Solve problems involving polynomials and polynomial equations.

Solve problems involving polynomial functions.

Matters for Students to Observe

(see above for notes for Lessons 1–5 and relevant to this lesson)

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) For Sequence A: $-1, 3, 7, \ldots,$

using $a_n = a_1 + (n-1)d$, where $a_1 = -1$, $d = 4$,

$a_n = -1 + (n-1)4$

$a_n = 4n - 5$

Using $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, where $a_1 = -1$, $d = 4$, $n = 6$,

$$S_6 = \frac{6}{2}(2(-1) + (6-1)(4))$$
$$= 3(18)$$
$$= 54$$

(ii) For Sequence B: $3, -1, \frac{1}{3}, \ldots,$

using $S_n = \frac{a_1(1-r^n)}{1-r}$ where $a_1 = 3, r = -\frac{1}{3}, n = 4$,

$$S_4 = \frac{3(1-(-\frac{1}{3})^4)}{1-(-\frac{1}{3})}$$
$$= 3 \times \frac{80}{81} \times \frac{3}{4}$$
$$= \frac{20}{9} \text{ (or } 2\frac{2}{9})$$

Using $S_\infty = \frac{a_1}{1-r}$ where $a_1 = 3, r = -\frac{1}{3}$,

$$S_\infty = \frac{3}{1-(-\frac{1}{3})}$$
$$= 3 \times \frac{3}{4}$$
$$= \frac{9}{4} \text{ (or } 2\frac{1}{4})$$

2. $P(x) = -9 + 4x - \frac{1}{3}x^2$

If $P(x) = 0, -9 + 4x - \frac{1}{3}x^2 = 0$.

Multiplying each term by -3: $x^2 - 12x + 27 = 0$.

Solving this equation, $(x-3)(x-9) = 0$

$x = 3$ or $x = 9$.

3. (i) The polynomial function $Q(x)$ is 0 or negative (i.e., $Q(x) \leq 0$) for the x-values of all points on its graph on or below the x-axis. For an inverted parabola, these x-values lie on the x-axis or 'outside' of the parabola i.e., 'outside' of where the parabola cuts the x-axis (for $Q(x)$ at $(4, 0)$ and $(10, 0)$).

∴ The required inequalities are $x \leq 4$ or $x \geq 10$.

(ii) Since $\mathrm{Q}(x)$ has vertex $(7, 9)$, maximum value of $P(x) = 9$.

Component 4

Part 4B Item 1

1. From the formula for the nth term of an arithmetic sequence, $a_n = a_1 + (n-1)d$, formula for finding the cost of constructing a particular storey of *Prism* is $a_n = 60\ 000 + 10\ 000(n-1)$.

   ∴ Cost of constructing the 15th storey (a_{15}). $= 60\ 000 + 10\ 000(15-1)$

   $= 200\ 000$ euros.

2. (i) Using $a_n = a_1 r^{n-1}$, where $a_1 = 40, r = \frac{3}{5}$,

   $a_n = 40(\frac{3}{5})^{n-1}$

   (ii) Using $a_n = 40(\frac{3}{5})^{n-1}$,

   $a_5 = 40(\frac{3}{5})^{5-1}$

   $= 40(\frac{3}{5})^4$

   $$= 40 \times \frac{81}{625} = \frac{648}{125} = 5\frac{23}{125}$$

   ∴ Jakes's ball attains a height of $5\frac{23}{125}$ meters on its 4th bounce.

3. (i) $P(x) = R(x) - C(x)$

   $$= 11x - \frac{1}{4}x^2 - (2x + 45)$$

   $$= 11x - \frac{1}{4}x^2 - 2x - 45$$

   $$= -45 + 9x - \frac{1}{4}x^2$$

   (ii) If $P(x) = 0$, then from Part 4B Question 3. (i),

   $$-45 + 9x - \frac{1}{4}x^2 = 0$$

   i.e., $x^2 - 36x + 180 = 0$

   $(x-6)(x-30) = 0$

   $x = 6$ or $x = 30$

   ∴ 'Break-even' production for The Whatsit Company in 2023 would have been 6000 or 30 000 doovalackies.

Part 4C Item 2

1. From the formula for the sum of n terms of an arithmetic sequence, $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, formula for finding the cost of constructing building is $S_n = \frac{n}{2}(2(60\ 000) + 10\ 000(n-1))$.

   Cost of constructing the building (S_{20}). $= \frac{20}{2}(2(60\ 000) + 10\ 000(20-1))$

   $$= 3\ 100\ 000 \text{ euros}$$

2. (i) Using $S_n = \frac{a_1(1-r^n)}{1-r}$ where $a_1 = 40, r = \frac{3}{5}, n = 5,$

   $S_5 = \frac{40(1-(\frac{3}{5})^5)}{1-\frac{3}{5}}$

   $= \frac{11\ 528}{125}$ (or $92\frac{28}{125}$)

   ∴ the sum of the heights (including its original height) attained by Jake's ball after its first four bounces is $92\frac{28}{125}$ meters.

   (ii) Using $S_\infty = \frac{a_1}{1-r}$ where $a_1 = 40, r = \frac{3}{5},$

   $S_\infty = \frac{40}{1-(\frac{3}{5})}$

   $= 40 \times \frac{5}{2}$

   $= 100$

   ∴ the limiting sum (or sum to infinity S_∞) of the sequence of heights of Jake's ball is 100 meters.

3. (i) From Part 4B Question 3. (i), $P(x) = -45 + 9x - \frac{1}{4}x^2$.

   Maximum possible profit each month in 2023 occurs when $x = 18$.

   $P(18) = -45 + 9(18) - \frac{1}{4}(18)^2 = 36$

   i.e., 36 000 euros each month

   (ii) The polynomial function $P(x) = -45 + 9x - \frac{1}{4}x^2$ is negative (i.e., $P(x) < 0$) for the x-values of all points on its graph below the x-axis. For an inverted parabola, these x-values lie on the x-axis 'outside' of the parabola or 'outside' of where the parabola cuts the x-axis (at $(6, 0)$ and $(30, 0)$). (Note solutions of $P(x) = 0$ in answer for Part 4B Question 3. (ii).)

   ∴ The required inequalities are $x < 6$ or $x > 30$.

Lesson 7: Solving Problems involving Circles

Syllabus Code: M10GE-IIf-2

Quarter: Grade 10 – Second Quarter

Content Section: Geometry

Content Standard

The learner demonstrates understanding of key concepts of circles and coordinate geometry.

Performance Standard

The learner is able to:

1. formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.
2. formulate and solve problems involving geometric figures on the rectangular coordinate plane with perseverance and accuracy.

Most Essential Learning Competency

The learner solves problems involving circles.

Key Idea

Solve problems involving circles.

Matters for Students to Observe

Students need to:

- have sound knowledge and ready recall of geometric theorems and results.
- know well the parts of a circle and their geometric relationships.
- have ready recall of the circumference and area formulae for a circle.
- know the terms and be able to explain the meaning of product, intercept, subtended, arc, minor sector, major sector, quadrant.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. An angle of 170 degrees forms a *complete revolution* with an angle of 190 degrees.

   The angle at the center of a circle is twice the angle at the circumference standing on the same arc.

2. (i) Circumference of circle with radius 20 meters $= 2 \times \pi \times 20$ (using $C = 2\pi r$)

   $= 40\pi$ meters

   $\therefore$ Arc length of quadrant of circle with radius 20 meters$= \frac{1}{4} \times 40\pi = 10\pi$ meters.

   Area of circle with radius 20 meters $= \pi \times 20 \times 20$ (using $A = \pi r^2$)

   $= 400\pi$ square meters

   $\therefore$ Area of quadrant of circle with radius 20 meters$= \frac{1}{4} \times 400\pi = 100\pi$ square meters.

   (ii) When two chords intersect within a circle, the products of the intercepts are equal.

3. (i) The product of the intercepts on a secant from an external point is equal to the square of the tangent from that point.

   Tangents to a circle from an external point have equal length.

   (ii) A tangent from an external point meets a circle at right angles to the radius drawn to the point of contact.

If a tangent of length 15 meters from an external point T meets a circle with center O and radius 8 meters at the point P on its circumference, by Pythagoras' theorem in triangle TPO:

$$OT^2 = OP^2 + PT^2$$

$$= 8^2 + 15^2$$

$$= 289$$

$\therefore OT = \sqrt{289} = 17$ meters

Component 4

Part 4B Item 1

1. (i) Central reflex angle $\angle EOF = 360°$ (angle of complete revolution)$-72° \ = \ 288°$

   (ii) Using 'The angle at the center of a circle is twice the angle at the circumference standing on the same arc.',

   $\angle EOF = 2 \times \angle EDF$

   $\frac{72°}{2} = \angle EDF$

   $\therefore \angle EDF = 36°$

2. (i) The circular park area has a circumference of 100π meters and an area of 2500π square meters.

   The angle at the center of the circle subtended by arc $EF = 72° = \frac{1}{5} \times 360°$

   $\therefore$ Arc $EF = \frac{1}{5} \times$ length of circumference of circle

   $= \frac{1}{5} \times 100\pi = 20\pi$ meters

   The sector angle of minor sector $EOF = 72° = \frac{1}{5} \times 360°$

   $\therefore$ Area of the minor sector $EOF = \frac{1}{5} \times$ area of circle

   $= \frac{1}{5} \times 2500\pi = 500\pi$ square meters

   (ii) Using 'When two chords intersect within a circle, the products of the intercepts are equal.',

   $DK \times \ FK = \ EK \times KJ$

   $DK \times \ 6 = 12 \times 8$

   $$DK = \frac{96}{6} = 16$$

   $\therefore$ Length of pathway section $DK = 16$ meters

3. $AF^2 = AE \times AJ$ (The product of the intercepts on a secant from an external point is equal to the square of the tangent from that point.)

