10

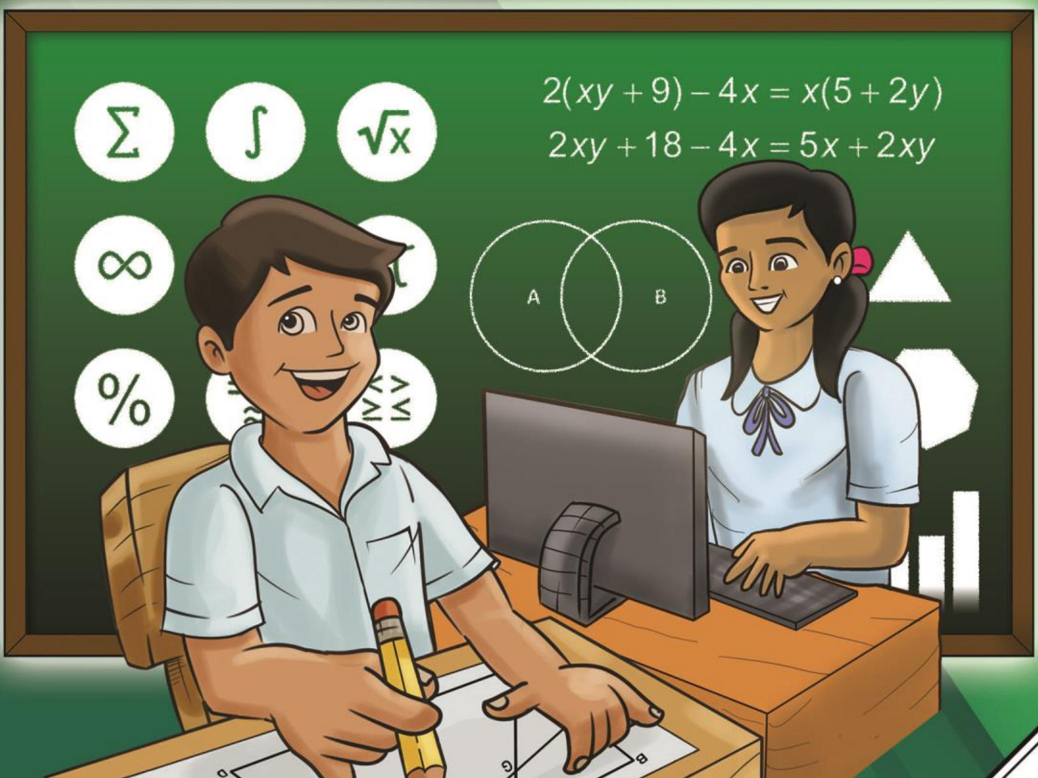
NATIONAL LEARNING CAMP

DepED DEPARTMENT OF EDUCATION

Mathematics

Enhancement Learning Camp

Student Workbook



Enhancement Learning Camp
Student Workbook

Mathematics Grade 10

Weeks 1 to 3

Contents

Introduction for Students

Welcome to the National Learning Camp. You are probably aware that this Camp is only open to students like you who have just completed Grade 9 or Grade 10 across the country.

You have chosen to be part of this important national program. Our focus this year is on: English, Mathematics, and Science.

The Plan

You are to attend school on three days each week: Tuesday, Wednesday, and Thursday.

You will take part in six special lessons each day. These lessons review subject content you have completed. This will help you further strengthen your learning.

There will be opportunities in each lesson for you to practice talking with other students and your teacher, and applying the knowledge you have gained in:

- understanding (comprehending) what you are reading in *English*,
- solving *Mathematics* problems, and
- interpreting the natural world through applying *Science* evidence.

Time in Class

How you use your time in lessons is very important. Every minute is valuable. It is critical that you work with the teacher and your classmates as closely as you can.

This means you will be expected to:

- start each lesson as quickly as possible,
- recognize the lesson pattern and help the teacher as you move from one part of the lesson to another,
- pay attention when the teacher or students in your class are talking about work, and
- try your best with all the different activities that make up the lesson.

You will have opportunities to write your answers down, explain to the teacher or classmates your reasons for your responses or thinking. There will be time to work on your own and at other times you will work with your classmates and report to the class.

