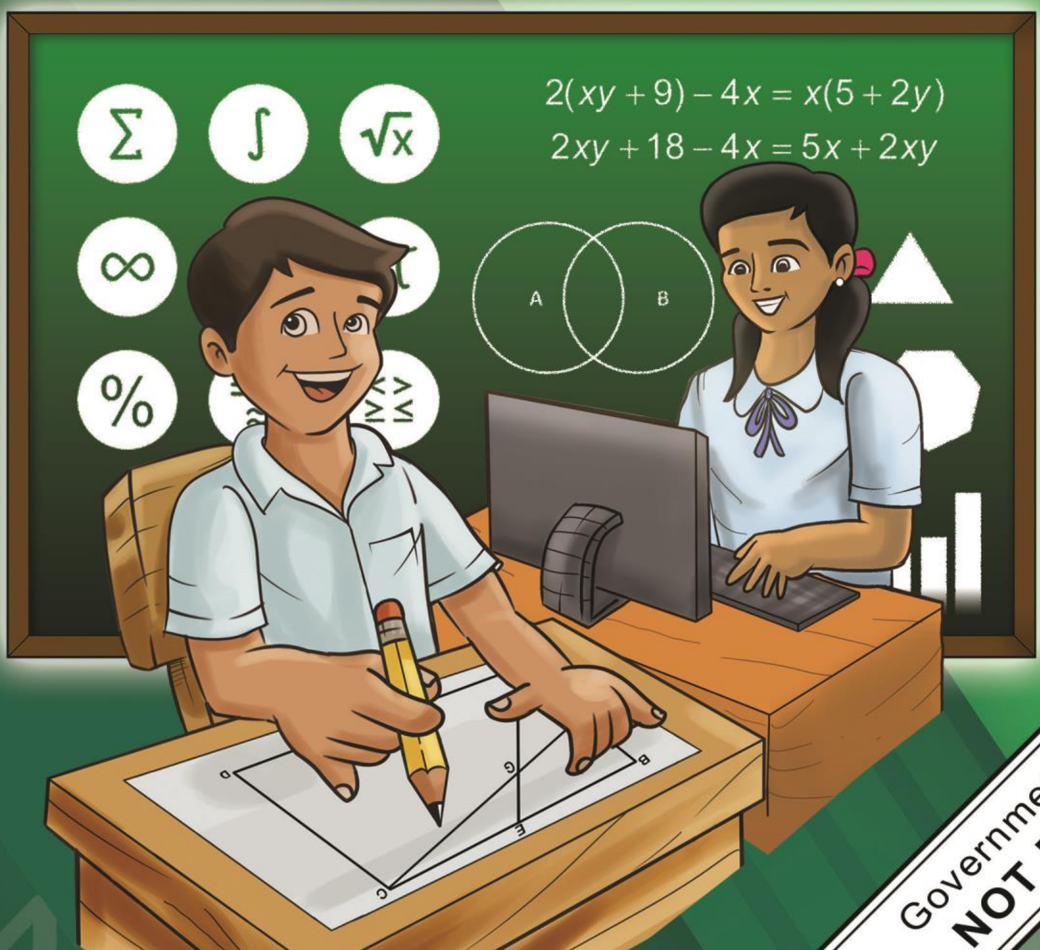
10

NATIONAL LEARNING CAMP

Mathematics

Enhancement Learning Camp

Lesson Plans



Enhancement Learning Camp
Lesson Plans Booklet

Mathematics Grade 10

Weeks 1 to 3

Contents

Dear Reader,

Every care has been taken to ensure the accuracy of the information provided in this Booklet. Nevertheless, if you identify a mistake, error, or issue, or wish to provide a comment, we would appreciate you informing the **Office of the Director of the Bureau of Learning Delivery** via telephone numbers (02) 8637-4346 and 8637-4347 or by email at bld.od@deped.gov.ph

Thank you for your support.

National Learning Camp Overview

Overview

The National Learning Camp (NLC) aims to enhance student and teacher learning through interactive lessons based on prior educational content. The program focuses on consolidating student knowledge, updating and expanding teacher expertise, and applying research-based strategies to improve learning teaching outcomes.

The NLC offers grade-level review lessons that are directed by the teacher and designed to be highly interactive among:

(i) students with their teacher; and
(ii) students with their peers.

The Camp lessons are grounded in the 'Science of Learning' framework, focusing on cognitive research and practical applications to enhance learning outcomes. Lessons are structured to reinforce foundational knowledge and skills, involve real-world problem-solving activities, and encourage higher-order thinking. The Camps also offer teachers opportunities for reflection and professional growth, encouraging the adoption of new teaching approaches and the extension of student learning through systematic review and application of knowledge.

Design Basis

A strength of the design is the focus on both student and teacher learning. The intentions and expectations of the NLC are for:

- students to consolidate and enhance their thinking in topics already covered;
- teachers to update, strengthen and expand their subject knowledge in ways that encourage students to be involved in learning activities at different levels including those considered as higher order.
- teachers to enhance their pedagogical practices by focusing on selected skills, which include 21st century skills.

Under the framework of 'Science of Learning', research-evidence is used to ground teaching and learning decisions around cognition research and features of a learning brain such as working memory demands, cognitive load, valuing errors, and domain specific skills. This framework highlights a *learning-focused approach* where teachers go beyond what might be considered current practice in the Philippines and incorporate brain-based ideas and approaches, including 21st Century skills, to make teaching more effective in enhancing learning for all.

To further support this direction, teachers are provided with resources, time and the opportunity to further extend their skills, knowledge and understandings of teaching and how students learn. The review lessons are designed to apply subject content already encountered by students. Because of this, lessons do not contain repetitive, routine questions of a particular subject aspect.

Review lessons

The review lessons are based on content already encountered the students in their current grade. All lessons *involve an exploration of ideas, concepts and content*. The purpose of the review lessons is two-fold:

(i) to establish in students a stronger basis for future learning development (prior to enrolling in a new Grade after the summer break); and

(ii) to enable teachers to strengthen and enrich their teaching practice in a research-based, learning-focused professional program (prior to a new academic year).

The primary focus of the review lessons concerns revising, clarifying and then applying previously-taught subject content with real-world problem-solving and/or comprehension activities. Each lesson begins with a focused content review and clarification of material needed in the lesson to come. For students, this initial review enables them to exercise retrieving and practicing important basics relevant to the lesson to come.

For the teachers, this information is designed to help determine the learners' subject background, knowledge, and skills relevant to the lesson as well as help teachers identify where to build on previous learning. This approach is different to 'teaching' students anew as if they have not been taught previously.

Lesson Overview

All lessons in each of the three subjects, English, Mathematics and Science, contain five components. These are 1. Short Review, 2. Purpose/Intention, 3, Language Practice, 4, Activity and 5, Conclusion.

Timing

Approximate component timings are indicated as advice to guide the teacher in pacing the lessons. Time management involves:

- moving through components at a pace that is appropriate for the learners;
- ensuring that all components are completed in a timely, efficient and constructive manner.

Research on student-learning quality and 'time' are related through student 'time-on-task'. Time-on-task refers to when students are actively involved (engaged) in some aspect of the learning process. The suggested times for each component are intended to maximize the time available for student involvement. This will encourage the student and teacher to work efficiently, timewise, through the lesson without jeopardizing the importance of student activities such as to:

- answer routine and non-routine questions,
- respond to verbal questions and explanations,
- interpret and use appropriate terminology,
- discuss aspects with their peers,
- explain or justify his/her approaches and thinking,
- work productively on their own, and
- listen carefully to the teacher or peers.

Establishing what is on-task time is more problematic when the teacher talks and students passively listen, such as in didactic teaching. With such an approach it is difficult to determine whether students are listening or even paying attention. Often in lessons identifying time-on task can also be problematic in case of problem-solving or intense reading and comprehension. Here, student activity is often more subtle and cerebral as students need to think quietly by themselves.

Ultimately, however, the time allocated to components will be determined by learners' needs and strengths, but not completely. There needs to be practical limits on the duration of the components to prevent major disruption to lessons which can have a detrimental impact on student learning. Often, teacher should not expect too much learning to occur on an initial meeting of unfamiliar content. It is repeated exposure associated with elaboration, addressing errors, and deliberately practicing key aspects where most learning occurs.

When times are allocated appropriately, and students become familiar with the approach and teacher expectations, concept development and student skill levels are improved as well as student engagement.

Note: Care needs to be exercised in determining what engagement means. Engagement is clearer when **students are doing the learning** through answering questions, writing, discussing and reading.

Key Ideas and Questioning

Critical aspects of the NLC for the teacher include questions related to learning areas, based around a *key idea*. The questions are offered at different levels of difficulty involving lower- to higher-order thinking, starting with questions of modest complexity up to those that require more developed reasoning.

In the lessons, students are provided with opportunities to practice solving non-routine questions to help improve their conceptual understanding by applying known content to subject-related problems.

Teacher Reflection

Teacher reflection on the lessons offer important insights to stimulate teachers and their peers to enhance their own practice and the learning of their students. This includes:

- new teaching approaches encouraged by lesson components that can contribute in different ways to student learning and lesson success;
- the use of review lessons that help review learnt material and extend student abilities in problem solving by utilizing known information;
- a focus on student concept and skill acquisition, pedagogical approaches, student errors, time-on-task, deliberate practice and working memory demands.

Enhancement and Consolidation Camps

The Enhancement Camp and the Consolidation Camp offer students the chance to review their subject background knowledge by consolidating previously taught material. The intention is:

- for students to have opportunities to review past work and to applying this knowledge of concepts and ideas through grade-related sets of questions of developing difficulty; and
- for teachers to follow the given format of components with some flexibility to adjust parts of a lesson to meet the learning needs of students in their class, particularly, if students are having difficulties.

Camp Differences

In the case of lessons for students in either the Enhancement Camp or Consolidation Camp, the materials, including the lesson plans and the sets of questions, are, on the surface, the same. These questions range from those of modest difficulty to those which require more insights and more knowledge and understanding.

There are important reasons for both Camps sharing the same content. Exploring and answering these question sets has value to students from both Camps, albeit in different ways. It enables students to work through a range of ideas on their own before hearing from their peers and teacher concerning the same questions – a very rich learning environment. Also, similar questions mean that expectations for students in both Camps is not limited and students have the same potential for growth.

The difference between Camps concerns the teaching focus, which is related to the breadth and depth of conceptual knowledge of students. It is anticipated that based on student performance within a lesson, the teacher will decide whether the class needs more practice and discussion of straightforward questions or whether extension material is more appropriate for the class.

In particular, questions marked as **Optional** (typically high-order questions) are more likely to be addressed in the Enhancement Camp than the Consolidation Camp, but not exclusively. It is the teacher who decides whether to include 'optional' questions and this will depend on student-learning success and understanding at that time.

If Optional questions are not used, teachers would spend that time productively. This includes reinforcing the concepts by increasing the focus on student errors and/or increasing student-student, and student-class directed conversations.

Lower- and Higher-order Skill and Knowledge Development

In all learning, lower-order thinking is a pre-requisite for higher-order skills and knowledge development. Many students are disadvantaged in their attempts to move forward in their learning through a lack of practice and conceptual development of needed lower-order skills, knowledge and understandings. Hence, *all* students benefit from a stock-take on relevant lower-order skills from previously addressed content. This helps establish a basis upon which student learning should build.

In both the Enhancement and Consolidation Camps important lower-order content skills, knowledge and understandings are re-visited at the beginning of each lesson. This helps ensure that potential learning obstacles are made visible to the student and the teacher. It also means that some errors in understanding or misconceptions are identified. This information is important to teachers in helping all students move forward regardless of their achievement levels.

As many questions posed are about applying content already encountered to a new problem, students have the opportunity to use their current knowledge, skill and understanding in a practical way at their level, further developing their conceptualization and understanding of the subject matter.

Both Camps offer students the opportunity to improve their learning and conceptual development by a stepped approach that involves:

(i) reminding students of relevant lower-order skills through practice,
(ii) having students use and discuss their knowledge in sets of graded questions with an emphasis on straightforward questions,
(iii) expecting students to apply their knowledge leading to more breadth in learning,
(iv) beginning an initial focused practice on higher-order skill development.

The approach advocated to solve problems or comprehend passages extends student learning beyond simple repetitive exercises sets. For these students the teaching part of the lesson requires teachers reviewing closely student solution attempts through student explanation, discussion and questioning of fundamental aspects of topics that are typically found in the earlier questions. Teachers should be sensitive to students' self-perceptions here as they may meet the ideas, presented in the lessons, maybe after many failures with these concepts in the past.

Nevertheless, these students should become aware of the more difficult questions as teachers allow them to consider links or connections between concepts previously taught. There is great value in problem solving for students to have time to read the problem and then be able to indicate in their own words, what the problem is about.

Finally, it is important that students in the Enhancement and Consolidation Camps become aware of what their students know, where it is progressing and how to build on student skills and knowledge. Teachers need to be nurturing and supportive of this development and continually look for evidence of success and growth. Teachers also need to encourage students to persist, continue to practice individual aspects, and use any mistakes/errors they make as an opportunity to learn more. These are important features of a successful learning journey.

Lesson Components: Short Overview

Lesson Component 1 (Lesson Short Review)

Component 1 offers teachers the chance to:

- settle the class quickly;
- review previously encountered information;
- address previous content in the form of a few targeted questions that are *relevant to the current lesson;*
- note what students already know;
- elicit answers from the class to reinforce the important content needed for the lesson; and
- address briefly issues that may arise.