   $\therefore AF^2 = 170 \times 150$

   $= \ 25\,500$

   $\therefore AF = \sqrt{25\,500}$

   $= 160$ meters (to nearest meter)

Part 4C Item 2

1. (i) $\angle DEO + \angle DFO + \angle EOF\ (\text{reflex}) + \angle EDF = 360°$ (angle sum of quadrilateral $EDFO$)

   $2\angle DEO + 288° + 36° = 360°$ (since $\angle DEO = \angle DFO$, $\angle EOF$ (reflex)$= 288°$ from Part 4B Question 1. (i) and $\angle EDF = 36°$ from Part 4B Question 1. (ii)).

   $\therefore 2\angle DEO = 36° \Rightarrow \angle DEO = 18° = \angle DFO$

   (ii) The circular park area has a circumference of 100π meters and an area of 2500π square meters.

   The angle at the center of the circle subtended by arc $EDF = 288° = \frac{4}{5} \times 360°$

   $\therefore$ Arc $EDF = \frac{4}{5} \times$ length of circumference of circle

   $= \frac{4}{5} \times 100\pi = 80\pi$ meters

   The sector angle of major sector $EOF = 288° = \frac{4}{5} \times 360°$

   Area of major sector $EOF = \frac{4}{5} \times$ area of circle

   $= \frac{4}{5} \times 2500\pi = 2000\pi$ square meters

2. (i) $AD = AF$ since 'Tangents to a circle from an external point have equal length.'

   ∴Triangle ADF has two equal sides AD and AF, which means that the triangle is an isosceles triangle.

   (ii) $\angle AFO = 90°$ since 'a tangent from an external point meets a circle at right angles to the radius drawn to the point of contact'.

   $\angle AFD = \angle AFO - \angle DFO = 90° - 18° = 72°$ (*)

   Since triangle ADF is isosceles (see Part 4C Question 2. (i)),

   $\angle ADF = \angle AFD = 72°$ (from (from *)

   $\angle DAF = 180° - (\angle AFD + \angle ADF)$ (angle sum of triangle ADF)

   $= 180° - (72° + 72°)$

   $= 36°$

3. Using Pythagoras' theorem in triangle AFO, which is right-angled because 'A tangent from an external point meets a circle at right angles to the radius drawn to the point of contact'.

   $OA^2 = AF^2 + OF^2$

   $= 25\,500 + 50^2$ (Note that $AF^2 = 25\,500$ from Part 4B Question 3.)

   $= 28\,000$

   $\therefore OA = \sqrt{28\,000}$

   Now, $AN = OA + ON$ (radius of circle)

   $= \sqrt{28\,000} + 50$

   $= 217$ meters (to nearest meter)

Lesson 8: Determining the Center and Radius of a Circle given its Equation, and vice versa

Syllabus Code: M10GE-IIh-2

Quarter: Grade 10 – Second Quarter

Content Section: Geometry

Content Standard

The learner demonstrates understanding of key concepts of circles and coordinate geometry.

Performance Standard

The learner is able to:

1. formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.
2. formulate and solve problems involving geometric figures on the rectangular coordinate plane with perseverance and accuracy.

Most Essential Learning Competency

The learner determines the center and radius of a circle given its equation, and vice versa.

Key Idea

Determine the center and radius of a circle given its equation, and vice versa.

Matters for Students to Observe

Students need to:

- have clear knowledge of the difference between center-radius form and general form of the equation of a circle.
- have a sound understanding of what taking a multiple of the radius of a circle means for the resulting area.
- know well that on the x-axis of the coordinate plane that $y = 0$ and on the y-axis that $x = 0$.
- know, and be able to explain the meanings of, the terms center-radius form, general form, concentric circles, and completing the square.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) Circles with equation of the form $x^2 + y^2 = r^2$ are centered on the origin with radius r.

   $\therefore$ the circle $x^2 + y^2 = 4$ has center $(0, 0)$ and radius 2 units.

   (ii) Circle with the same center and twice the radius of $x^2 + y^2 = 4$ is of form $x^2 + y^2 = r^2$ with radius $2 \times 2 = 4$ units.

   $\therefore$ equation of circle is $x^2 + y^2 = 4^2$ i.e., $x^2 + y^2 = 16$.

2. (i) Circle $x^2 + y^2 = 9$ is centered on the origin and has radius $\sqrt{9} = 3$ units.

   $\therefore$ the circle crosses the x-axis at $(-3, 0)$ and $(3, 0)$, and the y-axis at $(0, -3)$ and $(0, 3)$.

   (ii) Circle $(x + 1)^2 + y^2 = \frac{9}{4}$ has center $(-1, 0)$ and radius $\sqrt{\frac{9}{4}} = \frac{3}{2}$ units.

   $\therefore$ the circle crosses the x-axis at $(-1 - \frac{3}{2}, 0)$ and $\left(-1 + \frac{3}{2}, 0\right)$ i.e., $(-\frac{5}{2}, 0)$ and $\left(\frac{1}{2}, 0\right)$

3. (i) $(x+1)^2+y^2=\frac{9}{4} \Rightarrow x^2+2x+1+y^2=\frac{9}{4}$ (on expanding)

$\Rightarrow 4x^2+8x+4+4y^2=9$ (on multiplying each term by 4)

$\Rightarrow 4x^2+4y^2+8x-5=0$

$\therefore (x+1)^2+y^2=\frac{9}{4}$ (center-radius form) written in general form is $4x^2+4y^2+8x-5=0.$

(ii) On substitution of $x=-1, y=1\frac{1}{2}, LHS=4(-1)^2+4\left(\frac{3}{2}\right)^2+8(-1)-5=0=RHS$

$\therefore$ $(-1, 1\frac{1}{2})$ lies on the circle.

Component 4

Part 4B Item 1

1. (i) Circles with equation of the form $x^2+y^2=r^2$ are centered on the origin with radius r.

Circle A: $x^2+y^2=6\frac{1}{4}$ has center $(0,0)$ and radius $\sqrt{6\frac{1}{4}}=\sqrt{\frac{25}{4}}=\frac{5}{2}=2\frac{1}{2}$ units.

(ii) Circle B has center $(0,0)$ and radius $4\times 2\frac{1}{2}=10$ units.

$\therefore$ Equation of Circle B is $x^2+y^2=10^2=100$

2. (i) Circle C has center $(-\frac{3}{4}, 0)$ and radius $\frac{1}{2}\times\frac{5}{2}=\frac{5}{4}$ units.

$\therefore$ Equation of Circle C in center-radius form is $(x-(-\frac{3}{4}))^2+(y-0)^2=(\frac{5}{4})^2$

i.e., $(x+\frac{3}{4})^2+y^2=\frac{25}{16}$

(ii) Circle C crosses the y-axis when $x=0$.

i.e., when $(0+\frac{3}{4})^2+y^2=\frac{25}{16}$

Solving, $y^2=\frac{25}{16}-\frac{9}{16}\text{-}=\frac{16}{16}=1$

$\therefore y=\pm 1$

Circle C crosses y-axis at $(0,-1)$ and $(0,1)$.

3. (i) Circle D: equation is $(x-(-2))^2+(y-5)^2=\left(\sqrt{7}+1\right)^2=7+2\sqrt{7}+1=8+2\sqrt{7}$

$\therefore$ Circle D has equation $(x+2)^2+(y-5)^2=8+2\sqrt{7}$

If Circle E is nine times the area of Circle D, it must have three times the radius of Circle D.

$\therefore$ radius of Circle E is $3\times\left(\sqrt{7}+1\right)=(3\sqrt{7}+3)$ cm

$\therefore$ Circle E has equation $(x+2)^2+(y-5)^2=\left(3\sqrt{7}+3\right)^2=63+18\sqrt{7}+9=72+18\sqrt{7}$

i.e., $(x+2)^2+(y-5)^2=72+18\sqrt{7}$

(ii) Circle F: $x^2+y^2-6x+2y-15=0$

Completing squares, $x^2-6x+\left(-\frac{6}{2}\right)^2+y^2+2y+\left(\frac{2}{2}\right)^2=15+\left(-\frac{6}{2}\right)^2+\left(\frac{2}{2}\right)^2$

i.e., $(x-3)^2+(y+1)^2=25$

$\therefore$ center is $(3,-1)$; radius is 5

Part 4C Item 2

1. (i) Circle $G: 5x^2 + 5y^2 - 45 = 0 \Rightarrow x^2 + y^2 - 9 = 0$ (on dividing each term by 5) $\Rightarrow x^2 + y^2 = 9$

   $\therefore$ Circle G has center $(0, 0)$ and radius$= \sqrt{9} = 3$.

   (ii) Circle $H: 16(x + 1)^2 + 16(y - 5)^2 = 81 \Rightarrow (x + 1)^2 + (y - 5)^2 = \frac{81}{16}$ (on dividing each term by 16)

   $\therefore$ Circle H has center $(-1, 5)$ and radius$= \sqrt{\frac{81}{16}} = \frac{9}{4}$.