Mistakes

One important fact drawn from brain research on learning concerns making mistakes. **It might surprise you!**

Making mistakes while learning and trying to improve your skills and understanding is *part of the brain's process*. So, learning from mistakes is an important pathway of our learning journey. When a genuine mistake is made:

- do not be ashamed or embarrassed,
- do try to learn from your mistake,
- be willing to talk about your mistakes,
- try to understand why you committed a mistake, and
- find out how to correct the mistake.

Too often learners are embarrassed or feel they have failed because of errors/mistakes. **This should not be the case.** Everyone makes mistakes as they learn new material – **everyone.**

A very famous scientist, Niels Bohr, who won a Nobel Prize for Physics, said:

An expert is a person who has made all the mistakes that can be made in a very narrow field.

Everyone makes mistakes, even experts. **It is a vital part of learning.** If you make mistakes, it is a sign that you are moving your learning forward. You may need to return to earlier learning and fill in some gaps.

Mistakes and/or errors tell **you** and your **teacher** about your thinking and where you need help or practice (we call it deliberate practice) to do better. The **teacher** and **you** should celebrate finding the mistake as it will help you both know what new learning is needed.

You might be surprised, but if you do not make genuine mistakes and fix them, your learning will not move forward efficiently.

Practice

If you want to be good at something you must practice it. Practice alerts the brain that the information needs to be known and to store the information in your head.

This is the way the brain works; this is the way the brain learns. Learning, anything from sport, about your peers, and to learning subjects in school, requires effort and that means practice.

Effort requires persistence, but it is not supposed to be difficult and punishing. It may be continued until one learns. There are no tricks. This is what the brain needs to learn.

It is important that you try and try again

Learning is a competition with yourself, not others. It is recognizing how your effort results in showing you where and how you are doing better. To be as good as you can be will only be known if you try.

The Extensive Team of Educators and Teachers involved in the National Learning Camp wish you the very best in your education future. For the Learning Camp, and your work when you return to school, our hope is for you to take any new knowledge, skills and understandings you have acquired to learn more, and to use this knowledge to want to learn more.

Best Wishes

Student Workbook Mathematics Grade 10 Lesson 1

Determining Arithmetic Means, $n\text{th}$ term of an Arithmetic Sequence, and Sum of the Terms of an Arithmetic Sequence

Lesson Component 1 (Lesson Short Review)

<u>Questions</u>

1. Find the arithmetic mean of:

   (i) -7 and 9

   (ii) $2\sqrt{3}-3$ and $2\sqrt{3}+3$

2. Consider the arithmetic sequence: $2, -3, -8, \ldots$

   (i) Write down the next term of the sequence.

   (ii) Use the formula $a_n = a_1 + (n-1)d$ to write down a formula for the $n\text{th}$ term of the sequence and find its 15th term.

3. For the sequence in Question 2:

   (i) Calculate which term of the sequence is -93.

   (ii) Use the formula $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ to find the sum of 10 terms of the sequence.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

annual, arithmetic, formula, increase, mean, salary, sequence, term.

Lesson Component 4 (Lesson Activity)

Part 4A

<u>Stem for Items 1 and 2</u>

In each of the previous three years, Marcella has worked for a large company, ABC Enterprises. In her first year with the company, she was paid a salary of ₱2 000 000 . In her third year, Marcella was paid ₱2 600 000. In her second year, she was paid the arithmetic mean of these two amounts.

Marcella has recently taken a new job with XYZ Enterprises. She will be paid ₱2 500 000 in her first year, with annual salary increases of ₱200 000 in each future year.

Part 4B

Item 1

Questions

1. Calculate the amount that Marcella was paid in her second year with ABC Enterprises.

2. If Marcella had stayed with ABC Enterprises and received the same annual increase each year as during her first three years, find:

   (i) how much Marcella would have been paid in her tenth year with the company.

   (ii) in which of her years with the company she would have received a salary of ₱$7\ 100\ 000$.

3. Use the formula $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ to find the total amount that Marcella would have earned with ABC Enterprises if she had remained with the company for ten years.

Part 4C

Item 2

Questions

1. Insert 4 arithmetic means between Marcella's first year and third year salaries with XYZ Enterprises to obtain the salaries that she would have been paid each half year if the company had agreed to common half-yearly increases over the three years.