The questions set for the Short Review section of a lesson are designed to *remind* students of knowledge and skills developed when first studying the topic area, which are relevant to the lesson.

Lesson Component 2 (Lesson Purpose/Intention)

This component offers teachers a chance to acquaint students with the purpose/intention of the lesson. It is valuable if students see a link here with their prior knowledge or experience, especially if the teacher can connect it to the responses and levels of student understanding evident in Component 1.

In addition, this component is an appropriate time to address what students might expect/aim to achieve, i.e., their lesson goal(s). Teachers should clarify, in clear language, the learning intention for the students as well as what success will look like. (Note**:** The degree of success or partial success of student learning in the lesson should occur as part of Component 5.)

Lesson Component 3 (Lesson Language Practice)

Component 3 concerns language use – speaking, hearing, listening and comprehending. The focus is on words or phrases that are to be used in the lesson.

The language practice suggested has been identified by considering the whole lesson and identifying those words/phrases that have the potential to cause difficulties for students through speech, or listening, or understanding. Typically, the language identified is restricted to less than 6 words/phrases so that there is enough time to use a variety of approaches of practice within the time available.

Lesson Component 4 (Lesson Activity)

Component 4 has three aspects, 4A, 4B, and 4C.

In the case of the Learning Camp activity, Component 4 addresses the key idea for the lesson. It is about students applying known content to solve real-world problems. This requires students to interpret/understand the correct meaning of the 'stem', a stimulus, (such as a passage/text or diagram or the first part of the problem or story) before answering questions of differing degrees of complexity related to the stem.

Students are first presented with the stem in 4A and are given the time/chance to interpret its meaning. Then in 4B and 4C, two separate sets of questions related to the same stem are asked.

4A Reading and Understanding the Stem

4A involves understanding the language of the stem. The purposes here are for the teacher:

- to model fluent reading of the stem (first)
- to identify any unfamiliar language for the student (possibly addressed in Component 3)
- to read the passage or describe the figure, etc.
- to hear and experience fluency in reading the stem.

4B Solving the First Set of Questions

4B involves a set of questions associated with the stem. Students will need to refer to the stem as they prepare to answer the set of questions. Students write down responses or attempts at each question. It is important that every student in the class is expected to have a response for each question. It is expected and acceptable that students would make errors, which provide teachers with important information concerning students' learning needs. A critical procedural action here for teachers is the importance of **all** students starting on the same set of questions, *at the same time*.

When the students are finished, or sufficient time has been allocated, the teacher marks the questions. This can be achieved by student answers or approaches to the questions and by explaining or justifying their reasons. Time should be allocated to student discussion, explanation, and reasoning about answers.

4C Solving the Second Set of Questions

4C offers a new start for students regardless of how they performed in Component 4B. The structure is very similar to Component 4B, i.e., undertaking a new set of questions related to the same stem. In addition, the lesson structure allows a refresh as 4C presents a new starting point for the student. This structure also allows all students in the class to start a new activity at the same time.

This approach serves two purposes for teachers. *First,* it enables teachers to bring all students back together to proceed as a group with issues able to be directed to and considered by every student at the same time. *Second,* it offers teachers a way to extend their students problem solving practice where *a different set of questions* can be used with a single Stem. This is an efficient way to incorporate more problem-solving or comprehension practice on specific content into a lesson.

Lesson Component 5 Lesson Conclusion

Component 5 has a high metacognitive aspect for students – students thinking about their own thinking – which can be further enhanced by teacher modelling. Component 5 is designed to offer a student-focused overview to the main intentions of the lesson. In particular, the focus is about helping students reflect on their progress and achievement (or partial achievements) of the lesson intention as well as their understanding development during the lesson.

It builds on comments from Component 2 about teacher expectations. There is the chance here to confirm student progress during the lesson. A teacher may use a diagram, picture, or some aspect of the lesson as a catalyst to stimulate student discussion and reflection.

NOTE: A fuller description of the Components and features of the lessons is provided in the **Learning Camp – Notes to Teachers Booklet.** It is recommended that these notes are read and discussed by teachers as they provide a further basis to understanding the structure of lessons and the pedagogy.

Mathematics Grade 10 Lesson Plan 1

Determining Arithmetic Means, $n\text{th}$ term of an Arithmetic Sequence, and Sum of the Terms of an Arithmetic Sequence

Key Idea

Determine arithmetic means, $n\text{th}$ term of an arithmetic sequence, and sum of the terms of an arithmetic sequence.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Find the arithmetic mean of:

   (i) -7 and 9 (ii) $2\sqrt{3} - 3$ and $2\sqrt{3} + 3$

2. Consider the arithmetic sequence: $2, -3, -8, \ldots$

   (i) Write down the next term of the sequence.

   (ii) Use the formula $a_n = a_1 + (n-1)d$ to write down a formula for the $n\text{th}$ term of the sequence and find its 15th term.

3. For the sequence in Question 2:

   (i) Calculate which term of the sequence is -93.

   (ii) Use the formula $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ to find the sum of 10 terms of the sequence.

Answers

1. (i) 1 (ii) $2\sqrt{3}$
2. (i) -13 (ii) $a_n = 7 - 5n,\ a_{15} = -68$
3. (i) 20th term (ii) $S_{10} = -205$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned about arithmetic sequences to help us find solutions to real-world problems. Today we will employ arithmetic sequences in finding solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

annual, arithmetic, formula, increase, mean, salary, sequence, term

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

In each of the previous three years, Marcella has worked for a large company, ABC Enterprises. In her first year with the company, she was paid a salary of ₱2 000 000 . In her third year, Marcella was paid ₱2 600 000 . In her second year, she was paid the arithmetic mean of these two amounts.

Marcella has recently taken a new job with XYZ Enterprises. She will be paid ₱2 500 000 in her first year, with annual salary increases of ₱200 000 in each future year.

Part 4B

Item 1

Questions

1. Calculate the amount that Marcella was paid in her second year with ABC Enterprises.
2. If Marcella had stayed with ABC Enterprises and received the same annual increase each year as during her first three years, find:
   (i) how much Marcella would have been paid in her tenth year with the company.
   (ii) in which of her years with the company she would have received a salary of ₱7 100 000 .
3. Use the formula $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ to find the total amount that Marcella would have earned with ABC Enterprises if she had remained with the company for ten years.

Answers to Item 1

1. ₱2 300 000
2. (i) ₱4 700 000
   (ii) 18th year
3. ₱33 500 000

Part 4C

Item 2

Questions

1. Insert 4 arithmetic means between Marcella's first year and third year salaries with XYZ Enterprises to obtain the salaries that she would have been paid each half year if the company had agreed to common half-yearly increases over the three years.
2. (i) Find how much Marcella will be paid in her 15th year with XYZ Enterprises.
   (ii) In which of her years with XYZ Enterprises will Marcella first receive a salary of over ₱4 000 000 ?
3. (Optional) How much more in total would Marcella have earned after 20 years at ABC Enterprises, if she had stayed with the company, than she will earn at XYZ Enterprises if she stays for 20 years?

Answers to Item 2

1. The 4 arithmetic means between 2 500 000 and 2 900 000 (the salaries (in PHP) that Marcella would have been paid each half year between the first half-year and sixth half-year) are 2 580 000, 2 660 000, 2 740 000, 2 820 000.
2. (i) ₱5 300 000
   (ii) 9th year
3. ₱9 000 000

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 2

Determining Geometric Means, nth term of a Geometric Sequence,
and Sum of the Terms of a Finite or Infinite Geometric Sequence

Key Idea

Determine geometric means, nth term of a geometric sequence, and sum of the terms of a finite or infinite geometric sequence.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Find the geometric mean of -4 and -16.
2. Consider the geometric sequence: $45, 15, 5, \ldots$
   (i) Write down the next term of the sequence.
   (ii) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the nth term of the sequence and find its 6th term.
3. (i) Use your formula in Question 2. (ii) for the nth term of the sequence to find which term is $\frac{5}{9}$.
   (Hint: Write both sides of the equation that you need to solve as a power of the same fraction.)
   (ii) Use the formulae $S_n = \frac{a_1(1-r^n)}{1-r}$ and $S_\infty = \frac{a_1}{1-r}$ to find the sum of five terms and the sum to infinity of the sequence.

Answers

1. ± 8
2. (i) $\frac{5}{3} (= 1\frac{2}{3})$
   (ii) $a_n = 45(\frac{1}{3})^{n-1};\ a_6 = \frac{5}{27}$
3. (i) 5th term (ii) $S_8 = 67\frac{2}{9};\ S_\infty = 67\frac{1}{2}$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned about geometric sequences to help us find solutions to real-world problems. Today we will employ geometric sequences in finding solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

common ratio, formula, geometric, infinity, mean, sequence

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Sonny has a ball that he drops on a hard flat surface from a point 60 meters above the surface. The ball bounces to a certain fraction of its original height and continues to bounce to the same fraction of each previous height. Sonny knows that on the ball's fourth bounce it attains a height of $\frac{15}{4}$.(or $3\frac{3}{4}$) meters.

Cher has a 'bouncier' ball that she drops from the same point. The ball bounces to $\frac{2}{3}$ of its original height and continues to bounce to $\frac{2}{3}$ of each previous height. Cher knows that on the ball's second bounce it attains a height of $\frac{80}{3}$.(or $26\frac{2}{3}$) meters.

Part 4B

Item 1

Questions

1. Insert 3 geometric means between 60 and $\frac{15}{4}$ by using the formula $a_n = a_1 r^{n-1}$ to first find r (positive value only is valid), which represents the fraction of each previous height that Sonny's ball attains on each bounce. (Note that the 3 geometric means represent the heights that the ball attains on its first, second and third bounces.)
2. (i) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the sequence in Question 1 (Sequence S).

   (ii) Use your formula for the nth term of Sequence S in Question 2 (i) to write down the 7th term of the sequence (the height in meters that Sonny's ball attains on its 6th bounce).
3. (i) Use the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ to find the sum of five terms of Sequence S.

   (ii) Find the limiting sum (or sum to infinity S_∞) of Sequence S.

Answers to Item 1

1. $r = \frac{1}{2}$; The 3 geometric means are: 30, 15, and $\frac{15}{2}$ (or $7\frac{1}{2}$).
2. (i) $a_n = 60(\frac{1}{2})^{n-1}$ (ii) $a_7 = \frac{15}{16}$
3. (i) $S_5 = \frac{465}{4}$ (or $116\frac{1}{4}$) (ii) $S_\infty = 120$

Part 4C

Item 2

Questions

1. Find the positive geometric mean of 60 and $\frac{80}{3}$, using that the positive geometric mean x of a and b is $x = \sqrt{ab}$. (This represents the height in meters that Cher's ball attains on its first bounce.)
2. (i) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the sequence of heights (Sequence T) described for Cher's ball.

   (ii) (Optional) Find on which bounce that Cher's ball attains a height of $\frac{320}{27}$ $(= 11\frac{23}{27})$ meters. (Hint: Write both sides of the equation that you need to solve as a power of $\frac{2}{3}$.)
3. (i) Use the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ to find the sum of five terms of Sequence T.

   (ii) (Optional) How many times greater than the limiting sum (S_∞) of Sequence S is the limiting sum of Sequence T?

Answers to Item 2

1. 40

2. (i) $a_n = 60(\frac{2}{3})^{n-1}$

   (ii) $n = 5$, $\therefore$ on the ball's fourth bounce.

3. (i) $S_5 = \frac{4220}{27}$ (or $156\frac{8}{27}$)

   (ii) $1\frac{1}{2}$.times greater

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 3

Solving Problems involving Sequences

Key Idea

Solve problems involving sequences.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Find the arithmetic mean and the geometric mean of -3 and -27.
2. For Sequence A: $-2, 1, 4, \ldots$ and Sequence B: $2, -1, \frac{1}{2}, \ldots$, find the:
   (i) 8th term
   (ii) sum of 8 terms.
3. For Sequence B in Question 2, find:
   (i) which term is $-\frac{1}{16}$.
   (Hint: Write both sides of the equation that you need to solve as a power of the same fraction.)
   (ii) S_∞

Answers

1. arithmetic mean: -15; geometric mean: ± 9
2. Sequence A: (i) $a_8 = 19$ (ii) $S_8 = 68$
   Sequence B: (i) $a_8 = -\frac{1}{64}$ (ii) $S_8 = \frac{85}{64} = 1\frac{21}{64}$
3. (i) 6th term
   (ii) $\frac{4}{3}$

Lesson Component 2 (Lesson Purpose/Intention)
Time: 3 minutes
The teacher states:

We can use what we have learned about arithmetic and geometric sequences to help us find solutions to real-world problems. Today we will employ arithmetic and geometric sequences in finding solutions to such problems.

Lesson Component 3 (Lesson Language Practice)
Time: 5 minutes
Key words/terms

arithmetic mean/geometric mean, increase, initial, net, period, sequence, term

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Henrietta managed a plantation of pine trees which were grown and sold for their timber. The plantation was developed from an initial planting of 45 000 trees, with a net increase, after sales and plantings, of 2500 trees each year over a period of 15 years.

At the beginning of the 16th year of the plantation, it was decided to sell-off 40% of the trees remaining on the plantation each year for timber, without any further plantings, so that the land could ultimately be used for another purpose.