2. (i) Circle I: Circle concentric with $(x - 7)^2 + (y + 24)^2 = 625$ (Circle J) and one twenty-fifth its area (and therefore one fifth its radius) has center $(7, -24)$ and radius $\frac{1}{5} \times$ radius of Circle $J = \frac{1}{5} \times \sqrt{625} = \frac{1}{5} \times 25 = 5$

   $\therefore$ Circle I has equation $(x - 7)^2 + (y + 24)^2 = 25$

   (ii) Circle J, $(x - 7)^2 + (y + 24)^2 = 625$, crosses the x-axis when $y = 0$.

   i.e., when $(x - 7)^2 + (0 + 24)^2 = 625$

   Solving,$(x - 7)^2 = 625 - 576 \Rightarrow (x - 7)^2 = 49 \Rightarrow x - 7 = \pm 7 \Rightarrow x = 0$ or 14.

   Circle J crosses x-axis at $(0, 0)$ and $(14, 0)$.

3. Circle K has equation $(x - \left(-\frac{1}{2}\right))^2 + \left(y - \frac{7}{2}\right)^2 = (\frac{11}{2})^2 \Rightarrow (x + \frac{1}{2})^2 + \left(y - \frac{7}{2}\right)^2 = \frac{121}{4} \Rightarrow$

   $x^2 + x + \frac{1}{4} + y^2 - 7y + \frac{49}{4} = \frac{121}{4} \Rightarrow 4x^2 + 4y^2 + 4x - 28y - 71 = 0$ (in general form)

   On substitution of $x = -1, y = -2$, $LHS = 4(-1)^2 + 4(-2)^2 + 4(-1) - 28(-2) - 71 = 1 \neq 0 = RHS$

   $\therefore$ Point $(-1, -2)$ does not lie on the circle.

Lesson 9: Graphing and Solving Problems involving Circles and other Geometric Figures on the Coordinate Plane

Syllabus Code/s: —

Quarter: Grade 10 – Second Quarter

Content Section: Geometry

Content Standard

The learner demonstrates understanding of key concepts of circles and coordinate geometry.

Performance Standard

The learner is able to:

1. formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.
2. formulate and solve problems involving geometric figures on the rectangular coordinate plane with perseverance and accuracy.

Most Essential Learning Competency

The learner graphs and solves problems involving circles and other geometric figures on the coordinate plane.

Key Idea

Graph and solve problems involving circles and other geometric figures on the coordinate plane.

Matters for Students to Observe

Students need to:

- have ready recall of the midpoint formula and the distance formula from coordinate geometry.
- know well how to find the length of horizontal and vertical intervals in the coordinate plane from the x-coordinates (for horizontal intervals) or the y-coordinates (for vertical intervals) of their endpoints, rather than through the use of the distance formula.
- have well developed knowledge and skills in relation to the use of Pythagoras' theorem and in finding the area of triangles.
- have sound knowledge and ready recall of geometric theorems and results.
- know well the parts of a circle and their geometric relationships.
- know the terms and be able to explain the meaning of coordinate plane, inscribe, perpendicular, point of contact, horizontal, vertical.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. The midpoint (x, y) of the point (x_1, y_1) and (x_2, y_2) can be found using $M(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

   The distance between the points (x_1, y_1) and (x_2, y_2) can be found using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

2. (i) Length of the vertical interval joining $(3, 1\frac{1}{2})$ and $(3, 6\frac{1}{2})$ is the difference between the y-coordinates.

   $\therefore$ Length of interval$= 6\frac{1}{2} - 1\frac{1}{2} = 5$ units

(ii) Using Pythagoras' theorem in the triangle, $10^2 = x^2 + 8^2$.

Solving, $x^2 = 10^2 - 8^2 = 100 - 64 = 36$.

$\therefore x = \sqrt{36} = 6$ units

Area of triangle= $\frac{1}{2}$ ×base × height= $\frac{1}{2} \times 6 \times 8 = 24$ square units

3. $x^2 = 2 \times (2 + 9)$ (The product of the intercepts on a secant from an external point is equal to the square of the tangent from that point.)

$\therefore x^2 = 22 \Rightarrow x = \sqrt{22}$ units.

Component 4

Part 4B Item 1

1. H is midpoint of AB. $\therefore H$ has coordinates $\left(\frac{5}{2}, 0\right)$.

Center of circle C is midpoint of diameter GH.

Midpoint of $GH = (\frac{5}{2}, \frac{\frac{5\sqrt{3}}{3}+0}{2})$ (Note that C is on same vertical line as G and H and so has same x-coordinate.)

$\therefore$ Coordinates of C are $(\frac{5}{2}, \frac{5\sqrt{3}}{6})$

2. (i) Length of CH is difference of y-coordinates of C and H since CH is a vertical line.

$\therefore$ Length of $CH = \frac{5\sqrt{3}}{6} - 0 = \frac{5\sqrt{3}}{6}$ (radius of the circle).

(ii) Using Pythagoras' theorem in triangle BDH,

$DH^2 = 5^2 - (\frac{5}{2})^2 = 25 - \frac{25}{4} = \frac{75}{4}$

$\therefore DH = \sqrt{\frac{75}{4}} = \frac{\sqrt{75}}{2} = \frac{5\sqrt{3}}{2}$

$\therefore$ Coordinates of D are $(\frac{5}{2}, \frac{5\sqrt{3}}{2})$.

3. Length of $CD =$ difference between y-coordinates of C and D.

$$= \frac{5\sqrt{3}}{2} - \frac{5\sqrt{3}}{6} = \frac{5\sqrt{3}}{3}$$

Using the distance formula, $AC = \sqrt{(\frac{5}{2} - 0)^2 + (\frac{5\sqrt{3}}{6} - 0)^2} = \sqrt{(\frac{25}{4} + \frac{75}{36})} = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$.

Similarly, using the distance formula, $BC = \frac{5\sqrt{3}}{3}$.

$\therefore$ Length of $AC =$ Length of $BC =$ Length of $CD = \frac{5\sqrt{3}}{3}$.

Part 4C Item 1

1. (i) Length of DG is difference of y-values of D and G as DG is a vertical line.

   $\therefore$ Length of $DG = \frac{5\sqrt{3}}{2} - \frac{5\sqrt{3}}{3} = \frac{5\sqrt{3}}{6}$ = radius of circle from Part 4B Question 2. (i).

   (ii) $\angle CFD$ is angle between tangent DF to circle and radius CF of circle.

   $\therefore \angle CFD$ is a right angle (A tangent is perpendicular to the radius drawn to its point of contact.)

   Using Pythagoras' theorem in triangle CFD,

   $DF^2 = CD^2 - CF^2 = (\frac{5\sqrt{3}}{3})^2 - (\frac{5\sqrt{3}}{6})^2$ ($CD = \frac{5\sqrt{3}}{3}$ from Part 4B Question 3 and CF is radius of circle from Part 4B Question 2. (i))

   $$= \frac{75}{9} - \frac{75}{36} = \frac{225}{36} = \frac{25}{4}$$

   $$\therefore DF = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}$$

2. $CF = R$; $CD = 2R$; By Pythagoras' theorem: $DF^2 = (2R)^2 - R^2 = 3R^2$

   $$\therefore DF = \sqrt{3}R$$

   Side length of the triangle (5 units) is $2 \times DF = 2 \times \sqrt{3}R = 2\sqrt{3}$ times the radius ($\frac{5\sqrt{3}}{6}$ units) of the circle.

3. The product of the intercepts on a secant from an external point equals the square of the tangent from that point.

   $$DF^2 = DH.DG$$
   $$= \frac{5\sqrt{3}}{2} \times \frac{5\sqrt{3}}{6}$$
   $$= \frac{25}{4}$$

   $\therefore DF = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}$, which is consistent with the answer obtained in Part 4C Question 1. (ii).

   Area of triangle $CFD = \frac{1}{2} \times CF \times DF$

   $$= \frac{1}{2} \times \frac{5\sqrt{3}}{6} \times \frac{5}{2}$$

   $= \frac{25\sqrt{3}}{24}$ square units

   Area of triangle $ABD = \frac{1}{2} \times AB \times DH$

   $$= \frac{1}{2} \times 5 \times \frac{5\sqrt{3}}{2}$$

   $= \frac{25\sqrt{3}}{4}$ square units

   $\therefore$ Area of triangle $CFD = \frac{1}{6}$ of area of triangle ABD

Lesson 10: Differentiating Permutation from Combination of Objects taken r at a time

Syllabus Code: M10SP-IIIc-2

Quarter: Grade 10 – Third Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of combinatorics and probability.

Performance Standard

The learner is able to use precise counting techniques and probability in formulating conclusions and making decisions.

Most Essential Learning Competency

The learner differentiates permutation from combination of objects taken r at a time.

Key Idea

Differentiate permutation from combination of objects taken r at a time.

Matters for Students to Observe

Students need to:

- have a sound understanding of 'the basic counting theorem' i.e., if one event can happen in a different ways, and then another event can happen in b different ways, then the two events can occur in succession in ab different ways.
- have a sound understanding of the terms and be able to explain the meaning of ordered selection, unordered selection, permutation, combination, taken r at a time, with/without repetition.
- have ready recall of the formulae for calculating permutations and combinations.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) If order within a pair is important, the number of ordered selections of the symbols taken two at a time is needed. Hence, ▲⬟,▲⬢,⬟▲,⬟⬢,⬢▲,⬢⬟ i.e., 6 pairs

   (ii) If order within a pair is not important, the number of unordered selections of the symbols taken two at a time is needed: Hence, ▲⬟,▲⬢,⬟⬢ i.e., 3 pairs

2. Completing the formulae:

   (i) ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

   (ii) ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$

3. (i) ${}^{5}P_{2} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \times 4$ (on cancelling)$= 20$

   (ii) ${}^{7}C_{3} = \frac{7!}{3!\,(7-3)!} = \frac{7!}{3!\,4!} = 7 \times 5$ (on cancelling)$= 35$

Component 4

Part 4B Item 1

1. (i) $\{ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc\}$

   (ii) Set A has 4 elements, therefore $n = 4$, and the elements are to be taken two at a time i.e., $r = 2$

   Using ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$, where $n = 4, r = 2$,

   ${}^{4}C_{2} = \frac{4!}{2!\,(4-2)!} = \frac{4!}{2!\,2!} = 6$

2. Using ${}^{n}P_{r} = \frac{n!}{(n-r)!}$, where $n = 4, r = 3$,

   ${}^{4}P_{3} = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$

   $\therefore$ 24 different permutations are possible.

   Using ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$, where $n = 4, r = 3$,

   ${}^{4}C_{3} = \frac{4!}{3!\,(4-3)!} = \frac{4!}{3!\,1!} = 4$

   $\therefore$ 4 different combinations are possible.

3. (i) If Ralph wishes to create three-digit numbers from the digits in Set B, using no digit more than once:

   there are 6 choices for the first digit, 5 choices for the second digit, and 4 choices for the third digit.

   $\therefore$ Number of three-digit numbers that he can create$= 6 \times 5 \times 4 = 120$.

   (ii) If Ralph wishes to create three-digit numbers that are less than 500 from the digits in Set B, using no digit more than once:

   there are 4 choices for the first digit (numbers less than 500 must have first digit 1, 2, 3, or 4),

   5 choices for the second digit, and 4 choices for the third digit.

   $\therefore$ Number of three-digit numbers that are less than 500 that he can create$= 4 \times 5 \times 4 = 80$.

Part 4C Item 2

1. (i) Using ${}^{n}P_{r} = \frac{n!}{(n-r)!}$, where $n = 4, r = 4$,

   ${}^{4}P_{4} = \frac{4!}{(4-4)!} = \frac{4!}{0!} = 24$ (Note that $0! = 1$)

   $\therefore$ 24 different permutations are possible.

   (ii) Using ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$, where $n = 4, r = 4$,

   ${}^{4}C_{4} = \frac{4!}{4!\,(4-4)!} = \frac{4!}{4!\,0!} = 1$

   $\therefore$ 1 combination is possible (i.e., the combination $abcd$).

2. (i) If Ralph wishes to create two-digit numbers from the digits in Set B, using no digit more than once:

   there are 6 choices for the first digit, and 5 choices for the second digit.

   $\therefore$ Number of two-digit numbers that he can create$= 6 \times 5 = 30$.

   (ii) If Ralph wishes to create four-digit numbers from the digits in Set B, using no digit more than once:

   there are 6 choices for the first digit, 5 choices for the second digit, 4 choices for the third digit, and 3 choices for the fourth digit.

   $\therefore$ Number of four-digit numbers that he can create$= 6 \times 5 \times 4 \times 3 = 360$.

3. (i) If Ralph wishes to create four-digit numbers from the digits in Set B, with repetition of digits allowed:

   there are 6 choices for the first digit, 6 choices for the second digit, 6 choices for the third digit, and 6 choices for the fourth digit.

   $\therefore$ Number of four-digit numbers that he can create $= 6^4 = 1296$.

   (ii) Ralph selecting two digits from the six digits, placing them on separate cards, and placing the two cards in a separate envelope, is equivalent to the number of unordered selections of two digits that can be made from the six digits in Set B i.e., ${}^{6}C_{2}$

   ${}^{6}C_{2} = \frac{6!}{2!\,(6-2)!} = \frac{6!}{2!\,4!} = 15$

   $\therefore$ Ralph will need 15 envelopes.

Lesson 11: Solving Problems involving Permutations and Combinations

Syllabus Code: M10SP-IIId-e-1

Quarter: Grade 10 – Third Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of combinatorics and probability.

Performance Standard

The learner is able to use precise counting techniques and probability in formulating conclusions and making decisions.

Most Essential Learning Competency

The learner solves problems involving permutations and combinations.

Key Idea

Solve problems involving permutations and combinations.

Matters for Students to Observe

Students need to:

- read and interpret carefully the general terminology used in this topic area.
- become familiar with the language used to describe particular scenarios and to ask particular types of questions in this topic area.
- have ready recall of the formulae for calculating permutations and combinations.
- have a sound understanding of factorials for positive integers and to be aware that $0! = 1$.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $^5P_3 = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$

   $^5C_3 = \frac{5!}{3!(5-3)!} = \frac{120}{6(2)} = 10$

   $\therefore$ Since $60 = 6 \times 10$, $^5P_3 = 6 \times {}^5C_3$, as required.

2. If objects W, X and Y are to be together at the front of the line, there are two remaining positions, with two choices for the fourth position and one choice for the last position. W, X and Y can be arranged in $3! = 6$ ways at the front of the line.

   $\therefore$ there are $6 \times 2 \times 1 = 12$ ways that the 5 objects can be arranged.

3. (i) Number of ways groups of 5 people can be selected from 6 men and 4 women if there are to be 3 men and 2 women in each group $= {}^6C_3 \times {}^4C_2 = \frac{6!}{3!3!} \times \frac{4!}{2!2!} = 20 \times 6 = 120$

   (ii) Number of ways groups of 5 people can be selected from 6 men and 4 women if there are to be 2 men and 3 women in each group $= {}^6C_2 \times {}^4C_3 = \frac{6!}{2!4!} \times \frac{4!}{3!1!} = 15 \times 4 = 60$

Component 4

Part 4B Item 1

1. (i) Number of permutations of the six students if taken only three students at a time $= {}^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 120$

   (ii) Number of ways all six students could be arranged if there were no restrictions $= 6! = 720$

2. If A and B were to be together in the line, they are regarded as one unit and the unit together with the other 4 students constitute a group of 5 which may be ordered in $5!$ ways. The two students A and B can be arranged in 2 ways.

   ∴ Number of ways that all six students could be arranged if A and B were to be together in the line $= 2 \times 5! = 240$.

3. (i) Number of ways groups of 4 students could be selected if all members of the group were to be boys $= {}^5C_4 = \frac{5!}{4!1!} = 5$

   (ii) Number of ways groups of 4 students could be selected if there were to be all three girls and one boy in the group$= {}^3C_3 \times {}^5C_1 = \frac{3!}{3!0!} \times \frac{5!}{1!4!} = 1 \times 5 = 5$

Part 4C Item 2

1. (i) Number of permutations of the six students if taken only five students at a time $= {}^6P_5 = \frac{6!}{(6-5)!} = \frac{6!}{1!} = 720$

   (ii) Number of groups of four students that could be selected if there were no restrictions on how they were to be selected $= {}^8C_4 = \frac{8!}{4!4!} = 70$

2. If D was to be second in the line and F was to be fifth in the line, these two positions were decided. The other four positions could be selected in $4!$ ways from the remaining 4 students.

   ∴ Number of ways $= 4! = 24$.

3. (i) Number of ways groups of 4 students could be selected if there were to be two boys and two girls in the group $= {}^5C_2 \times {}^3C_2 = \frac{5!}{2!3!} \times \frac{3!}{2!1!} = 10 \times 3 = 30$

   (ii) If a particular girl X was to be included, then the other three students in the group were selected from the remaining seven students.

   ∴ Number of ways $= {}^7C_3 = \frac{7!}{3!4!} = 35$.

Lesson 12 Deliberate Practice: Solving Problems involving Circles; Graphing and Solving Problems involving Circles and Other Geometric Figures on the Coordinate Plane, Solving Problems involving Permutations and Combinations

Syllabus Codes: M10GE-IIf-2, M10SP-IIId-e-1

Key Ideas

Solve problems involving circles.

Graph and solve problems involving circles and other geometric figures on the coordinate plane.

Solve problems involving permutations and combinations.

Matters for Students to Observe

(see above for notes for Lessons 7–11 and relevant to this lesson)

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) Length of the horizontal interval joining $(-4\frac{1}{2}, 1)$ and $(7, 1)$ is the difference between the x-coordinates.

$\therefore$ Length of interval $= 7 - (-4\frac{1}{2}) = 11\frac{1}{2}$ units.

(ii) A circle with center $(0, 0)$ has equation of the form $x^2 + y^2 = r^2$, where r is the radius of the circle.

$\therefore$ circle with center $(0, 0)$ and radius 12 units has equation $x^2 + y^2 = 12^2$ i.e., $x^2 + y^2 = 144$.

2. (i) Using the distance formula to find the length of the interval joining the points $(5, -2\sqrt{3})$ and $(2, 2\sqrt{3})$,

$$d = \sqrt{(2-5)^2 + (2\sqrt{3} - (-2\sqrt{3})^2} = \sqrt{(-3)^2 + (4\sqrt{3})^2} = \sqrt{(9+48)} = \sqrt{57}$$

$\therefore$ Length of interval $= \sqrt{57}$ units.

(ii) The area of a rhombus is given by the formula $Area\ of\ rhombus = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the rhombus.

3. Number of different groups of 7 people that can be selected from 5 men and 6 women if there are to be 3 men and 4 women in each group $= {}^5C_3 \times {}^6C_4 = \frac{5!}{3!2!} \times \frac{6!}{4!2!} = 10 \times 15 = 150$

Component 4

Part 4B Item 1

1. (i) D is midpoint of OA. $\therefore D$ has coordinates $(5, 0)$.

(ii) Required circle has center $(0, 0)$ and radius $OA = 10$ units.

A circle with center $(0, 0)$ and radius r has equation of the form $x^2 + y^2 = r^2$.

$\therefore$ Required equation is $x^2 + y^2 = 10^2$ i.e., $x^2 + y^2 = 100$.

2. (i) Since CD is a vertical line, point C has the same x-coordinate as point D $\therefore$ x-coordinate is 5.