Part 4B

Item 1

Questions

1. Use a formula for the nth term of a sequence to determine:
   (i) how many trees were on the plantation at the beginning of the 5th year (i.e., $n = 5$) of the plantation.
   (ii) at the beginning of which year of the plantation there were 65 000 trees on the plantation.
2. (i) Show that there were 82 500 trees on the plantation at the beginning of the 16th year of the plantation.
   (ii) Find the arithmetic mean of the initial (45 000) and final number of trees (82 500) on the plantation.
3. (Optional) Each year the plantation received a government subsidy of ₱100 for each tree that it had on the plantation at the beginning of that year. Calculate the total subsidy that the plantation had received by the beginning of the 16th year of the plantation.

Answers to Item 1

1. (i) 55 000
   (ii) 9th
2. (i) Using $a_n = 2500n + 42\,500$

$$a_{16} = 2500(16) + 42\,500$$
$$= 82\,500$$

   ∴ there were 82 500 trees on the plantation at the beginning of the 16th year of the plantation.

   (ii) 63 750
3. 102 000 000 PHP

Part 4C

Item 2

Questions

1. Calculate how many trees remained on the plantation at the beginning of the 17th year of the plantation (i.e., one year after beginning to sell-off trees).
2. (i) Use a formula for the nth term of a sequence (with $n = 5$) to determine how many trees remained on the plantation at the beginning of the 21st year of the plantation. (Note that a_1 will correspond to the answer obtained in Part 4C Question 1.)
   (ii) Show that there were less than 500 trees left on the plantation at the beginning of the tenth year after beginning to sell-off trees.

3. (Optional) Show that the limiting amount (S_∞) of government subsidy that the plantation could receive after beginning to sell-off trees is 12 375 000 PHP.

Answers to Item 2

1. 49 500

2. (i) approximately 6415

   (ii) Using $a_n = 49\,500(\frac{3}{5})^{n-1}$, where $a_1 = 49\,500, r = \frac{3}{5}, n = 10,$

$$a_{10} = 49\,500(\frac{3}{5})^{10-1}$$
$$\cong 499\ (< 500)$$

∴ there were less than 500 trees left on the plantation at the beginning of the tenth year after beginning to sell-off trees.

3. Limiting amount (S_∞) of subsidy $= \frac{49\,500}{1-0.6} \times 100$ PHP (using $\mathrm{S}_\infty = \frac{a_1}{1-r}$, where $a_1 = 49\,500, r = 0.6$).

$$= 123\,750\ \times 100\ \text{PHP}$$
$$= 12\,375\,000\ \text{PHP}.$$

∴ Limiting amount of government subsidy that the plantation could receive after beginning to sell-off trees is 12 375 000 PHP.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 4

Solving Problems involving Polynomials and Polynomial Equations

Key Idea

Solve problems involving polynomials and polynomial equations.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. What is the volume of a rectangular prism with length $(x + 5)$.meters, breadth $(x + 2)$ meters, and height x meters?
2. (i) The boxes in a set of boxes each have a height of $(x + 6)$ meters and a base area of $(2x^2 + x - 1)$ square meters. Find the height and base area of the box for which $x = 2$.

   (ii) What is the volume of the box for which $x = -4$?
3. Another box in the set of boxes in Question 2 (i), has a base area of 5 square meters. Solve the polynomial equation $2x^2 + x - 1 = 5$ to find the possible values of x for this box.

Answers

1. $(x^3 + 7x^2 + 10x)$ cubic meters
2. (i) height$= 8$ meters, base area $= 9$ square meters

   (ii) 54 cubic meters
3. $x = -2$ or $1\frac{1}{2}$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned about polynomials and polynomial equations to help us solve real-life problems. Today we will work with polynomials and polynomial equations to solve such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

binomial, long division, polynomial, polynomial equation, quadratic, rectangular prism, trinomial

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

A set of storage spaces (Set A), in the shape of a rectangular prism, have a height of $(2x - 1)$ meters and a volume of $(6x^3 - 13x^2 + x + 2)$ cubic meters.

Another set of storage spaces (Set B), again in the shape of a rectangular prism, also have a height of $(2x - 1)$ meters, with a base area of $(3x^2 - 7x - 6)$ square meters.

A third set of storage spaces (Set C), also in the shape of a rectangular prism, have a height of $(3x - 2)$ meters and a base area of $(2x^2 + x - 3)$ square meters.

Part 4B

Item 1

Questions

1. (i) For Set A, show that the base area of the storage spaces in square meters is $(3x^2 - 5x - 2)$.

   (ii) Hence, show that the volume in cubic meters of the storage spaces in Set A may be written as $(6x^3 - 13x^2 + x + 2) = (2x-1)(3x+1)(x-2)$.

2. (i) Find the height, base area, and volume of the storage space in Set A for which $x = 3$.

   (ii) By considering the height of the storage spaces in Set A, explain why the value of x for the storage spaces cannot be equal to or less than $\frac{1}{2}$.

3. (Optional)

   (i) One of the storage spaces in Set B has a base area of 14 square meters. Solve the polynomial equation $3x^2 - 7x - 6 = 14$ to show that the value of x for this storage space is $x = 4$.

   (ii) Hence, calculate the height and volume of this storage space.

Answers to Item 1

1. (i) Base area $= \frac{\text{Volume of storage spaces}}{\text{Height of storage spaces}}$

$$= \frac{6x^3 - 13x^2 + x + 2}{2x-1}$$

$$2x-1\,\overline{\smash{)}\,6x^3 - 13x^2 + x + 2}\quad\text{quotient: } 3x^2 - 5x - 2$$

(by 'long division' of polynomials)

Base area $= (3x^2 - 5x - 2)$ square meters

   (ii) From Question 1. (i) Volume in cubic meters $= 6x^3 - 13x^2 + x + 2$

$= (2x-1)(3x^2 - 5x - 2)$

$= (2x-1)(x-2)(3x+1)$ on factorizing $3x^2 - 5x - 2$.

2. (i) height $= 5$ meters; base area $= 10$ square meters; volume $= 50$ cubic meters

   (ii) The height (h) of the storage spaces in Set A is given by $h = 2x - 1$. On solving $2x - 1 \leq 0$, we obtain $x \leq \frac{1}{2}$. Therefore, if $x \leq \frac{1}{2}$ we would obtain a zero or negative height for the storage space.

3. (i) Since $3x^2 - 7x - 6 = 14$

$3x^2 - 7x - 20 = 0$

$(3x+5)(x-4) = 0$

$(3x+5) = 0$ or $(x-4) = 0$

$x = -\frac{5}{3}$ or $x = 4$

However, $x = -\frac{5}{3}$ is not a valid solution, since for $x = -\frac{5}{3}$, we would obtain a negative height for the storage space. $\therefore$ $x = 4$ is only solution.

   (ii) height= 7 meters; volume= 98 square meters

Part 4C

Item 2

Questions

1. For Set C, write down a polynomial that expresses the volume in cubic meters of each storage space in the set.
2. (i) Write the base area, $(2x^2 + x - 3)$ square meters, of the storage spaces in Set C in factored form.

   (ii) One of the storage spaces in Set C has a height of 16 meters. Show that the value of x for this storage space is $x = 6$, and hence find the base area and volume of the storage space.
3. (Optional)

   (i) A new Set B storage space is to be built that has a base area 6 square meters more than that of the Set C storage space with the same value of x. Solve a polynomial equation to show that this value of x is $x = 9$.

   (ii) Calculate the height, base area, and volume of the new storage space.

Answers to Item 2

1. $6x^3 - x^2 - 11x + 6$
2. (i) Base area $= 2x^2 + x - 3$

   $= (2x + 3)(x - 1)$

   (ii) Height $=\ 16$

   $\therefore 3x - 2 = 16$

   $3x = 18$

   $x = 6$

   Base area$=\ 75$ square meters; Volume $=\ 1200$ cubic meters.
3. (i) Polynomial equation:

   $(2x^2 + x - 3) + 6 = 3x^2 - 7x - 6$

   $0 = x^2 - 8x - 9$

   $x + 1 = 0$ or $x - 9 = 0$

   $\therefore\ x = -1$ or $x = 9$

   However, $x = -1$ is not a valid solution, since for $x = -1$, we would obtain a negative height for the storage space.

   $\therefore\ x = 9$ is only solution.

   (ii) height= 17 meters; base area= 174 square meters; volume = 2958 cubic meters

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material in this lesson high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 5

Solving Problems involving Polynomial Functions

Key Idea

Solve problems involving polynomial functions.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. (i) $U(x) = -10 + 7x - \frac{1}{3}x^2$ and $V(x) = 4x + 25$.

   If $W(x) = U(x) - V(x)$, show that $W(x) = -35 + 3x - \frac{1}{3}x^2$.

   (ii) Solve the polynomial ('quadratic') equation $-8 + 5x - \frac{1}{2}x^2 = 0$.

2. The equation of the axis of symmetry of the graph (a parabola) of the polynomial function $P(x) = ax^2 + bx + c$ is given by $x = -\frac{b}{2a}$.

   What is the equation of the axis of symmetry of the graph of $A(x) = -12 + 4x - \frac{1}{4}x^2$?

3. The graph of the polynomial function $Q(x) = -10 + 3x - \frac{1}{5}x^2$ is an inverted parabola that cuts the $x -$ axis at $(5, 0)$ and $(10, 0)$.

   Write down two inequalities in terms of x to represent the values of x for which $Q(x) < 0$.

   Write down the equation of the axis of symmetry of $Q(x)$ and the maximum value of $Q(x)$.

Answers

1. (i) $W(\mathrm{x}) = U(\mathrm{x}) - V(\mathrm{x})$

$$= -10 + 7x - \frac{1}{3}x^2 - (4x + 25)$$

$$= -10 + 7x - \frac{1}{3}x^2 - 4x - 25$$

$$= -35 + 3x - \frac{1}{3}x^2$$

   (ii) $x = 2$ or $x = 8$

2. $x = 8$

3. Inequalities: $x < 5$ or $x > 10$; Axis of symmetry of $Q(x)$: $x = 7\frac{1}{2}$; Maximum value of $Q(x) = 1\frac{1}{4}$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 5 minutes

Teacher states:

We can use what we have learned about polynomial equations and polynomial functions to help us solve real-life problems. Today we will work with polynomial equations and polynomial functions to solve such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

axis of symmetry, break-even, inverted, polynomial function, profit, revenue

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

GizmoCo is an exporting company that produces a product, the gizmo. The monthly profit that the company makes is determined by subtracting the cost of producing its gizmos from the revenue received from the sale of the gizmos.

In 2022, the cost (in thousands of U.S. dollars) of producing the company's gizmos each month was given by $C_1(x) = 2x + 30$, where x is the number of thousands of gizmos produced in a month, while the revenue received (in thousands of U.S. dollars) was given by $R_1(x) = 6x - \frac{1}{10}x^2$.

In 2023, GizmoCo found that it could reduce the cost of producing its gizmos each month, for $x > 30$, to $C_2(x) = x + 60$, while maintaining the same mathematical model for revenue i.e., $R_2(x) = 6x - \frac{1}{10}x^2$.

Part 4B

Item 1

Questions

1. (i) Show that GizmoCo's monthly profit (in thousands of U.S. dollars) in 2022 was given by the polynomial function $P_1(x) = -30 + 4x - \frac{1}{10}x^2$.

   (ii) Show, by solving the polynomial equation $P_1(x) = 0$, that in 2022 GizmoCo would only have 'broken even' (i.e., made a profit each month of 0 U.S. dollars) if it produced 10 thousand or 30 thousand gizmos.

2. (i) The graph of the polynomial function $P_1(x)$ is an inverted parabola with axis of symmetry $x = 20$. Use this information to find the maximum possible profit that GizmoCo could have made each month in 2022 from producing gizmos.

   (ii) Write down two inequalities in terms of x to represent the values of x for which the number of gizmos would have resulted in a loss each month for GizmoCo in 2022.

3. (i) Show that the polynomial function $P_2(x)$ that represented GizmoCo's profit each month in 2023 is given by $P_2(x) = -60 + 5x - \frac{1}{10}x^2$.

   (ii) (Optional) Find the amount of profit that GizmoCo would have made each month in 2023 if it produced the same number of gizmos (20 thousand) that would have maximized its profits in 2022.

Answers to Item 1

1. (i) $P_1(x) = R_1(x) - C_1(x)$

$$= 6x - \frac{1}{10}x^2 - (2x + 30)$$

$$= 6x - \frac{1}{10}x^2 - 2x - 30 = -30 + 4x - \frac{1}{10}x^2$$

   (ii) If $P_1(x) = 0$, then from Part 4B Question 1. (i),

$$-30 + 4x - \frac{1}{10}x^2 = 0$$

i.e., $x^2 - 40x + 300 = 0$

$(x - 10)(x - 30) = 0$

$x = 10$ or $x = 30$

∴ 'Break-even' production for GizmoCo in 2022 would have been 10 000 or 30 000 gizmos.

2. (i) Maximum possible profit each month in 2022 $= 10\,000$ U.S. dollars

(ii) $x < 10$ or $x > 30$

3. (i) $P_2(x) = R_2(x) - C_2(x)$

$$= 6x - \frac{1}{10}x^2 - (x + 60)$$

$$= 6x - \frac{1}{10}x^2 - x - 60$$

$$= -60 + 5x - \frac{1}{10}x^2$$

(ii) $P_2(20) = 0$

$\therefore$ GizmoCo would have only broken even each month in 2023.

Part 4C

Item 2

Questions

1. (i) Use the formulae for $P_1(x)$ and $P_2(x)$ to determine in which year, 2022 or 2023, GizmoCo would have made the greater loss by producing 5000 gizmos only.

(ii) By considering the information in Part 4B Question 1. (ii) and the symmetry of the polynomial function $P_1(x)$, write down the number of gizmos that, if produced in 2022, would have produced the same loss as for producing 5000 gizmos.

2. (i) Show, by solving the polynomial equation $P_2(x) = 0$, that GizmoCo would only have 'broken even' (i.e., made a profit of 0 U.S. dollars) if it produced 20 thousand or 30 thousand Gizmos in 2023.

(ii) Write down an inequality in terms of x to represent the values of x for which the number of gizmos would have resulted in a profit for GizmoCo in 2023.

3. (Optional)

(i) Show that the value of x that would have maximised GizmoCo's profit in 2023 was $x = 25$.

(ii) Find how much smaller GizmoCo's maximum possible profit each month was in 2023 compared to in 2022.

Answers to Item 2

1. (i) 2023

(ii) 35 000

2. (i) If $P_2(x) = 0$, then from Part 4B Question 3. (i),

$$-60 + 5x - \frac{1}{10}x^2 = 0$$

i.e., $x^2 - 50x + 600 = 0$

$(x - 20)(x - 30) = 0$

$x = 20$ or $x = 30$

$\therefore$ 'Break-even' production for GizmoCo in 2023 would have been 20 000 or 30 000 gizmos.

(ii) $20 < x < 30$

3. (i) The graph of the polynomial function $P_2(x)$ is an inverted parabola with axis of symmetry given by $x = -\frac{\text{coefficient of } x}{2\times\text{coefficient of } x^2} = -\frac{5}{2\times-\frac{1}{10}} = 25.$

(ii) Maximum possible profit in 2023 is 7500 U.S. dollars less than in 2022.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 6 Deliberate Practice

Solving Problems involving Sequences
Solving Problems involving Polynomials and Polynomial Equations
Solving Problems involving Polynomial Functions

Key Ideas

Solve problems involving sequences.

Solve problems involving polynomials and polynomial equations.

Solve problems involving polynomial functions.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Consider the following sequences: Sequence A: $-1, 3, 7, \ldots$ and Sequence B: $3, -1, \frac{1}{3}, \ldots$
   - (i) For Sequence A, write down a_n (the nth term of the sequence), and calculate S_6.
   - (ii) For Sequence B, calculate S_4 and S_∞.
2. If $P(x)$ is a polynomial function such that $P(x) = -9 + 4x - \frac{1}{3}x^2$, write $P(x) = 0$ in the form $ax^2 + bx + c = 0$, where $a = 1$, and solve $P(x) = 0$.
3. The graph of the polynomial function $Q(x)$ is an inverted parabola that cuts the x-axis at $(4, 0)$ and $(10, 0)$, and has vertex $(7,9)$.
   - (i) Write down two inequalities in terms of x that represent the values of x for which $Q(x) \leq 0$.
   - (ii) Find the maximum value of $Q(x)$.

Answers

1. (i) $a_n = 4n - 5$; $S_6 = 54$
   (ii) $S_4 = \frac{20}{9} = 2\frac{2}{9}$; $S_\infty = \frac{9}{4} = 2\frac{1}{4}$
2. $P(x) = 0$ can be written as $x^2 - 12x + 27 = 0$. On solving this equation, $x = 3$ or $x = 9$.
3. (i) $x \leq 4$ or $x \geq 10$
   (ii) 9

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We have been working on the solution of problems using arithmetic and geometric sequences, polynomials, polynomial equations, and polynomial functions. Today we are going to consolidate this work through the solution of such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

arithmetic sequence, euro, geometric sequence, polynomial equation, polynomial function, vertex

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Prism, a new type of modular multi-storey building, was constructed recently in Europe. The cost of constructing the first storey was $60\ 000$ euros, the second storey $70\ 000$ euros, and the third storey $80\ 000$ euros, with the cost continuing to increase by $10\ 000$ euros for each subsequent storey.

Jake works for The Whatsit Company in the new building and drops a ball from the top of the tenth storey to a hard flat surface on ground level, 40 meters below. The ball bounces to $\frac{3}{5}$ of its original height. Until it comes to rest, Jakes's ball continues to bounce to $\frac{3}{5}$ of each previous height.

The Whatsit Company produces a product called the doovalacky. Its monthly profit is determined by subtracting the cost of producing the doovalackies from the revenue received from the sale of the doovalackies. In 2023, this cost (in thousands of euros) was given by $C(x) = 2x + 45$, where x is the number of thousands of doovalackies produced in a month, while the revenue received (in thousands of euros) was given by $R(x) = 11x - \frac{1}{4}x^2$.

Part 4B

Item 1

Questions

1. Write down a formula for finding the cost of constructing a particular storey of *Prism*.

   Use the formula to find the cost of constructing the 15th storey of the building.

2. (i) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the sequence of heights (including the original height) of Jake's ball.

   (ii) Use your formula for the nth term of the sequence to write down the 5th term of the sequence (the height Jakes's ball attains on its 4th bounce).

3. (i) Show that The Whatsit Company's monthly profit (in thousands of euros) in 2023 was given by the polynomial function $P(x) = -45 + 9x - \frac{1}{4}x^2$.

   (ii) (Optional) Show, by solving the polynomial equation $P(x) = 0$, that in 2023 The Whatsit Company would only have 'broken even' (i.e., made a profit each month of 0 euros) if it produced 6 thousand or 30 thousand doovalackies.

Answers to Item 1

1. From the formula for the nth term of an arithmetic sequence, $a_n = a_1 + (n-1)d$, formula for finding the cost of constructing a particular storey of *Prism* is $a_n = 60\ 000 + 10\ 000(n-1)$.

   ∴ Cost of constructing the 15th storey (a_{15}). $= 200\ 000$ euros.

2. (i) $a_n = 40(\frac{3}{5})^{n-1}$

   (ii) $a_5 = 5\frac{23}{125}$

3. (i) $P(x) = R(x) - C(x)$

$$= 11x - \frac{1}{4}x^2 - (2x + 45)$$

$$= 11x - \frac{1}{4}x^2 - 2x - 45$$

$$= -45 + 9x - \frac{1}{4}x^2$$

(ii) If $P(x) = 0$, then from Part 4B Question 3. (i),

$$-45 + 9x - \frac{1}{4}x^2 = 0$$

i.e., $x^2 - 36x + 180 = 0$

$(x - 6)(x - 30) = 0$

$x = 6$ or $x = 30$

∴ 'Break-even' production for The Whatsit Company in 2023 would have been 6000 or 30 000 doovalackies.

Part 4C

Item 2

Questions

1. Write down a formula for finding the cost of constructing *Prism*.

   Use the formula to find the cost of constructing the building if it was built to a height of 20 storeys.

2. (i) Use the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ to find the sum of the heights (including its original height) attained by Jake's ball after its first four bounces.

   (ii) Find the limiting sum (or sum to infinity S_∞) of the sequence of heights of Jake's ball.

3. (i) The graph of the polynomial function $P(x)$ is an inverted parabola with axis of symmetry $x = 18$. Use this information to find the maximum possible profit that The Whatsit Company could have made each month in 2023 from producing doovalackies.

   (ii) (Optional) Write down two inequalities in terms of x to represent the values of x for which the number of doovalackies would have resulted in a loss each month for The Whatsit Company in 2023.

Answers to Item 2

1. From the formula for the sum of n terms of an arithmetic sequence, $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, formula for finding the cost of constructing building is $S_n = \frac{n}{2}(2(60\ 000) + 10\ 000(n-1))$.

   Cost of constructing the building $(S_{20}) = 3\ 100\ 000$ euros.

2. (i) $S_5 = \frac{11\ 528}{125}$ (or $92\frac{28}{125}$)

   ∴ the sum of the heights (including its original height) attained by Jake's ball after its first four bounces is $92\frac{28}{125}$ meters.

   (ii) $S_\infty = 100$

   ∴ the limiting sum (or sum to infinity S_∞) of the sequence of heights of Jake's ball is 100 meters.

3. (i) Maximum possible profit each month in 2023 occurs when $x = 18$.

   $P(18) = -45 + 9(18) - \frac{1}{4}(18)^2$

   $= 36$

   ∴ Maximum possible profit each month is 36 000 euros.

   (ii) $x < 6$ or $x > 30$

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 7

Solving Problems involving Circles

Key Idea

Solve problems involving circles.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Complete: An angle of 170 degrees forms a *complete revolution* with an angle of …………… degrees.

   Complete: The angle at the center of a circle is …………… the angle at the circumference standing on the same arc.

2. (i) Find, in terms of π, the arc length and area of a quadrant of the circle with radius 20 meters.

   (ii) Complete: When two chords intersect within a circle, the products of the intercepts are …………… .

3. (i) Complete: The product of the intercepts on a secant from an external point is equal to the …………… of the tangent from that point.

   Complete: Tangents to a circle from an external point have …………… length.

   (ii) A tangent of length 15 meters from an external point T meets a circle with center O and radius 8 meters at the point P on its circumference. Calculate the length of OT.

Answers

1. 190; twice
2. (i) Arc length of quadrant $= 10\pi$ meters; Area of quadrant $= 100\pi$ square meters

   (ii) equal
3. (i) square; equal

   (ii) 17 meters

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of geometry to help us solve real-life problems. Today we will use our knowledge of relationships involving circles in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

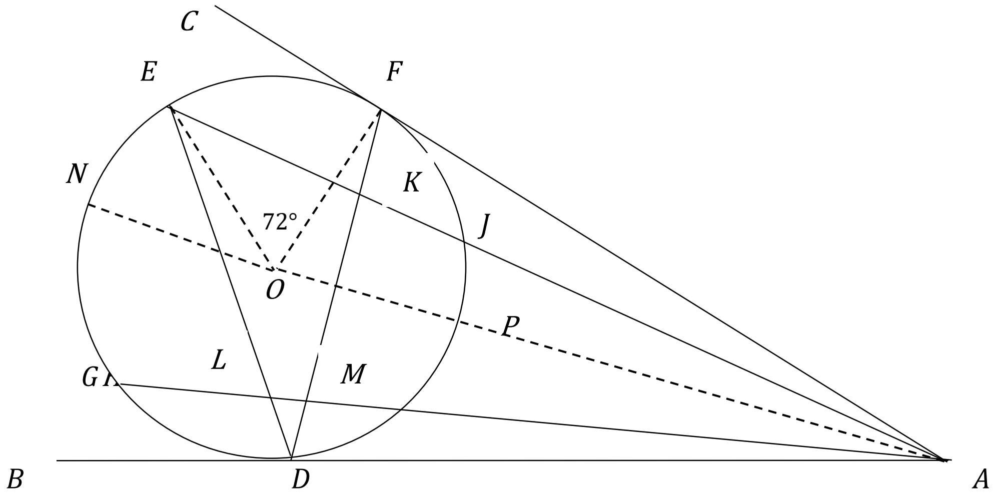
circumference, inscribed angle, intercept, intersecting, major/minor sector, secant, tangent, theorem

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

<u>Stem for Items 1 and 2</u>



A large park includes a circular area of radius 50 meters and a number of pathways that cross the area, as well as two pathways that are tangent to the area. Sebastian works for the council that manages the park and needs to check various measurements in relation to the circular park area for an upcoming project.

Sebastian knows that the circular park area, having a 50-meter radius, has an area of 2500π square meters and a circumference of 100π meters.

As part of the upcoming project, it is proposed that a new pathway AN (shown as a broken line) be constructed.

Part 4B

<u>Item 1</u>

<u>Questions</u>

1. (i) Show that the central reflex angle Ð EOF measures 288°.

   (ii) In the diagram, the central angle Ð EOF stands on the same arc EF as the inscribed angle Ð EDF. Sebastian has measured Ð EOF to be 72°. What measurement should he obtain for Ð EDF, the angle between pathways DE and DF?

2. (i) Given that the circular park area has a circumference of 100π meters and an area of 2500π square meters, calculate the length of the arc EF and the area of the minor sector EOF.

   (ii) The intersecting pathways DF and EJ are represented by intersecting chords in the diagram. Sebastian has obtained the measurements $EK = 12$ meters, $KJ = 8$ meters and $FK = 6$ meters. What length in meters should he obtain for the length of pathway section DK?

3. (Optional) For pathway AE and section AJ of pathway AE, Sebastian has obtained the measurements $AE = 170$ meters and $AJ = 150$ meters. Use this information to show that the length of pathway section AF, which is tangent to the circular park area, is 160 meters (to the nearest meter).

<u>Answers to Item 1</u>

1. (i) Central reflex angle Ð $EOF = 360°\text{ (angle of complete revolution)} - 72° = 288°$

   (ii) Ð $EDF = 36°$

2. (i) Length arc $EF = 20\pi$ meters; Area of minor sector $EOF = 500\pi$ square meters

   (ii) Length of pathway section $DK = 16$ meters

3. $AF^2 = AE \times AJ$ (Theorem: The product of the intercepts on a secant from an external point is equal to the square of the tangent from that point.)

$\therefore AF^2 = 170 \times 150$

$= 25\,500$

$\therefore AF = \sqrt{25\,500}$ =160 meters (to nearest meter).

Part 4C

Item 2

Questions

1. (i) Sebastian measures Đ DEO and Đ DFO and finds that they are equal. Show that Sebastian would have found that the two angles each measure 18°.