Since C lies on the circle $x^2 + y^2 = 100$, $(5)^2 + y^2 = 100$.

$$y^2 = 100 - 25 = 75 \Rightarrow y = \pm 5\sqrt{3}$$

Point C lies in first quadrant of coordinate plane, so has positive y-coordinate $(+5\sqrt{3})$.

$\therefore C$ is the point $(5, 5\sqrt{3})$.

(ii) Using the distance formula to find the length of AC, where A is point $(10, 0)$ and C is point $(5, 5\sqrt{3})$,

$$d = \sqrt{(5-10)^2 + (5\sqrt{3}-0)^2} = \sqrt{(-5)^2 + (5\sqrt{3})^2} = \sqrt{(25+75)} = \sqrt{100} = 10 \text{ units}$$

3. (i) Number of different committees that Sarah can form if there are no restrictions $= {}^8C_6 = \frac{8!}{6!2!} = 28$

(ii) Number of different committees that Sarah can form if there are to be 2 men and 4 women on the committee$= {}^3C_2 \times {}^5C_4 = \frac{3!}{2!1!} \times \frac{5!}{4!1!} = 3 \times 5 = 15$

Part 4C Item 2

1. (i) Since AB is parallel to OC and C is 5 units to right of O, then B is 5 units to right of A $\therefore$ B has x-coordinate $10 + 5 = 15$.

Since BC is parallel to OA (on the horizontal x-axis), B has same y-coordinate as C $\therefore$ B has y-coordinate $5\sqrt{3}$.

$\therefore$ B is the point $(15, 5\sqrt{3})$.

(ii) Using the distance formula to find the length of OB, where O is point $(0, 0)$ and B is point $(15, 5\sqrt{3})$,

$$d = \sqrt{(15-0)^2 + (5\sqrt{3}-0)^2} = \sqrt{15^2 + (5\sqrt{3})^2} = \sqrt{(225+75)} = \sqrt{300} = 10\sqrt{3} \text{ units}$$

2. (i) Rhombus $OABC$ has diagonals $AC = 10$ units (from Part 4B Question 2. (ii)) and $OB = 10\sqrt{3}$ units (from Part 4C Question 1. (ii)).

The area of a rhombus can be found using the formula $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the rhombus.

$\therefore$ Area rhombus $OABC = \frac{1}{2} \times AC \times OB = \frac{1}{2} \times 10 \times 10\sqrt{3} = 50\sqrt{3}$ square units.

(ii) Given that the sector angle $\angle AOC = 60° = \frac{360°}{6}$,

area of sector $AOC = \frac{1}{6} \times$ area of circle$= \frac{1}{6} \times \pi \times (10)^2 = \frac{50\pi}{3}$ square units

Area of section of rhombus outside sector AOC = area of rhombus $OABC$ − area of sector AOC

$= 50\sqrt{3} - \frac{50\pi}{3}$

$= 34.2$ square units (correct to 1 decimal place)

3. (i) Number of different committees Sarah can form if all members of the committee are to be women

${}^7C_5 = \frac{7!}{5!2!} = 21$

The other three positions are then selected from the remaining nine people.

$\therefore$ Number of different committees Sarah can form if a particular man and a particular woman are to be included on the committee $= {}^9C_3 = \frac{9!}{3!6!} = 84$.

Lesson 13: Illustrating and finding the Probability of a Union of Two Events $(\boldsymbol{A} \cup \boldsymbol{B})$

Syllabus Codes: M10SP-IIIg-1, M10SP-IIIg-h-1

Quarter: Grade 10 – Third Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of combinatorics and probability.

Performance Standard

The learner is able to use precise counting techniques and probability in formulating conclusions and making decisions.

Most Essential Learning Competencies

The learner:

1. illustrates the probability of a union of two events.
2. finds the probability of $(A \cup B)$.

Key Idea

Illustrate and find the probability of a union of two events $(A \cup B)$.

Matters for Students to Observe

Students need to:

- have a sound understanding of what is meant by a random event (or experiment) and the sample space, and that an event is a subset of the sample space.
- know well that the theory of probability is closely associated with the theory of sets.
- have clear knowledge of the difference between simple events and compound events.
- know well that the range of the probability of an event A is $0 \leq P(A) \leq 1$.
- have clear knowledge that the complement of an event E (denoted by $\bar{E}$) consists of all the simple events in the sample space that do not belong to E i.e., $\bar{E}$ is the event that 'E does not occur'.
- have a sound understanding of the union and intersection of sets A and B and have clear knowledge of the associated notation i.e., $A \cup B$ and $A \cap B$.
- know well the terms 'mutually exclusive' and 'not mutually exclusive' and be able to explain their meaning.
- have ready recall that for mutually exclusive events A and B that $P(A \cap B) = 0$.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $P(Y) = \frac{1}{4}$

$\therefore P(\bar{Y}) = \frac{3}{4}$.

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

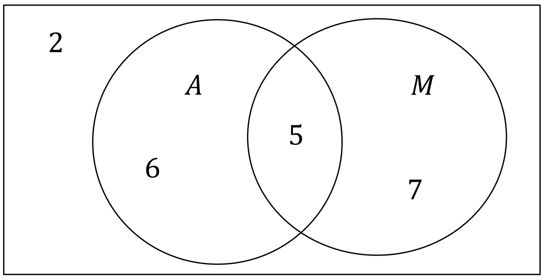
$= \frac{1}{2} + \frac{1}{4} - \frac{2}{5} = \frac{7}{20}$

2. Since the events are mutually exclusive, $P(V \cap W) = 0$

$P(V \cup W) = P(V) + P(W) = \frac{2}{3} + \frac{1}{10} = \frac{23}{30}$

$P(\overline{V \cup W}) = 1 - P(V \cup W) = 1 - \frac{23}{30} = \frac{7}{30}$

3. (i)



(ii) $P(A \cup M) = P(A) + P(M) - P(A \cap M)$

$= \frac{11}{20} + \frac{12}{20} - \frac{5}{20} = \frac{18}{20} = \frac{9}{10}$

Component 4
Part 4B Item 1

1. (i) $P(A) = \frac{3}{5}$

$P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{5} = \frac{2}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{5} + \frac{1}{5} - \frac{1}{10} = \frac{7}{10}$

(ii) $P(C \cup D) = P(C) + P(D)$ (Note that $P(C \cap D) = 0$ because the events are mutually exclusive.)

$= \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$

$P(\overline{C \cup D}) = 1 - P(C \cup D) = 1 - \frac{9}{10} = \frac{1}{10}$

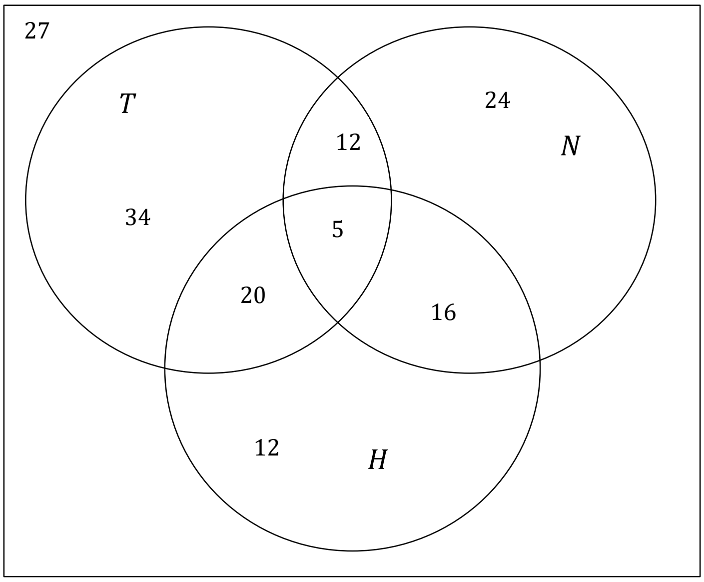
2. The set $M \cap S$ represents those students who have the same level of preference for Mathematics and Science. i.e., 2 students of the 30 students in the class.

$\therefore P(M \cap S) = \frac{2}{30} = \frac{1}{15}$

$P(M \cup S) = P(M) + P(S) - P(M \cap S)$

$= \frac{15}{30} + \frac{13}{30} - \frac{2}{30} = \frac{26}{30} = \frac{13}{15}$

3.



(i) P(one of girls selected at random plays one sport only) $= \frac{34+24+12}{150} = \frac{70}{150} = \frac{7}{15}$

(ii) The girls that play 'at least two sports' play two of the sports only or they play all three of the sports.

$\therefore$ P(one of girls selected at random plays at least two sports) $= \frac{12+20+16+5}{150} = \frac{53}{150}$.

Part 4C Item 2

1. (i) $P(R \cup S) = P(R) + P(S) - P(R \cap S) = \frac{3}{10} + \frac{2}{5} - \frac{1}{4} = \frac{9}{20}$

$P(Q \cup S) = P(Q) + P(S) - P(Q \cap S) = \frac{2}{5} + \frac{2}{5} - \frac{1}{10} = \frac{7}{10}$

$P(\overline{Q \cup S}) = 1 - P(Q \cup S) = 1 - \frac{7}{10} = \frac{3}{10}$

(ii) $P(U) = \frac{1}{3}$

$P(\overline{U}) = 1 - P(U) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(T \cup U) = P(T) + P(U)$ (Note that $P(T \cap U) = 0$ because the events are mutually exclusive.)