   (ii) Given that the circular park area has a circumference of 100π meters and an area of 2500π square meters, use the result in Part 4B Question 1. (i) to calculate the length in meters of the arc EDF and the area in square meters of the major sector EOF.

2. (i) Explain why pathway section AD is equal in length to pathway section AF and hence state the type of triangle represented by triangle ADF.

   (ii) Given that Đ $DFO = 18°$, find the sizes of Đ AFD and Đ DAF.

3. (Optional) In relation to the construction of the new pathway AN, Sebastian first measures OA. Use Pythagoras' theorem in triangle AFO to find the length of OA, and hence the length of AN (to the nearest meter).

Answers to Item 2

1. (i) Đ DEO + Đ DFO + Đ EOF (reflex) + Đ $EDF = 360°$ (angle sum of quadrilateral $EDFO$)

   2 Đ DEO + 288°+36°=360° (since Đ DEO = Đ DFO, Đ EOF (reflex) =288° from Part 4B Question 1. (i) and Đ $EDF = 36°$ from Part 4B Question 1. (ii)).

   $\therefore$ 2 Đ $DEO = 36° \Rightarrow$ Đ$DEO = 18°$ =Đ DFO

   (ii) Length arc $EDF = 80\pi$ meters; Area of major sector $EOF = 2000\pi$ square meters

2. (i) $AD = AF$ (Theorem: Tangents to a circle from an external point have equal length.)

   Triangle ADF is an isosceles triangle.

   (ii) Đ $AFD = 72°$; Đ DAF=36°

3. $OA = \sqrt{28\,000}$; AN=217 meters (to nearest meter)

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 8

Determining the Center and Radius of a Circle given its Equation, and vice versa

Key Idea

Determine the center and radius of a circle given its equation, and vice versa.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

<u>Questions</u>

1. (i) Write down the center and radius of the circle $x^2 + y^2 = 4$.

   (ii) Write down the equation of the circle with twice the radius of $x^2 + y^2 = 4$.

2. Determine the points where:

   (i) the circle $x^2 + y^2 = 9$ crosses the x- and y-axes.

   (ii) the circle $(x + 1)^2 + y^2 = \frac{9}{4}$ crosses the x-axis.

3. (i) Write the equation of the circle in Question 2 (ii) in general form.

   (ii) Use the equation to determine whether the point $(-1, 1\frac{1}{2})$ lies on the circle.

<u>Answers</u>

1. (i) center: $(0, 0)$; radius: 2 units

   (ii) $x^2 + y^2 = 16$

2. (i) $x^2 + y^2 = 9$ crosses the x-axis at $(-3, 0)$ and $(3, 0)$, and the y-axis at $(0, -3)$ and $(0, 3)$.

   (ii) $(x + 1)^2 + y^2 = \frac{9}{4}$ crosses the x-axis at $(-\frac{5}{2}, 0)$ and $\left(\frac{1}{2}, 0\right)$

3. (i) $4x^2 + 4y^2 + 8x - 5 = 0$

   (ii) On substitution of $x = -1, y = 1\frac{1}{2}$, $LHS = 4(-1)^2 + 4\left(\frac{3}{2}\right)^2 + 8(-1) - 5 = 0 = RHS$

   $\therefore$ $(-1, 1\frac{1}{2})$ lies on the circle.

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of geometry and algebra to help us solve real-life problems. Today we will use our knowledge of circles, equations, and the coordinate plane, in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

center-radius form, concentric, construct, general form, radii, radius

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Eliza has been studying circles, including finding their centers, radii, and equations, whether particular points lie on the circles being considered, as well as constructing a number of circles on graph paper.

She has also become interested recently in studying concentric circles (i.e., circles with the same center, but of different radius) and is constructing such circles for a range of artistic designs.

Part 4B

Item 1

Questions

1. Eliza has constructed on graph paper a circle (Circle A) with equation (in center-radius form) $x^2 + y^2 = 6\frac{1}{4}$.
   (i) What is the center and radius of Circle A?
   (ii) Write down the equation of the circle (Circle B) with the same center as Circle A and four times the radius.
2. Eliza constructs on graph paper a circle (Circle C) with center$(-\frac{3}{4}, 0)$ and with radius half of that of Circle A.
   (i) Write down the equation of Circle C in center-radius form.
   (ii) Determine the points where Circle C crosses the y-axis.
3. (i) Eliza constructs two concentric circles (Circle D and Circle E) with center $(-2, 5)$. Circle D has radius $(\sqrt{7} + 1)$ cm, while Circle E is nine times the area of Circle D.

   Find the equation of Circle D (in center-radius form), and the equation of Circle E (in center-radius form) after first finding the radius of Circle E.

   (ii) (Optional) To graph the circle, Circle F, with equation (in general form) $x^2 + y^2 - 6x + 2y - 15 = 0$, Eliza needs first to write the equation in center-radius form. By completing squares, write the equation in center-radius form and use it to state the center and radius of the circle.

Answers to Item 1

1. (i) Circle A: center $(0, 0)$; radius $2\frac{1}{2}$
   (ii) Equation Circle B: $x^2 + y^2 = 100$
2. (i) Equation Circle C: $(x + \frac{3}{4})^2 + y^2 = \frac{25}{16}$
   (ii) Circle C crosses y-axis at $(0, -1)$ and $(0, 1)$.
3. (i) Radius of Circle $E = (3\sqrt{7} + 3)$ cm

   Equation Circle D: $(x + 2)^2 + (y - 5)^2 = 8 + 2\sqrt{7}$

   Equation Circle E: $(x + 2)^2 + (y - 5)^2 = 72 + 18\sqrt{7}$

   (ii) Equation Circle F: $(x - 3)^2 + (y + 1)^2 = 25$

   Center: $(3, -1)$; radius 5

Part 4C

Item 2

Questions

1. Write down the center and radius of the circles:

   (i) Circle G: $5x^2 + 5y^2 - 45 = 0$

   (ii) Circle H: $16(x+1)^2 + 16(y-5)^2 = 81$.

2. (i) Write down the equation (in center-radius form) of the circle (Circle I) concentric with the circle (Circle J)$(x-7)^2 + (y+24)^2 = 625$ and one twenty-fifth its area.

   (ii) Determine the points where Circle J crosses the x-axis.

3. (Optional) Write the equation in general form of the circle, Circle K, with center $(-\frac{1}{2}, 3\frac{1}{2})$ and radius $5\frac{1}{2}$units and use the equation to determine whether the point $(-1,-2)$ lies on the circle.

Answers to Item 2

1. (i) Circle G: center $(0,0)$; radius 3

   (ii) Circle H: center $(-1,5)$; radius $\frac{9}{4}$

2. (i) Equation Circle I: $(x-7)^2 + (y+24)^2 = 25$

   (ii) Circle J crosses x-axis at $(0,0)$ and $(14,0)$.

3. Equation Circle K (in general form) : $4x^2 + 4y^2 + 4x - 28y - 71 = 0$

   Point $(-1,-2)$ does not lie on the circle.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area(   )(   )?

Mathematics Grade 10 Lesson Plan 9

Graphing and Solving Problems involving Circles and other Geometric Figures on the Coordinate Plane

Key Idea

Graph and solve problems involving circles and other geometric figures on the coordinate plane.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

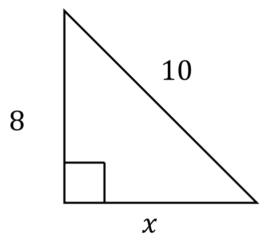
Questions

1. Complete: The midpoint (x, y) of the point (x_1, y_1) and (x_2, y_2) can be found using $M(x, y) = (\_\_\_, \_\_\_)$

   Complete: The distance between the points (x_1, y_1) and (x_2, y_2) can be found using the formula $d = \sqrt{\_\_\_\_\_\_}$

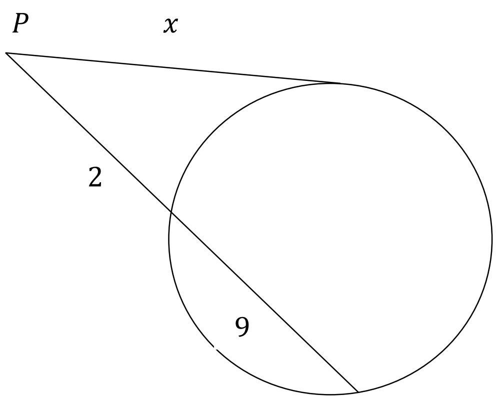
2. (i) Without using the distance formula, write down the length of the vertical interval joining $(3, 1\frac{1}{2})$ and $(3, 6\frac{1}{2})$.

   (ii)



   Find the value of x and the area of the triangle.

3.



   The diagram shows a tangent and a secant to a circle from the point P.

   Find the value of x.

Answers

1. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$; $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. (i) 5 units (ii) $x = 6$ units; Area of triangle$=$ 24 square units
3. $x = \sqrt{22}$ units

Lesson Component 2 (Lesson Purpose/Intention)

Time: 5 minutes

The teacher states:

We can use what we have learned in our study of geometry and algebra to help us solve real-life problems. Today we will use our knowledge of circles, other geometric figures, and the coordinate plane, in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

coordinate plane, geometric, interval, perpendicular, point of intersection, Pythagoras, vertical

Lesson Component 4 (Lesson Activity)

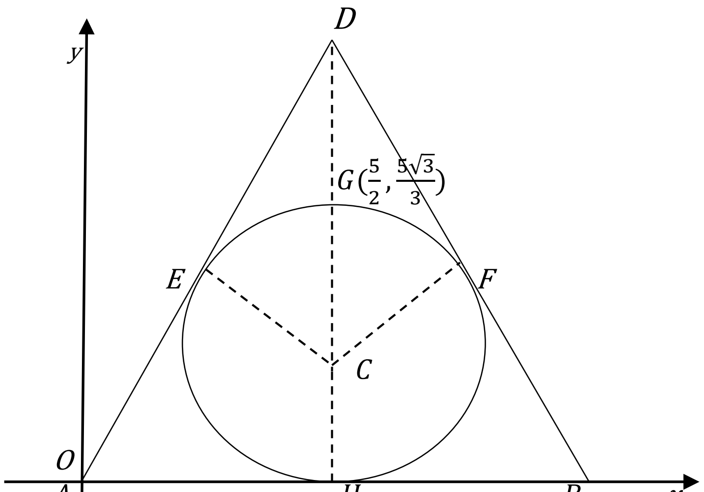
Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Jason is designing a company logo. He has inscribed a circle in an equilateral triangle of side length 5 units. Vertex A of the triangle is at the origin $(0, 0)$ and vertex B is at $(5, 0)$ on the x-axis. Jason has drawn the straight line DH which divides the triangle ABD in half and passes through the center of the circle C and the point G on the circumference of the circle. He knows that the coordinates of G are $(\frac{5}{2}, \frac{5\sqrt{3}}{3})$.

The diagram below shows Jason's design on the coordinate plane.



Part 4B

Item 1

Questions

1. Write down the coordinates of the point H on the x-axis.
   Use the coordinates of G and H to show that the coordinates of point C, the center of the circle, are $(\frac{5}{2}, \frac{5\sqrt{3}}{6})$.

2. (i) Show that the length of CH is $\frac{5\sqrt{3}}{6}$ i.e., the radius of the circle.

(ii) Use Pythagoras' theorem in triangle BDH ($\angle BHD$ is a right angle) to find the length of DH. Hence, write down the coordinates of point D.

3. (Optional) Use the coordinates of C and D to write down the length of CD, and use the distance formula to find the length of AC and BC. Hence show that $AC = BC = CD$. (This means that the point C, the center of the circle, is equidistant from the vertices of the triangle ABD.)

<u>Answers to Item 1</u>

1. H has coordinates $\left(\frac{5}{2}, 0\right)$.

   Center of circle C is midpoint of diameter GH.

   Midpoint of $GH = (\frac{5}{2}, \frac{\frac{5\sqrt{3}}{3}+0}{2})$ (Note that C is on same vertical line as G and H and so has same x-coordinate.)

   $\therefore$ Coordinates of C are $(\frac{5}{2}, \frac{5\sqrt{3}}{6})$.

2. (i) Length of CH is the difference of y-values of C and H as CH is a vertical line.

   $\therefore$ Length of $CH = \frac{5\sqrt{3}}{6} - 0 = \frac{5\sqrt{3}}{6}$ (radius of the circle).

   (ii) Length of $DH = \frac{5\sqrt{3}}{2}$; $\therefore$ Coordinates of D are $(\frac{5}{2}, \frac{5\sqrt{3}}{2})$.

3. Length of $AC = \frac{5\sqrt{3}}{3}$; Length of $BC = \frac{5\sqrt{3}}{3}$

   Length of $CD = \frac{5\sqrt{3}}{3}$

   $\therefore$ Length of $AC =$ Length of $BC =$ Length of $CD = \frac{5\sqrt{3}}{3}$.

Part 4C

<u>Item 2</u>

<u>Questions</u>

1. (i) Use the coordinates of D and G to find the length of DG and hence show that this length is equal to the radius of the circle.

   (ii) Write down the geometric reason that $\angle CFD$ is a right angle.

   Use Pythagoras' theorem in triangle CFD to find the length of DF.

2. Let the radius of the circle be equal to R units. Write CF and CD in terms of R and use Pythagoras' theorem to express DF in terms of R.

   State what multiple of the radius of the circle is the side length of the triangle.

3. (Optional)

   Jason knows that $m(DF)^2 = m(DH)\ m(DG)$ from his knowledge of geometry theorems involving tangents and secants. Complete the relevant theorem: The product of the intercepts on a secant from an external point equals
   the _____________ from that point.

   Use $DF^2 = DH.DG$ to find the length of DF and check that it is consistent with your answer in Part 4C Question 1. (ii).

   Find the area of triangle CFD and what fraction it is of the area of triangle ABD.

Answers to Item 2

1. (i) Length of $DG = \frac{5\sqrt{3}}{6}$ = radius of circle from Part 4B Question 2. (i)

   (ii) $\angle CFD$ is angle between tangent DF to circle and radius CF of circle.

   $\therefore \angle CFD$ is a right angle (A tangent is perpendicular to the radius drawn to its point of intersection.)

   Length of $DF = 2\frac{1}{2}$

2. $CF = R; CD = 2R; DF = \sqrt{3}R$

   Side length of the triangle (5 units) is $2\sqrt{3}$ times the radius ($\frac{5\sqrt{3}}{6}$ units) of the circle.

3. The product of the intercepts on a secant from an external point equals the square of the tangent from that point.

   $DF = 2\frac{1}{2}$, which is consistent with the answer obtained in Part 4C Question 1. (ii).

   Area of triangle $CFD = \frac{25\sqrt{3}}{24}$ square units

   Area of triangle $CFD = \frac{1}{6}$ of area of triangle ABD.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 10

Differentiating Permutation from Combination of Objects taken r at a time

Key Idea

Differentiate permutation from combination of objects taken r at a time.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. How many different pairs of symbols can be taken from the set {▲,⬟,⬢} if:
   (i) order within a pair is important?
   (ii) order within a pair is not important?
2. Complete the formulae:
   (i) ${}^nP_r = \frac{\quad}{(n-r)!}$
   (ii) ${}^nC_r = \frac{\quad}{r!\,(\ldots\ldots)}$
3. Calculate:
   (i) ${}^5P_2$
   (ii) ${}^7C_3$

Answers

1. (i) 6 (ii) 3
2. (i) ${}^nP_r = \frac{n!}{(n-r)!}$
   (ii) ${}^nC_r = \frac{n!}{r!\,(n-r)!}$
3. (i) 20 (ii) 35

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of probability to help us solve real-life problems. Today we will use our knowledge of differentiating permutation from combination of objects taken r at a time in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

combination, digit, element, ordered selection, permutation, repetition, unordered selection

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Ralph is learning about permutations (ordered selections of items) and combinations (unordered selections of items) and the different results obtained when making ordered and unordered selections.

He is considering:

(i) the set (Set A) of the first four letters of the English alphabet i.e., Set $A = \{a, b, c, d\}$.

(ii) the set (Set B) of the whole numbers from 1 to 6 i.e., Set $B = \{1, 2, 3, 4, 5, 6\}$.

Part 4B

Item 1

Questions

1. (i) List the possible permutations of the elements of Set A if Ralph takes the elements two at a time (Hint: ab is one of the possible permutations).

   (ii) Of the possible permutations in Part B Question 1. (i), find how many are the different possible combinations of the elements of Set A when the elements are taken two at a time, using the formula ${}^{n}C_r = \frac{n!}{r!\,(n-r)!}$

2. If Ralph takes the elements of Set A three at a time, how many different permutations and how many different combinations are possible?

3. Ralph wishes to create three-digit numbers from the digits in Set B, using no digit more than once.

   (i) How many three-digit numbers can he create?

   (ii) (Optional) How many three-digit numbers that are less than 500 can he create?

Answers to Item 1

1. (i) $\{ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc\}$ (ii) 6

2. 24; 4

3. (i) 120 (ii) 80

Part 4C

Item 2

Questions

1. (i) How many permutations of the elements of Set A are possible if Ralph takes the elements four at a time?

   (ii) Of the possible permutations in Part 4C Question 1. (i), find how many are the different possible combinations of the elements of Set A where the elements are taken four at a time.

2. How many different numbers are possible if Ralph uses no digit more than once and takes the elements of Set B:

   (i) two at a time to create two-digit numbers?

   (ii) four at a time to create four-digit numbers?

3. (Optional) Using the digits in Set B, Ralph wishes to:

   (i) create four-digit numbers, with repetition of digits allowed. How many numbers can he create?

   (ii) select two digits, place them on separate cards, and place the two cards in a separate envelope. How many envelopes will he need for the possible different pairs?

Answers to Item 2

1. (i) 24 (ii) 1
2. (i) 30 (ii) 360
3. (i) 1296 (ii) 15

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 11

Solving Problems involving Permutations and Combinations

Key Idea

Solve problems involving permutations and combinations.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Show that ${}^5P_3 = 6\ ({}^5C_3)$
2. In how many ways can 5 objects $\{V, W, X, Y, Z\}$ be arranged in a line if objects W, X and Y are to be together at the front of the line?
3. Groups of 5 people are to be selected from 6 men and 4 women.

   How many different groups can be selected if:

   (i) there are to be 3 men and 2 women in each group?

   (i) there are to be 2 men and 3 women in each group?

Answers

1. ${}^5P_3 = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$

   ${}^5C_3 = \frac{5!}{3!(5-3)!} = \frac{120}{6(2)} = 10$

   $\therefore$ Since $60 = 6 \times 10$, ${}^5P_3 = 6 \times {}^5C_3$, as required.
2. 12
3. (i) 120

   (ii) 60

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of probability to help us solve real-life problems. Today we will use our knowledge of permutations and combinations in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

arrange, combination, permutation, restriction, selection

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Suellen is a teacher who needs to arrange her students in different groups for various class activities.

On a recent day she needed to:

(i) arrange six of her students $\{A, B, C, D, E, F\}$ in a line for a morning activity.

(ii) select groups of four of her students from five boys and three girls for an afternoon activity.

Part 4B

Item 1

Questions

1. (i) How many permutations of the six students could Suellen have obtained for the morning activity if she took only three students at a time?

   (ii) In how many ways could Suellen have arranged all six students for the morning activity if there were no restrictions on how they were to be arranged?

2. In how many ways could Suellen have arranged all six students for the morning activity if A and B were to be together in the line?

3. How many groups of four of her students could Suellen have selected for the afternoon activity if there were the following restrictions:

   (i) all members of the group were to be boys?

   (ii) (Optional) there were to be all three girls and one boy in the group?

Answers to Item 1

1. (i) 120

   (ii) 720

2. 240

3. (i) 5

   (ii) 5

Part 4C

Item 2

Questions

1. (i) How many permutations of the six students could Suellen have obtained for the morning activity if she took only five students at a time?

   (ii) How many groups of four of her students could Suellen have selected for the afternoon activity if there were no restrictions on how they were to be selected?

2. In how many ways could Suellen have arranged all six students for the morning activity if D was to be second in the line and F was to be fifth in the line?

3. How many groups of four of her students could Suellen have selected for the afternoon activity if there were the following restrictions:

   (i) there were to be two boys and two girls in the group?

   (ii) (Optional) a particular girl X was to be included in the group?

Answers to Item 2

1. (i) 720

   (ii) 70

2. 24

3. (i) 30

   (ii) 35

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 12 Deliberate Practice

Solving Problems involving Circles
Graphing and Solving Problems involving Circles and Other Geometric Figures on the Coordinate Plane
Solving Problems involving Permutations and Combinations

Key Ideas

Solve problems involving circles.

Graph and solve problems involving circles and other geometric figures on the coordinate plane.

Solve problems involving permutations and combinations.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

<u>Questions</u>

1. (i) Without using the distance formula, write down the length of the horizontal interval joining $(-4\frac{1}{2}, 1)$ and $(7, 1)$.

   (ii) Find the equation of the circle with center $(0, 0)$ and radius 12 units.

2. (i) Use the distance formula to find the length of the interval joining the points $(5, -2\sqrt{3})$ and $(2, 2\sqrt{3})$.

   (ii) Complete: The area of a rhombus is given by the formula $Area\ of\ rhombus = \frac{1}{2}xy$, where x and y are the lengths of the …………… of the rhombus.

3. Groups of 7 people are to be selected from 5 men and 6 women.

   How many different groups can be selected if there are to be 3 men and 4 women in each group?

<u>Answers</u>

1. (i) $11\frac{1}{2}$ (ii) $x^2 + y^2 = 144$
2. (i) $\sqrt{57}$ units (ii) diagonals
3. 150

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We have been working on the solution of problems involving circles, other geometric figures on the coordinate plane, and permutations and combinations. Today we are going to consolidate this work through the solution of such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

arc, coordinate plane, diagonal, rhombus, sector, vertex/vertices, vertical

Lesson Component 4 (Lesson Activity)

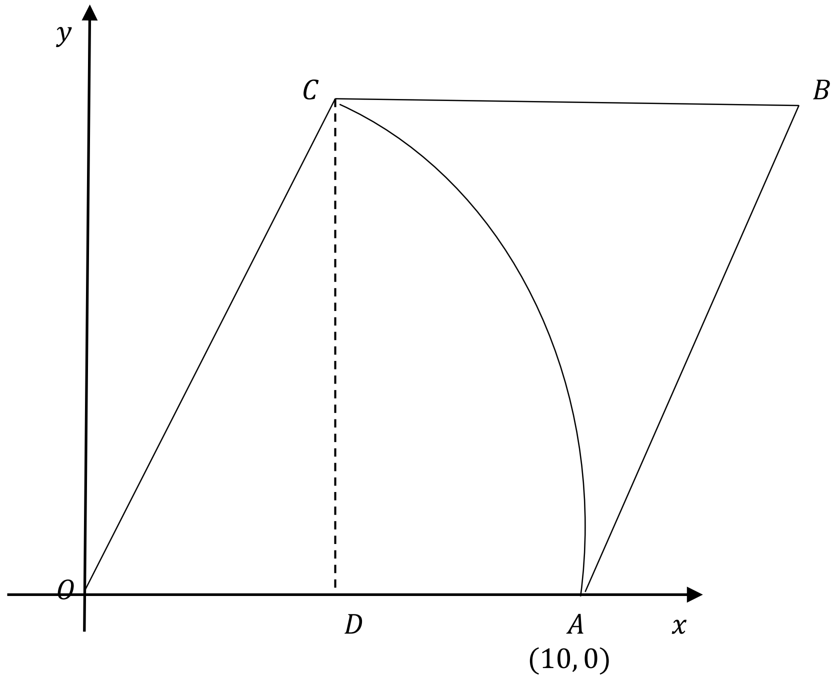
Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Sarah is a member of a professional association of graphic designers. The first stage of her latest design involves a rhombus of side length 10 units and a sector of a circle with center at the vertex O of the rhombus, which corresponds with the origin of the coordinate plane. The vertices A and C of the rhombus lie at the ends of the arc of the sector. The point D on the x-axis is the midpoint of the side OA of the rhombus and lies at one end of the vertical line CD.

The diagram below shows the first stage of Sarah's design.



Sarah has also recently become responsible for establishing a number of committees for her professional association.

Part 4B

Item 1

Questions

1. (i) Find the coordinates of the point D on the x-axis.

   (ii) Sector AOC is part of the circle with' center O and radius OA.

   Write down the equation of this circle.

2. (i) Use the equation of the circle found in Part 4B Question 1. (ii) and the fact that CD is a vertical line to find the coordinates of the point C.

   (ii) Use the distance formula to find the length of AC, the shorter diagonal of the rhombus $OABC$.

3. Sarah is forming Committee A, a committee of six graphic designers from a group of three men and five women.

   (i) How many different committees can she form if there are no restrictions?

   (ii) (Optional) How many different committees can she form if there are to be 2 men and 4 women on the committee?

Answers to Item 1

1. (i) $(5, 0)$ (ii) $x^2 + y^2 = 100$

2. (i) $(5, 5\sqrt{3})$ (ii) 10 units

3. (i) 28 (ii) 15

Part 4C

Item 2

Questions

1. (i) Find the coordinates of the vertex B of the rhombus.

   (ii) Use the distance formula to find the length of OB, the longer diagonal of the rhombus.

2. (i) Sarah knows that the area of a rhombus is given by the formula $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the rhombus.

   Use the formula to find the area of the rhombus $OABC$.

   (ii) (Optional) Given that $\angle AOC = 60°$, find the area of the sector AOC, and the area of the section of the rhombus outside the sector (in square units correct to 1 decimal place).

3. Sarah is forming Committee B, a committee of five graphic designers from a group of four men and seven women.

   (i) How many different committees can she form if all members of the committee are to be women?

   (ii) (Optional) How many different committees can she form if a particular man and a particular woman are to be included on the committee?

Answers to Item 2

1. (i) $(15, 5\sqrt{3})$ (ii) $10\sqrt{3}$ units

2. (i) $50\sqrt{3}$ square units (ii) Area of sector $AOC = \frac{50\pi}{3}$square units; Area of the section of the rhombus outside sector $AOC = 34.2$ square units (correct to 1 decimal place)

3. (i) 21 (ii) 84

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 13

Illustrating and finding the Probability of a Union of Two Events $(A \cup B)$, including for when the events are mutually exclusive

Key Idea

Illustrate and find the probability of a union of two events $(A \cup B)$, including for when the events are mutually exclusive.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. For two events X and Y, $P(X) = \frac{1}{2}, P(Y) = \frac{1}{4}$ and $P(X \cap Y) = \frac{2}{5}$.