$= \frac{2}{5} + \frac{1}{3} = \frac{11}{15}$

$P(\overline{T \cup U}) = 1 - P(T \cup U) = 1 - \frac{11}{15} = \frac{4}{15}$

$P(T \cup U \cup W) = P(T) + P(U) + P(W) = \frac{2}{5} + \frac{1}{3} + \frac{1}{5} = \frac{14}{15}$

2. $P(G) = \frac{10}{27}, P(H) = \frac{13}{27}, P(G \cap H) = \frac{4}{27}$

$P(\overline{G \cap H}) = 1 - P(G \cap H) = 1 - \frac{4}{27} = \frac{23}{27}$

$P(G \cup H) = P(G) + P(H) - P(G \cap H) = \frac{10}{27} + \frac{13}{27} - \frac{4}{27} = \frac{19}{27}$

3. (i) P(one of girls selected at random plays none of the three sports) $= \frac{27}{150}$

(ii) The girls that play 'at most one sport' play one of the sports only or they play none of the sports.

$\therefore$ P(one of girls selected at random plays at most one sport) $= \frac{34+24+12+27}{150} = \frac{97}{150}$.

Lesson 14: Solving Problems involving Probability

Syllabus Code: M10SP-IIIi-j-1

Quarter: Grade 10 – Third Quarter

Content Section: Statistics and Probability

Content StandardM10SP-IIIj-1

The learner demonstrates understanding of key concepts of combinatorics and probability.

Performance Standard

The learner is able to use precise counting techniques and probability in formulating conclusions and making decisions.

Most Essential Learning Competency

The learner solves problems involving probability.

Key Idea

Solve problems involving probability.

Matters for Students to Observe

Students need to:

- be aware that in work on probability that mistakes are often made due to hasty or incorrect interpretation of events or experiments and/or their description.
- know well the terms ‘in succession’, ‘without replacement’, and ‘with replacement’, and be able to explain their meaning.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) $P(\text{first marble drawn is blue}) = \frac{3}{5}$

   (ii) $P(\text{both marbles drawn are blue}) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

2. $P(\text{both marbles are red}) = (\frac{1}{3} \times \frac{2}{5} \times \frac{1}{4}) + (\frac{1}{3} \times \frac{1}{6} \times \frac{0}{5}) + (\frac{1}{3} \times \frac{3}{5} \times \frac{2}{4})$

   $= \frac{1}{30} + 0 + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$

3. (i) Number of ways three different colors can be drawn in three draws in succession from Bag $4 = 3!$ (there are 3 choices for the first marble, 2 for the second, and 1 for the third)

   $= 6$

   (ii) As in Question 3. (i), there are 6 possible orders in which the three colors can be drawn. The probabilities of obtaining any one of the 6 orders is the same.

   $P(\text{one white, one yellow, and one green marble in any order}) = 6 \times(\frac{3}{9} \times \frac{4}{9} \times \frac{2}{9}) = \frac{16}{81}$

Component 4

Part 4B Item 1

1. $P(\text{both marbles drawn are red}) = \frac{2}{6} \times \frac{5}{8} = \frac{5}{24}$

2. (i) $P(\text{marble is white}) = \left(\frac{1}{2} \times \frac{2}{6}\right) + \left(\frac{1}{2} \times \frac{3}{9}\right) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

   (ii) $P(\text{both marbles are blue}) = \left(\frac{1}{2} \times \frac{4}{6} \times \frac{3}{5}\right) + \left(\frac{1}{2} \times \frac{6}{9} \times \frac{5}{8}\right) = \frac{1}{5} + \frac{5}{24} = \frac{49}{120}$

3. $P(\text{one black } (B)\text{, one purple } (P)\text{, and one green } (G) \text{ marble in any order}) = P(BPG) + P(BGP) + \cdots\text{etc})$

   $=(\left(\frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}\right) + \left(\frac{5}{12} \times \frac{3}{11} \times \frac{4}{10}\right) + \cdots\text{etc}) = 6 \times \left(\frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}\right) = \frac{3}{11}$

Part 4C Item 2

1. (i) $P(\text{both marbles drawn are yellow}) = \frac{4}{6} \times \frac{3}{8} = \frac{1}{4}$

   (ii) $P(\text{the marbles are the same color})$

   $= P(\text{both marbles are red } (R) \text{ or both marbles are yellow } (Y))= P(RR \text{ or } YY)$

$$= \left(\frac{2}{6} \times \frac{5}{8}\right) + \left(\frac{4}{6} \times \frac{3}{8}\right)$$

$$= \frac{5}{24} + \frac{1}{4} = \frac{11}{24}$$

2. (i) $P(\text{marble is blue}) = \left(\frac{1}{2} \times \frac{4}{6}\right) + \left(\frac{1}{2} \times \frac{6}{9}\right) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

   (ii) $P(\text{the marbles are of different colors}) = P(BW \text{ or } WB) = \left(\frac{1}{2} \times \frac{4}{6} \times \frac{2}{6}\right) + \left(\frac{1}{2} \times \frac{3}{9} \times \frac{6}{9}\right) = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

3. $P(\text{one black } (B)\text{, one purple } (P)\text{, and one green } (G) \text{ marble in any order}) = P(BPG) + P(BGP) + \cdots\text{etc})$

   $= (\left(\frac{5}{12} \times \frac{4}{12} \times \frac{3}{12}\right) + \left(\frac{5}{12} \times \frac{3}{12} \times \frac{4}{12}\right) + \cdots\text{etc}) = 6 \times \left(\frac{5}{12} \times \frac{4}{12} \times \frac{3}{12}\right) = \frac{5}{24}$

Lesson 15: Calculating and Interpreting Measures of Position (quartiles, deciles, and percentiles) of a Set of Data

Syllabus Codes: M10SP-IVb-1, M10SP-IVc-1

Quarter: Grade 10 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of measures of position.

Performance Standard

The learner is able to conduct systematically mini research, applying the different statistical methods.

Most Essential Learning Competencies

The learner:

1. calculates a specified measure of position (e.g., 90th percentile) of a set of data.
2. interprets measures of position.

Key Idea

Calculate and interpret measures of position (quartiles, deciles, and percentiles) of a set of data,

Matters for Students to Observe

Students need to:

- know well the terms: measure of position, quartile, decile, and percentile; and be able to explain their meaning.
- have clear knowledge that the median is equivalent to the 2nd quartile, the 5th decile, and the 50th percentile.
- have clear knowledge that the first quartile represents the value below which one-quarter, or 25%, of the scores in a data set lie; the second quartile represents the value below which one-half, or 50%, of the scores lie; and the third quartile represents the value below which three-quarters, or 75%, of the scores lie.
- have a sound understanding of Tukey's Method for finding the quartiles of a data set.
- have ready recall of the formulae for finding the position in a data set of deciles and percentiles.
- have a sound understanding of the use of the cumulative frequency column in a tabulated set of data to find the location of specific scores.

Worked Answers to Component 1 and Component 4 Questions

Component 1 th

1. (i) The median is equivalent to the second quartile and the fiftieth percentile.

   (ii) A: 10; B: $n + 1$

   Hence the formulae are:

   $Position\ of\ D_i = \frac{i}{10}(n+1)$ (For locating the position of deciles in a data set.)

   $Position\ of\ P_i = \frac{i}{100}(n+1)$ (For locating the position of percentiles in a data set.)

2. (i) For the set of scores $14, 15, 16, 16, 17, 18, 18, 19, 20$ (in ascending order) there are four scores below the score 17 and four scores above. Therefore, 17 is the middle score i.e., the median.

   (ii) To find Q_1.(the lower quartile) for the set of scores listed in part (i) using Tukey's Method, we consider the four scores below the median (17) and take the average of the two middle scores of these four scores $(14, 15, 16, 16)$. $\therefore Q_1 = \frac{15+16}{2} = \frac{31}{2} = 15.5$

3. Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 55$ and $n = 60$,

   $Position\ of\ P_{55} = \frac{55}{100}(60+1) = 0.55 \times 61 = 33.55 = 34$ (on rounding up)

   ∴ The position of the fifty-fifth percentile in a set of 60 scores is the 34th score.

Component 4

Part 4B Item 1

1. (i) Set A: $\{2, 3, 3, 5, 5, 6, 7, 7, 8, 9, 9, 9, 10, 10, 11, 12\}$.

   There are 16 scores in Set A, so the median is the average of the 8th and 9th scores (7 and 8).

   ∴ Median $= \frac{7+8}{2} = 7.5$.

   (ii) The median represents the middle of the data set and is equivalent to the 2nd quartile.

2. (i) The first quartile represents the value below which one-quarter, or 25%, of the scores in the data set lie. The third quartile represents the value below which three-quarters, or 75%, of the scores lie.

   (ii) There are 8 scores below the median (7.5) of Set A.

   By Tukey's Method, the first quartile Q_1 is the average of the two middle scores within the 8 scores below the median. $\therefore\ Q_1 = \frac{5+5}{2} = 5$.

3. (i) Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 60$ and $n = 200$,

   $Position\ of\ P_{60} = \frac{60}{100}(200+1) = 0.6 \times 201 = 120.6 = 121$ (on rounding up)

   P_{60} is represented by the 121st score in Set B i.e., 28 (on locating the 121st score using the Cumulative Frequency column).

   $Position\ of\ P_{90} = \frac{90}{100}(200+1) = 0.9 \times 201 = 180.9 = 181$ (on rounding up)

   P_{90} is represented by the 181st score in Set B i.e., 33 (on locating the 181st score using the Cumulative Frequency column).

   (ii) The 75th percentile is midway between the 60th percentile and 90th percentile and is equivalent to the upper quartile (Q_3).

Part 4C Item 2

1. The median of the 200 scores in Set B is the average of the 100th and 101st scores. Using the Cumulative Frequency column of the Set B data table, the 100th and 101st scores are both 27.