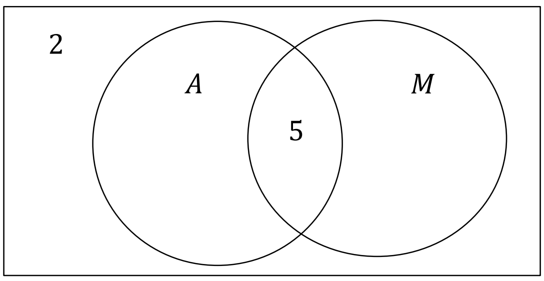
   Find $P(Y')$; $P(X \cup Y)$.

2. For two mutually exclusive events V and W , $P(V) = \frac{2}{3},\ P(W) = \frac{1}{10}$.

   Find $P(V \cap W)$; $P(V \cup W)$; $P\ (V \cup W)'$.

3. In a class of 20 students, 11 students study Art (A), 12 study Music (M), 5 study both subjects, and 2 study neither subject.

   (i) Complete the diagram to show this information.

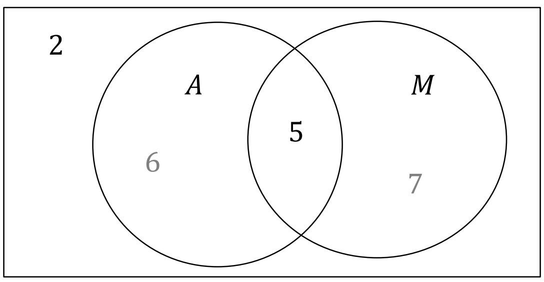


   (ii) One student is to be selected from the class. A is the event of selecting a student who studies Art and M is the event of selecting a student who studies Music.

   Find $P(A \cup M)$.

Answers

1. $\frac{3}{4}$; $\frac{7}{20}$
2. 0; $\frac{23}{30}$; $\frac{7}{30}$
3. (i)



   (ii) $\frac{9}{10}$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of probability to help us solve real-life problems. Today we will use the probability of a union of two events in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

at random, event, mutually exclusive, probability, respectively, union.

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Mary-Jane attends Southwood Girls' High School. In her Mathematics class she has been illustrating and finding the probability of a union of two events $(A \cup B)$, including for when the events are mutually exclusive.

Mary-Jane is in a class of 30 students for Mathematics and Science. In a recent vote, 15 students voted Mathematics as their preferred subject, 13 students voted Science as their preferred subject, while 2 students said that they had no preference and liked both subjects the same. In Mary-Jane's class of 27 students for Geography and History, 10 students voted Geography as their preferred subject, 13 students voted History as their preferred subject, while 4 students said that they had no preference and liked both subjects the same.

There are 150 students in Mary-Jane's Grade at Southwood Girls' High School. For sport, 71 play tennis, 57 play netball, 53 play hockey, 21 play hockey and netball, 17 play tennis and netball, 25 play tennis and hockey, while 5 students play all three sports.

Part 4B

Item 1

Questions

1. (i) Mary-Jane is considering events A and B which are not mutually exclusive. $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$.and $P(A \cap B) = \frac{1}{10}$.

   What should she obtain for: $P(A)'$, $P\,(A \cup B)'$?

   (ii) Mary-Jane then considers events C and D which are mutually exclusive, where:

   $P(C) = \frac{1}{2}$, $P(D) = \frac{2}{5}$.

   What should she obtain for: $P(C \cup D)$, $P\,(C \cup D)'$?

2. If M and S denote the events that Mathematics and Science (respectively) are preferred subjects, find $P(M \cap S)$, $P(M \cup S)$.

3. If one of the girls in Mary-Janes' Grade is selected at random, what is the probability that she plays:

   (i) one sport only?

   (ii) at least two sports?

   (Hint: First complete a Venn diagram for the sports information for Mary-Jane's Grade.)

Answers to Item 1

1. (i) $P(A)' = \frac{2}{5}; P(A \cup B) = \frac{7}{10}$

   (ii) $P(C \cup D) = \frac{9}{10},\ P(C \cup D)' = \frac{1}{10}$

2. $P(M \cap S) = \frac{1}{15};\ P(M \cup S) = \frac{13}{15}$

3. (i) $\frac{7}{15}$ (ii) $\frac{53}{150}$

Part 4C

Item 2

Questions

1. (i) Mary-Jane is considering events Q, R and S which are not mutually exclusive. She knows that $P(Q) = \frac{2}{5}, P(R) = \frac{3}{10}, P(S) = \frac{2}{5}, P(Q \cap R) = \frac{1}{5}, P(Q \cap S) = \frac{1}{10}, P(R \cap S) = \frac{1}{4}$.

   What should she obtain for: $P(R \cup S)$; $P\ (Q \cup S)'$?

   (ii) Mary-Jane then considers the mutually exclusive events T, U and W, where:

   $P(T) = \frac{2}{5}, P(U)' = \frac{1}{3}, P(W) = \frac{1}{5}$.

   What should she obtain for: $P(U)'$; $P\ (T \cup U)'$; $P(T \cup U \cup W)$?

2. If G and H denote the events that Geography and History (respectively) are preferred subjects, find $P\ (G \cap H)'$; $P(G \cup H)$.

3. (Optional) If one of the girls in Mary-Jane's class is selected at random, what is the probability that she plays:

   (i) none of the three sports?

   (ii) at most one sport?

Answers to Item 2

1. (i) $P(R \cup S) = \frac{9}{20}; P\ (Q \cup S)' = \frac{3}{10}$

   (ii) $(P(U)' = \frac{2}{3}; P(T \cup U)' = \frac{4}{15}; P(T \cup U \cup W) = \frac{14}{15}$

2. $P(G \cap H)' = \frac{23}{27}; P(G \cup H) = \frac{19}{27}$

3. (i) $\frac{27}{150}$; (ii) $\frac{97}{150}$

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 14

Solving Problems involving Probability

Key Idea

Solve problems involving probability.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Bag 1 contains 3 blue and 2 red marbles. Two marbles are drawn one at a time from the bag, without replacement. What is the probability that:
   (i) the first marble drawn is blue?
   (ii) both marbles drawn are blue?
2. Bag 2 contains 5 blue marbles and 1 red marble, and Bag 3 contains 2 blue marbles and 3 red marbles. If one of Bags $1, 2,$ and 3 is selected at random and then two marbles are drawn in succession without replacement, what is the probability that both marbles are red?
3. Bag 4 contains 3 white (W), 4 yellow (Y), and 2 green (G) marbles.
   (i) In how many ways can three different colors be drawn in three draws in succession from the bag? (As examples, two of these ways are the sequences W, Y, G and W, G, Y.)
   (ii) If, after each draw, the marble drawn is replaced in the bag, what is the probability that one white, one yellow, and one green marble, are drawn in any order?

Answers

1. (i) $\frac{3}{5}$ (ii) $\frac{3}{10}$
2. $\frac{2}{15}$
3. (i) 6 (ii) $\frac{16}{81}$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of probability to help us solve real-life problems. Today we will use permutations, combinations, and the probability of a union of two events, in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

at random, experiment, in succession, marble, with/without replacement

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Don and Phil are in Grade 10 at Everly High School. They are conducting different experiments in their study of probability in their Mathematics class.

For Experiment A, they have two bags of marbles: Bag 1 contains 2 red (R) marbles and 4 yellow(Y) marbles, while Bag 2 contains 5 red marbles and 3 yellow marbles.

For Experiment B, they also have two bags of marbles: Bag 1 contains 4 blue (B) marbles and 2 white(W) marbles, while Bag 2 contains 6 blue marbles and 3 white marbles.

For Experiment C, they have one bag of marbles that contains 5 black (B) marbles, 4 purple (P) marbles, and 3 green (G) marbles.

Part 4B

Item 1

Questions

1. Don conducts Experiment A first and for this experiment draws a marble from each bag. What is the probability that the marbles he draws are both red?
2. Phil conducts Experiment B first and for this experiment chooses a bag at random and then draws:
   (i) one marble from that bag. What is the probability that the marble is white?
   (ii) two marbles in succession without replacing the first marble drawn in the bag. What is the probability that both marbles are blue?
3. Don conducts Experiment C first and for this experiment draws three marbles at random from the bag. What is the probability of drawing one black, one purple, and one green marble (in any order) if the marbles are drawn without replacement?

Answers to Item 1

1. $\frac{5}{24}$
2. (i) $\frac{1}{3}$ (ii) $\frac{49}{120}$
3. $\frac{3}{11}$

Part 4C

Item 2

Questions

1. When Phil conducts Experiment A, he also draws a marble from each bag. What is the probability that the marbles he draws are:
   (i) both yellow?
   (ii) the same color?
2. When Don conducts Experiment B, he also chooses a bag at random and then draws:
   (i) one marble from that bag. What is the probability that the marble is blue?
   (ii) two marbles in succession, replacing the first marble drawn before drawing the second marble. What is the probability that the marbles are of different colors?

3. (Optional) When Phil conducts Experiment C, he also draws three marbles at random from the bag. What is the probability of drawing one black, one purple and one green marble (in any order) if the marbles are drawn with replacement? (i.e., after each draw, the marble selected is replaced in the bag.)

<u>**Answers to Item 2**</u>

1. (i) $\frac{1}{4}$; (ii) $\frac{11}{24}$
2. (i) $\frac{2}{3}$; (ii) $\frac{2}{9}$
3. $\frac{5}{24}$

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 15

Calculating and Interpreting Measures of Position (quartiles, deciles, and percentiles) of a Set of Data

Key Idea

Calculate and interpret measures of position (quartiles, deciles, and percentiles) of a set of data.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. (i) Complete: The median is equivalent to the ______________quartile and the ____________ percentile.

   (ii) What value or expression do A and B represent in the following formulae:

   $Position\ of\ D_i = \frac{i}{A}(n+1);\qquad Position\ of\ P_i = \frac{i}{100}(B)$

2. For the set of scores $14, 15, 16, 16, 17, 18, 18, 19, 20$:

   (i) Find the median.

   (ii) Use Tukey's Method to find Q_1.(the lower quartile).

3. Using the second formula in Question 1. (ii), what will be the position of the fifty-fifth percentile in a set of 60 scores?

Answers

1. (i) second; fiftieth

   (ii) A: 10; B: $n+1$

2. (i) Median $= 17$

   (ii) $Q_1 = 15.5$

3. 34th score

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of statistics to help us explore sets of data. Today we will calculate and interpret measures of position (quartiles, deciles, and percentiles) in exploring such sets of data.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

cumulative frequency, decile, frequency, measure of position, median, percentile, quartile, Tukey's Method

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Magdalena has collected a small set of data, Set A, and has arranged the data in ascending order:

Set A: $\{2, 3, 3, 5, 5, 6, 7, 7, 8, 9, 9, 9, 10, 10, 11, 12\}$.

She has also collected a larger set of data, the set of scores Set B, and has tabulated the data as follows:

Set B scores x	Frequency f	Cumulative Frequency
20	6	6
21	8	14
22	9	23
23	14	37
24	15	52
25	19	71
26	20	91
27	24	115
28	19	134
29	13	147
30	11	158
31	10	168
32	9	177
33	8	185
34	7	192
35	6	198
36	2	200
	$\Sigma f = 200$	

Part 4B

Item 1

Questions

1. Magdalena wishes to calculate the median for the data in Set A.
   (i) What value should she obtain for the median?
   (ii) What does this value represent for the data set and to which quartile is it equivalent?
2. Magdalena wishes to use Tukey's Method to calculate the other quartiles for the data in set A.
   (i) What do the first quartile and the third quartile represent?
   (ii) What value should she obtain for the first quartile (Q_1)?
3. Magdalena wishes to use the formula $Position\ of\ P_i = \frac{i}{100}(n+1)$ to calculate different percentiles for the data in Set B.
   (i) What values should she obtain for the 60th percentile and 90th percentile?
   (ii) What does the percentile midway between the 60th percentile and 90th percentile represent?

Answers to Item 1

1. (i) 7.5 (ii) Middle of the data set; equivalent to the 2nd quartile
2. (i) The first quartile represents the value below which one-quarter, or 25%, of the scores in the data set lie. The third quartile represents the value below which three-quarters, or 75%, of the scores lie.

   (ii) 5
3. (i) $P_{60} = 28; P_{90} = 33;$ (ii) The 75th percentile is equivalent to the upper quartile (Q_3).

Part 4C

Item 2

Questions

1. In order to find the median of the set of scores in Set B, Magdalena knows that she can use the Cumulative Frequency column of the table to help her locate its value. What value should she obtain for the median?
2. Magdalena wishes to use the formula $Position\ of\ D_i = \frac{i}{10}(n+1)$ to calculate different deciles for the data in Set B.

   (i) What values should she obtain for the third decile and seventh decile?

   (ii) What does the decile midway between the third decile and seventh decile represent?
3. (Optional) Magdalena uses Tukey's Method and the formula $Position\ of\ P_i = \frac{i}{100}(n+1)$ to calculate the first quartile (25th percentile) for the data in Set B.

   (i) What two results should she obtain?

   (ii) How do the results compare? Why won't this always be true?

Answers to Item 2

1. 27
2. (i) third decile: 25; seventh decile: 29

   (ii) The decile midway between the third decile and seventh decile is the fifth decile, which represents the median.
3. (i) $24;\ 24$

   (ii) In this case, the results are the same using the two methods. The process of the second method involves 'rounding' in order to always identify one of the members of the data set. Such identification is not a requirement of Tukey's Method.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 16

Solving Problems involving Measures of Position

Key Idea

Solve problems involving measures of position.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

Score x	Frequency f	Cumulative Frequency
5	2	
6	3	
7	4	
8	4	
9	5	
10	3	
	$\sum f$=21	

For the data in the table above:

1. (i) Complete the Cumulative Frequency column.