   $\therefore$ Median$=27$ (average of two scores of 27).

2. (i) Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 3$ and $n = 200$,

   $Position\ of\ D_3 = \frac{3}{10}(200+1) = 0.3 \times 201 = 60.3 = 61$ (on rounding up)

   D_3 is represented by the 61st score in Set B i.e., 25 (on locating the 61st score using the Cumulative Frequency column).

   $Position\ of\ D_7 = \frac{7}{10}(200+1) = 0.7 \times 201 = 140.7 = 141$ (on rounding up)

   D_7 is represented by the 141st score in Set B i.e., 29 (on locating the 141st score using the Cumulative Frequency column).

   (ii) The decile midway between the third decile and seventh decile is the fifth decile, which represents the median.

3. (i) There are 100 scores below the median (27) of Set B.

   By Tukey's Method, the first quartile $Q_1$is the average of the two middle scores within the100 scores below the median. i.e., the average of the 50th and 51st scores. Using the Cumulative Frequency column of the Set B data table, the 50th and 51st scores are both 24.

   $\therefore Q_1 = 24$ (average of two scores of 24).

   Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 25$ and $n = 200$,

   $Position\ of\ P_{25} = \frac{25}{100}(200+1) = 0.25 \times 201 = 50.25 = 51$ (on rounding up)

   P_{25} is represented by the 51st score in Set B i.e., 24 (on locating the 51st score using the Cumulative Frequency column).

   (ii) In this case, the results are the same (both 24) using the two methods. The process of the second method involves 'rounding' in order to always identify one of the members of the data set. Such identification is not a requirement of Tukey's Method.

Lesson 16: Solving Problems involving Measures of Position

Syllabus Code: M10SP-IVd-e-1

Quarter: Grade 10 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of measures of position.

Performance Standard

The learner is able to conduct systematically mini research, applying the different statistical methods.

Most Essential Learning Competency

The learner solves problems involving measures of position.

Key Idea

Solves problems involving measures of position.

Matters for Students to Observe

Students need to:

- have ready recall of the meaning of the terms quartile, decile, and percentile, and be able to explain their meaning.
- have a sound understanding of Tukey's Method for finding the quartiles of a data set and develop the ability to apply the method accurately in solving problems.
- have ready recall of the 'position formulae' for finding the position in a data set of deciles and percentiles and develop the ability to apply the formulae accurately in solving problems.
- develop fluency in the processes associated with the application of Tukey's Method and the 'position formulae' and know well how to interpret the results obtained.
- have a sound understanding of the use of the cumulative frequency column in a tabulated set of data to find the location of specific scores.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i)

Score x	Frequency f	Cumulative Frequency
5	2	2
6	3	5
7	4	9
8	4	13
9	5	18
10	3	21
	$\sum f = 21$	

(ii) Median (the middle score of the 21 scores) is the 11th score, which from the Cumulative Frequency column is 8.

2. For the set of scores in Question 1, there are ten scores above the 11th score (which is the median 8).

   To find Q_3.(the third quartile) for the set of scores using Tukey's Method, we consider the ten scores above the median and take the average of the two middle scores (the 16th and 17th scores of the data set, which from the Cumulative Frequency column are both equal to 9). $\therefore Q_3 = \frac{9+9}{2} = 9$

3. Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 4$ and $n = 21$,

   $Position\ of\ D_4 = \frac{4}{10}(21+1) = 0.4 \times 22 = 8.8 = 9$ (on rounding up)

   $\therefore D_4$ is represented by the 9th score in the data set.

   Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 45$ and $n = 21$,

   $Position\ of\ P_{45} = \frac{45}{100}(21+1) = 0.45 \times 22 = 9.9 = 10$ (on rounding up)

   $\therefore P_{45}$ is represented by the 10th score in the data set.

Part 4B Item 1

1. There are 50 scores in Set L. $\therefore$ the median is the average of the 25th and 26th scores (the two middle scores of the data set), which from the Cumulative Frequency column are both equal to 5. $\therefore$ the median is 5.

2. There are 25 scores below the median score (5) of Set L.

   By Tukey's Method, the first quartile Q_1 is the 13th score (the middle score of the 25 scores below the median). Using the Cumulative Frequency column of the Set L data table, the 13th score is 3.

   $\therefore Q_1 = 3$.

3. (i) Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 9$ and $n = 50$,

   $Position\ of\ D_9 = \frac{9}{10}(50+1) = 0.9 \times 51 = 45.9 = 46$ (on rounding up)

   $\therefore D_9$ is represented by the 46th score in Set L i.e., 7 (on locating the 46th score using the Cumulative Frequency column).

   (ii) Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 35$ and $n = 50$,

   $Position\ of\ P_{35} = \frac{35}{100}(50+1) = 0.35 \times 51 = 17.85 = 18$ (on rounding up)

   $\therefore P_{35}$ is represented by the 18th score in Set L i.e., 4 (on locating the 18th score using the Cumulative Frequency column).

Part 4C Item 1

1. (i) There are 50 scores in Set T. $\therefore$ the median is the average of the 25th and 26th scores (the two middle scores of the data set), which from the Cumulative Frequency column are 6 and 7 (respectively).

   $\therefore$ Median$= \frac{6+7}{2} = 6.5$

   (ii) $\frac{\text{Median of Set } T}{\text{Median of Set } L} = \frac{6.5}{5} = 1.3 = 1.3 \times 100\% = 130\%$ i.e., median of Set T is 130% of median of Set L

   $\therefore$ Median of Set T is 30% greater than that of Set L.

2. There are 25 scores above the median score (6.5) of Set T.

   By Tukey's Method, the third quartile Q_3 is the 38th score (the middle score of the 25 scores above the median). Using the Cumulative Frequency column of the Set T data table, the 38th score is 8.

   $\therefore Q_3 = 8$.

3. (i) Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 2$ and $n = 50$,

   $Position\ of\ D_2 = \frac{2}{10}(50+1) = 0.2 \times 51 = 10.2 = 11$ (on rounding up)

   $\therefore D_2$ is represented by the 11th score in Set T i.e., 5 (on locating the 11th score using the Cumulative Frequency column).

   $\therefore$ second decile is 5.

   (ii) Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 65$ and $n = 50$,

   $Position\ of\ P_{65} = \frac{65}{100}(50+1) = 0.65 \times 51 = 33.15 = 34$ (on rounding up)

   $\therefore P_{65}$ is represented by the 34th score in Set T i.e., 4 (on locating the 34th score using the Cumulative Frequency column).

   $\therefore$ 65th percentile is 7.

Lesson 17: Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data

Syllabus Code: M10SP-IVh-j-1

Quarter: Grade 10 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of measures of position.

Performance Standard

The learner is able to conduct systematically mini research, applying the different statistical methods.

Most Essential Learning Competency

The learner uses appropriate measures of position and other statistical methods in analyzing and interpreting data.

Key Idea

Use appropriate measures of position and other statistical methods in analyzing and interpreting data.

Matters for Students to Observe

Students need to:

- have a sound understanding of what it means to 'analyze' and 'interpret' statistical data, as well as what statistical measures, including measures of position, are appropriate in analyzing and interpreting specific sets of statistical data.
- be aware that a key aspect of this work is to be able to make decisions about whether a particular statistical measure is a better indicator than another of an attribute or characteristic being investigated.
- have a sound understanding of why mean, median, and mode are described as 'measures of central tendency'.
- have a sound understanding of what is meant by dispersion and variability and why the range and standard deviation are described as 'measures of variability'. It is also important that they know well how to calculate each measure of variability, and the role of the mean in the calculation of the 'deviation' of each score.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. (i) Range $=$ highest score $-$ lowest score $= 29 - 23 = 6$

   (ii) There are ten scores in the set of scores. $\therefore$ the median is the average of the 5th (26) and 6th (27) scores (the two middle scores in the set). $\therefore$ Median$= \frac{26+27}{2} = 26.5$.

2. (i) Mean $= \frac{\text{sum of scores}}{\text{number of scores}} = \frac{263}{10} = 26.3$

   (ii) $\sqrt{\frac{\Sigma(fd^2)}{\Sigma f}}$, where for each score in a set of scores f is the frequency and d is the deviation from the mean, is used to calculate the standard deviation for a set of scores.

3. (i) There are 5 scores below the median (26.5) in the set of scores.

   By Tukey's Method, the lower quartile Q_1 is the middle score of the 5 scores below the median.

   $\therefore\ Q_1 = 25$.

(ii) Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 7$ and $n = 10$,

$Position\ of\ D_7 = \frac{7}{10}(10+1) = 0.7 \times 11 = 7.7 = 8$ (on rounding up)

$\therefore D_7$ is represented by the 8th score in the set of scores.

$\therefore$ seventh decile is 28.

Component 4

Part 4B Item 1

1. (i) Mean of Buddy's scores $= \frac{\text{sum of scores}}{\text{number of scores}}$

$$= \frac{42}{6}$$

$$= 7$$

Mean of Holly's scores $= \frac{\text{sum of scores}}{\text{number of scores}}$

$$= \frac{39}{6}$$

$$= 6.5$$

Buddy's scores arranged from lowest to highest are:

$6, 7, 7, 7, 7, 8$

As there is an even number of scores (6), the median is the average of the two middle scores (the 3rd and 4th scores, which are both 7).

Median of Buddy's scores $= \frac{7+7}{2}$

$$= 7$$

Holly's scores arranged from lowest to highest are:

$1, 5, 7, 8, 9, 9$

As there is an even number of scores (6), the median is the average of the two middle scores (the 3rd and 4th scores, 7 and 8).