   (ii) Use the Cumulative Frequency column to find the median.

2. Use Tukey's Method to find the third quartile.

3. Use the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ and $Position\ of\ P_i = \frac{i}{100}(n+1)$ to find the position of the fourth decile and the forty-fifth percentile.

Answers

1. (i) Cumulative Frequency column: $2, 5, 9, 13, 18, 21$

   (ii) Median$= 8$

2. $Q_3 = 9$

3. Position of the fourth decile: 9th score; Position of the forty-fifth percentile:10th score

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of statistics to help us solve real-life problems. Today we will use measures of position (quartiles, deciles, and percentiles) for sets of data in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms are:

decile, median, percentile, quartile, survey, tabulate, Tukey's Method

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Alessandro has surveyed 50 locomotive drivers (Set L) and 50 drivers of large haulage trucks (Set T) that are based in his local area, regarding the average number of hours (to the nearest hour) that they drive on each of their workdays. He has then tabulated the data obtained in the tables below:

Data for Locomotive Drivers (Set L)

Average number of hours driving locomotive per workday x	Frequency f	Cumulative Frequency
1	2	2
2	5	7
3	6	13
4	8	21
5	9	30
6	9	39
7	7	46
8	4	50
	$\sum f$=50	

Data for Truck Drivers (Set T)

Average number of hours driving truck per workday x	Frequency f	Cumulative Frequency
3	3	3
4	5	8
5	7	15
6	10	25
7	9	34
8	6	40
9	6	46
10	4	50
	$\sum f$=50	

Part 4B

Item 1

Questions

1. Alessandro has calculated the median for the set of average hours that the locomotive drivers drive their vehicles each workday. What should he have obtained for the median for Set L?
2. Alessandro has used Tukey's Method to find the first quartile for Set L. What value should he have obtained?
3. Alessandro has used the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) and $Position\ of\ P_i = \frac{i}{100}(n+1)$ (for calculating position of percentiles) to calculate the:
   (i) ninth decile for set L. What value should he have obtained?
   (ii) thirty-fifth percentile for Set L. What value should he have obtained?

Answers to Item 1

1. 5
2. 3
3. (i) 7

   (ii) 4

Part 4C

Item 2

Questions

1. Alessandro has calculated the median for the average hours that the truck drivers drive their vehicles each workday. What should he have obtained for the median for:

   (i) Set T?

   (ii) How much greater in percentage terms is the median for Set T than that for Set L?

2. Alessandro has used Tukey's Method to find the third quartile for Set T. What value should he have obtained?

3. (Optional) Alessandro has used the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) and $Position\ of\ P_i = \frac{i}{100}(n+1)$ (for calculating position of percentiles) to calculate the:

   (i) second decile for Set T. What value should he have obtained?

   (ii) sixty-fifth percentile for Set T. What value should he have obtained?

Answers to Item 2

1. (i) 6.5 (ii) 30% greater
2. 8
3. (i) second decile is 5

   (ii) sixty-fifth percentile is 7

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 17

Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data

Key Idea

Use appropriate measures of position and other statistical methods in analyzing and interpreting data.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

A class of Mathematics students is analyzing and interpreting the following set of scores:

$$23, 24, 25, 26, 26, 27, 27, 28, 28, 29$$

1. For the set of scores above, calculate the:

   (i) range (ii) median.

2. (i) For the set of scores above, calculate the mean.

   (ii) Complete: $\sqrt{\frac{\Sigma(fd^2)}{\Sigma f}}$, where for each score in a set of scores f is the frequency and d is the deviation from the mean, is used to calculate the _____________for a set of scores.

3. For the set of scores above:

   (i) use Tukey's Method to calculate the lower quartile Q_1.

   (ii) use the formula $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) to calculate the seventh decile.

Answers

1. (i) 6 (ii) 26.5
2. (i) 26.3 (ii) standard deviation
3. (i) 25 (ii) seventh decile: 28

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We can use what we have learned in our study of statistics to help us solve real-world problems. Today we will use appropriate measures of position and other statistical methods to analyze and interpret data in finding the solutions to such problems.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

decile, modal, percentile, quartile, standard deviation, statistical

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Buddy and Holly are in the same Grade 10 Mathematics class. In their last six Mathematics class tests for the year (results of each reported as a mark out of 10), Buddy scored $6, 7, 7, 7, 8, 7$ while Holly scored $9, 1, 8, 9, 7, 5$.

Their teacher, Mr. Statz, has calculated the mean, median, lower (first) quartile, upper (third) quartile, and standard deviation for Buddy's and for Holly's test scores.

At the end of the year, the 24 students in the class sat for their final examination in Mathematics (results reported as a mark out of 100). Mr Statz has listed below their results in increasing order:

$$41, 43, 47, 51, 52, 55, 55, 57, 58, 60, 64, 65, 67, 67, 68, 71, 74, 78, 80, 83, 86, 88, 92, 96$$

Part 4B

Item 1

Questions

1. (i) Calculate the mean score and median score that Mr. Statz calculated for Buddy and for Holly.

   (ii) Explain why one of the measures Is the better measure of their Mathematics ability.

2. For the last six Mathematics class tests, find the lower quartile that Mr Statz calculated for:

   (i) Buddy's scores (ii) Holly's scores.

3. (i) If Buddy wished to score higher than the sixth decile in the final examination, what is the lowest of the marks that Mr Statz listed that Buddy would have had to achieve?

   (ii) (Optional) If Holly wished to score higher than the eightieth percentile in the final examination, what is the lowest of the marks that Mr. Statz listed that Holly would have had to achieve?

Answers to Item 1

1. (i) Buddy: Mean $= 7$, Median $= 7$; Holly: Mean $= 6\frac{1}{2}$, Median $= 7\frac{1}{2}$

   (ii) The median is the better measure of their Mathematics ability because the mean for Holly's marks is heavily affected by one very low mark.

2. (i) Lower quartile for Buddy's scores $= 7$ (ii) Lower quartile for Holly's scores $= 5$

3. (i) 71 (ii) 86

Part 4C

Item 2

Questions

1. (i) Calculate the median score and the range of the scores for Mr Statz' class in the final examination.

   (ii) What are the modal scores for the final examination?

2. For the last six Mathematics class tests, find the:

   (i) upper quartile for Holly's scores.

   (ii) standard deviation for Buddy's scores.

3. (Optional)

   (i) If Mr. Statz tells Buddy that he scored the first mark higher than the seventh decile in the final examination, what mark did Buddy achieve?

   (ii) If Mr. Statz tells Holly that she scored the first mark higher than the eighty-fifth percentile in the final examination, what mark did Holly achieve?

<u>Answers to Item 2</u>

1. (i) Median $= 66$; Range $= 55$

   (ii) Modal scores are 55 and 67.

2. (i) Upper quartile for Holly's scores $= 9$

   (ii) Standard deviation for Buddy's scores $= 0.58$ (correct to 2 decimal places)

3. (i) 80 (ii) 92

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

Mathematics Grade 10 Lesson Plan 18 Deliberate Practice

Solving Problems involving Probability
Solving Problems involving Measures of Position
Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data

Key Ideas

Solve problems involving probability.

Solve problems involving measures of position.

Use appropriate measures of position and other statistical methods in analyzing and interpreting data.

Lesson Component 1 (Lesson Short Review)

Time: 7 minutes

Questions

1. Bag A contains 3 yellow marbles and 2 black marbles and Bag B contains 4 yellow and 2 black marbles. If one of the bags is selected at random and then two marbles are drawn in succession without replacement, what is the probability that both marbles are black?
2. For the set of scores: $1, 2, 2, 1, 4, 2$
   (i) Show that the mean for the set of scores is $\bar{x} = 2$.
   (ii) Complete the columns for the frequency, deviation from the mean ($\bar{x} = 2.$), the square of the deviation, and the frequency × the square of the deviation.

Score x	Frequency f	d	d^2	fd^2
1				
2				
4				
	$\Sigma f=$			$\Sigma(f\,d^2)=$

Use the formula, Standard Deviation $= \sqrt{\frac{\Sigma(fd^2)}{\Sigma f}}$, to find the standard deviation for the scores.

3. Use Tukey's Method to find the Interquartile Range (IQR) for the set of scores. (The Interquartile Range is the difference between the Upper Quartile (Q_3) and the Lower Quartile (Q_1) for a set of scores.)

Answers

1. $\frac{1}{12}$
2. (i) $\text{Mean } (\bar{x}) = \frac{\text{sum of scores}}{\text{number of scores}}$
$= \frac{1+2+2+1+4+2}{6}$
$= \frac{12}{6}$
$= 2$

(ii)

Score x	Frequency f	d	d^2	fd^2
1	2	-1	1	2
2	3	0	0	0
4	1	2	4	4
	$\sum f$=6			$\sum(f\,d^2)$=6

Standard Deviation $= 1$

3. $IQR = 1$

Lesson Component 2 (Lesson Purpose/Intention)

Time: 3 minutes

The teacher states:

We have been working on the solution of problems involving probability and measures of position, as well as the use of appropriate measures of position and other statistical methods in analyzing and interpreting data. Today we are going to consolidate this work through the solution of such problems and the analysis and interpretation of such data.

Lesson Component 3 (Lesson Language Practice)

Time: 5 minutes

Key words/terms

at random, disc, in succession, interquartile range, standard deviation, survey, with/without replacement.

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

<u>Stem for Items 1 and 2</u>

Melinda is the Population Data Officer for a city's Municipal Council. The city is divided into two large sections called the Northern Division, which has 4 Eastern Suburbs and 3 Western Suburbs, and the Southern Division, which has 3 Eastern Suburbs and 5 Western Suburbs.

One of Melinda's recent tasks was to select one of the city's suburbs at random in order to conduct a survey of 100 families in the suburb to collect data on the number of children per household.

Melinda wants to analyze the survey data and has partly completed the table below:

Number of children per household in survey suburb x	Frequency f	$f \times x$	Cumulative Frequency	d	d^2	fd^2
0	25					
1	18					
2	21					
3	17					
4	9					
5	7					
6	3					
	$\sum f$=	$\sum(f \times x)$ =				$\sum(f\,d^2)$=

Part 4B

Item 1

Questions

1. To select the suburb for the survey, Melinda placed the names of the suburbs on small discs in one of two bags, with the Northern Division suburbs being placed in Bag 1 and the Southern Division suburbs in Bag 2. She then selected one bag at random and then one disc from the bag selected.

   (i) What was the probability of Melinda selecting a Western Suburb from the Southern Division?

   (ii) What was the probability of selecting any of the city's Eastern Suburbs?

2. (i) Write down the mode and range for the survey data.

   (ii) Complete the $f \times x$ column and Cumulative Frequency column of the table and use them to find the mean (using Mean $= \frac{\Sigma(f \times x)}{\Sigma f}$) and median number of children per household in the surveyed suburb.

3. (Optional) Melinda used Tukey's Method to find the first and third quartiles for the survey data. What values should she have obtained?

Answers to Item 1

1. (i) $\frac{5}{16}$ (ii) $\frac{53}{112}$

2. (i) Mode$= 0$ (ii) Range $= 6$

   (ii)

Number of children per household in survey suburb x	Frequency f	$f \times x$	Cumulative Frequency
0	25	0	25
1	18	18	43
2	21	42	64
3	17	51	81
4	9	36	90
5	7	35	97
6	3	18	100
	$\Sigma f = 100$	$\Sigma(f \times x) = 200$	

   Mean $= 2$ Median $= 2$

3. First quartile $(Q_1) = 0.5$ Third quartile $(Q_3) = 3$

Part 4C

Item 2

Questions

1. For the suburb selection process described in Part 4B Question 1:

   (i) What was the probability of Melinda selecting an Eastern Suburb from the Northern Division?

   (ii) What was the probability of selecting any of the city's Western Suburbs?

2. (i) Complete Columns 5 (using $\bar{x} = 2$), 6 and 7 of the table for the survey.

   (ii) Show that the standard deviation for the data is approximately 1.68.

3. (Optional) Melinda used the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) and $Position\ of\ P_i = \frac{i}{100}(n+1)$ (for calculating position of percentiles) to calculate the fourth decile and ninety-fifth percentile for the survey data. What values should she have obtained?

Answers to Item 2

1. (i) $\frac{2}{7}$ (ii) $\frac{59}{112}$

2. (i)

Number of children per household in survey suburb x	Frequency f	$d = x - \bar{x}$	d^2	fd^2
0	25	−2	4	100
1	18	−1	1	18
2	21	0	0	0
3	17	1	1	17
4	9	2	4	36
5	7	3	9	63
6	3	4	16	48
	$\sum f$=100			$\sum(fd^2)$ =282

(ii) Standard Deviation = $\sqrt{\frac{\sum(fd^2)}{\sum f}} = \sqrt{\frac{282}{100}} \cong 1.68$

3. fourth decile is 1; ninety-fifth percentile is 5.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Time: 5 minutes

The teacher facilitates student reflection and discussion that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you gain further insight into aspects of the material covered that represent strengths or represent weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Delivery (DepEd-BLD)
Contact Numbers: 8637-4366; 8637-4347; 8633-9347

Department of Education - Bureau of Learning Resources (DepEd-BLR)
Contact Numbers: 8634-1072; 8631-6922

Email Address: blr.od@deped.gov.ph

WITH SUPPORT FROM Australian Aid