Median of Holly's scores $= \frac{7+8}{2}$

$$= 7.5$$

(ii) The median is the better measure of their Mathematics ability because the mean for Holly's marks is heavily affected by one very low mark.

2. (i) For Buddy's scores, there are 3 scores below the median (7).

By Tukey's Method, the lower quartile Q_1 is the middle score of the 3 scores below the median.

$\therefore\ Q_1 = 7.$

(ii) For Holly's scores, there are 3 scores below the median (7.5).

By Tukey's Method, the lower quartile Q_1 is the middle score of the 3 scores below the median.

$\therefore\ Q_1 = 5.$

3. (i) Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 6$ and $n = 24$,

$Position\ of\ D_6 = \frac{6}{10}(24+1) = 0.6 \times 25 = 15$

∴ D_6 is represented by the 15th score in the set of scores.

∴ sixth decile is 68.

∴ the lowest of the marks that Mr Statz listed that Buddy would have had to achieve is 71 (lowest mark higher than the sixth decile).

(ii) Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 80$ and $n = 24$,

$Position\ of\ P_{80} = \frac{80}{100}(24+1) = 0.8 \times 25 = 20$

∴ P_{80} is represented by the 20th score in the set of scores.

∴ the eightieth percentile is 83.

∴ the lowest of the marks that Mr Statz listed that Holly would have had to achieve is 86 (lowest mark higher than the eightieth percentile).

Part 4C Item 2

1. (i) As there is an even number of scores (24), the median is the average of the two middle scores (the 12th and 13th scores, 65 and 67).

∴ Median of the scores for Mr Statz' class in the final examination$= \frac{65+67}{2}$

$= 66.$

Range of the scores for Mr Statz' class in the final examination = highest score − lowest score

$= 96 - 41 = 55$

(ii) The modal scores are the scores with the highest frequency. There are two scores with a frequency of 2 (the scores 55 and 67). All the other scores have a frequency of 1.

∴ The modal scores are 55 and 67.

2. (i) For Holly's scores, there are 3 scores above the median (7.5).

By Tukey's Method, the upper quartile Q_3 is the middle score of the 3 scores above the median.

∴ $Q_3 = 9$.

(ii) Tabulating Buddy's scores for the last six Mathematics class tests:

Score x	Frequency f	Deviation from mean ($\bar{x} = 7$) d	d^2	fd^2
6	1	-1	1	1
7	4	0	0	0
8	1	1	1	1
	$\Sigma f = 6$			$\Sigma(fd^2)$ =2

Using the formula, Standard Deviation $= \sqrt{\frac{\Sigma(fd^2)}{\Sigma f}}$

Standard deviation for Buddy's scores $= \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}} = 0.58$ (correct to 2 decimal places)

3. (i) Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 7$ and $n = 24$,

$Position\ of\ D_7 = \frac{7}{10}(24+1) = 0.7 \times 25 = 17.5 = 18$ (on rounding up)

$\therefore D_7$ is represented by the 18th score in the set of scores.

∴ seventh decile is 78.

Buddy achieved first mark higher than the seventh decile (78), which is 80.

(ii) Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 85$ and $n = 24$,

$Position\ of\ P_{85} = \frac{85}{100}(24+1) = 0.85 \times 25 = 21.25 = 22$ (on rounding up)

$\therefore P_{85}$ is represented by the 22nd score in the set of scores.

∴ the eighty-fifth percentile is 88.

Holly achieved first mark higher than the eighty-fifth percentile (88), which is 92.

Lesson 18 Deliberate Practice: Solving Problems involving Probability, Solving Problems involving Measures of Position, Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data

Syllabus Codes: M10SP-IIIj-1, M10SP-IVd-e-1, M10SP-IVh-j-1

Quarter: Grade 10 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of combinatorics and probability.

The learner demonstrates understanding of key concepts of measures of position.

Performance Standard

The learner is able to use precise counting techniques and probability in formulating conclusions and making decisions.

The learner is able to conduct systematically mini research, applying the different statistical methods.

Most Essential Learning Competencies

The learner solves problems involving probability.

The learner solves problems involving measures of position.

The learner uses appropriate measures of position and other statistical methods in analyzing and interpreting data.

Key Ideas

Solve problems involving probability.

Solve problems involving measures of position.

Use appropriate measures of position and other statistical methods in analyzing and interpreting data.

Matters for Students to Observe

(see above for notes for Lessons 13–17 and relevant to this lesson)

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $P(\text{both marbles are black}) = \left(\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{2}{6}\times\frac{1}{5}\right)=\frac{1}{20}+\frac{1}{30}=\frac{5}{60}=\frac{1}{12}$

2. (i) $$\begin{aligned}\text{Mean }(\bar{x}) &= \frac{\text{sum of scores}}{\text{number of scores}}\\ &= \frac{1+2+2+1+4+2}{6}\\ &= \frac{12}{6}\\ &= 2\end{aligned}$$

(ii)

Score x	Frequency f	d	d^2	fd^2
1	2	-1	1	2
2	3	0	0	0
4	1	2	4	4
	$\sum f=6$			$\sum(f\,d^2)=6$

Using the formula, Standard Deviation $= \sqrt{\frac{\sum(fd^2)}{\sum f}}$

Standard deviation $= \sqrt{\frac{6}{6}} = \sqrt{1} = 1$

3. Listing the set of scores $1, 2, 2, 1, 4, 2$ in ascending order we obtain:

   $1, 1, 2, 2, 2, 4$

   The median is the average of the two middle scores (both equal to 2) ∴ the median is 2.

   There are 3 scores below the position of the median.

   By Tukey's Method, the lower quartile $Q_1$is the middle score of these 3 scores.

   ∴ $Q_1 = 1$

   Again, by Tukey's Method, the upper quartile Q_3 is the middle score of the 3 scores above the position of the median.

   ∴ $Q_3 = 2$

   Interquartile Range (IQR) = Upper Quartile (Q_3) – Lower Quartile $(Q_1) = 2 - 1 = 1$

Component 4

Part 4B Item 1

1. (i) P(selecting a Western Suburb from the Southern Division)

   = probability of selecting Bag 2 and then a Western Suburb from the bag$= \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$

   (ii) P(selecting any of the city's Eastern Suburbs) $= \left(\frac{1}{2} \times \frac{4}{7}\right) + \left(\frac{1}{2} \times \frac{3}{8}\right) = \frac{2}{7} + \frac{3}{16} = \frac{53}{112}$

2. (i) Mode is score with the highest frequency. From the table, the score with the highest frequency (25) is 0.

   ∴ Mode $= 0$.

   Range is the difference between the highest and lowest score. From the table, highest score is 6 and lowest score is 0.

   ∴ Range $= 6 - 0 = 6$.

(ii)

Number of children per household in survey suburb x	Frequency f	$f \times x$	Cumulative Frequency
0	25	0	25
1	18	18	43
2	21	42	64
3	17	51	81
4	9	36	90
5	7	35	97
6	3	18	100
	$\sum f = 100$	$\sum(f \times x)$ =200	

Mean $= \frac{\sum(f \times x)}{\sum f} = \frac{200}{100} = 2$

There are 100 scores. ∴ The median is the average of the 50th and 51st scores.

Using the Cumulative Frequency column, the 50th and 51st scores are both 2.

∴ Median = 2.

3. To find Q_1.(the first quartile) for the set of scores using Tukey's Method, we consider the 50 scores below the position of the median and take the average of the two middle scores (the 25th and 26th scores of the data set, which from the Cumulative Frequency column are 0 and 1). ∴ $Q_1 = \frac{0+1}{2} = 0.5$.

   To find Q_3.(the third quartile) for the set of scores using Tukey's Method, we consider the 50 scores above the position of the median and take the average of the two middle scores (the 75th and 76th scores of the data set, which from the Cumulative Frequency column are both equal to 3). ∴ $Q_3 = \frac{3+3}{2} = 3$.

Part 4C Item 1

1. (i) P(selecting an Eastern Suburb from the Northern Division)

   $=$ probability of selecting Bag 1 and then an Eastern Suburb from the bag$= \frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$

   (ii) P(selecting any of the city's Western Suburbs) $= \left(\frac{1}{2} \times \frac{3}{7}\right) + \left(\frac{1}{2} \times \frac{5}{8}\right) = \frac{3}{14} + \frac{5}{16} = \frac{59}{112}$

2. (i)

Number of children per household in survey suburb x	Frequency f	$d = x - \bar{x}$	d^2	fd^2
0	25	-2	4	100
1	18	-1	1	18
2	21	0	0	0
3	17	1	1	17
4	9	2	4	36
5	7	3	9	63
6	3	4	16	48
	$\sum f$=100			$\sum(fd^2)$ =282

   (ii) Standard Deviation $= \sqrt{\frac{\sum(fd^2)}{\sum f}} = \sqrt{\frac{282}{100}} \cong 1.68$

3. Using $Position\ of\ D_i = \frac{i}{10}(n+1)$, where $i = 4$ and $n = 100$,

   $Position\ of\ D_4 = \frac{4}{10}(100+1) = 0.4 \times 101 = 40.4 = 41$ (on rounding up)

   $\therefore D_4$ is represented by the 41st score in the set of scores.

   $\therefore$ fourth decile is 1.

   Using $Position\ of\ P_i = \frac{i}{100}(n+1)$, where $i = 95$ and $n = 100$,

   $Position\ of\ P_{95} = \frac{95}{100}(100+1) = 0.95 \times 101 = 95.95 = 96$ (on rounding up)

   $\therefore P_{95}$ is represented by the 96th score in the set of scores.

   $\therefore$ the ninetieth-fifth percentile is 5.

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