2. (i) Find how much Marcella will be paid in her 15th year with XYZ Enterprises.

   (ii) In which of her years with XYZ Enterprises will Marcella first receive a salary of over ₱$4\ 000\ 000$?

3. (Optional) How much more in total would Marcella have earned after 20 years at ABC Enterprises, if she had stayed with the company, than she will earn at XYZ Enterprises if she stays for 20 years?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 2

Determining Geometric Means, nth term of a Geometric Sequence, and Sum of the Terms of a Finite or Infinite Geometric Sequence

Lesson Component 1 (Lesson Short Review)

Questions

1. Find the geometric mean of -4 and -16.

2. Consider the geometric sequence: $45, 15, 5, \ldots$
   (i) Write down the next term of the sequence.
   (ii) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the nth term of the sequence and find its 6th term.

3. (i) Use your formula in Question 2. (ii) for the nth term of the sequence to find which term is $\frac{5}{9}$.
   (Hint: Write both sides of the equation that you need to solve as a power of the same fraction.)
   (ii) Use the formulae $S_n = \frac{a_1(1-r^n)}{1-r}$ and $S_\infty = \frac{a_1}{1-r}$ to find the sum of five terms and the sum to infinity of the sequence.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

common ratio, formula, geometric, infinity, mean, sequence.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Sonny has a ball that he drops on a hard flat surface from a point 60 meters above the surface. The ball bounces to a certain fraction of its original height and continues to bounce to the same fraction of each previous height. Sonny knows that on the ball's fourth bounce it attains a height of $\frac{15}{4}$.(or $3\frac{3}{4}$) meters.

Cher has a 'bouncier' ball that she drops from the same point. The ball bounces to $\frac{2}{3}$ of its original height and continues to bounce to $\frac{2}{3}$ of each previous height. Cher knows that on the ball's second bounce, it attains a height of $\frac{80}{3}$.(or $26\frac{2}{3}$) meters.

Part 4B

Item 1

Questions

1. Insert 3 geometric means between 60 and $\frac{15}{4}$ by using the formula $a_n = a_1 r^{n-1}$ to first find r (positive value only is valid), which represents the fraction of each previous height that Sonny's ball attains on each bounce. (Note that the 3 geometric means represent the heights that the ball attains on its first, second, and third bounces.)

2. (i) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the sequence in Question 1 (Sequence S).

   (ii) Use your formula for the nth term of Sequence S in Question 2 (i) to write down the 7th term of the sequence (the height in meters that Sonny's ball attains on its 6th bounce).

3. (i) Use the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ to find the sum of five terms of Sequence S.

   (ii) Find the limiting sum (or sum to infinity S_∞) of Sequence S.

Part 4C

Item 2

Questions

1. Find the positive geometric mean of 60 and $\frac{80}{3}$, using that the positive geometric mean x of a and b is $x = \sqrt{ab}$. (This represents the height in meters that Cher's ball attains on its first bounce.)

2. (i) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the sequence of heights (Sequence T) described for Cher's ball.

   (ii) (Optional) Find on which bounce that Cher's ball attains a height of $\frac{320}{27}$ $(= 11\frac{23}{27})$ meters. (Hint: Write both sides of the equation that you need to solve as a power of $\frac{2}{3}$.)

3. (i) Use the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ to find the sum of five terms of Sequence T. (ii) (Optional) How many times greater than the limiting sum (S_∞) of Sequence S is the limiting sum of Sequence T?	
Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)	

Student Workbook Mathematics Grade 10 Lesson 3

Solving Problems involving Sequences

Lesson Component 1 (Lesson Short Review)

Questions

1. Find the arithmetic mean and the geometric mean of -3 and -27.

2. For Sequence A: $-2, 1, 4, \ldots$ and Sequence B: $2, -1, \frac{1}{2}, \ldots$, find the:

   (i) 8th term

   (ii) sum of 8 terms.

3. For Sequence B in Question 2, find:

   (i) which term is $-\frac{1}{16}$.

   (Hint: Write both sides of the equation that you need to solve as a power of the same fraction.)

   (ii) S_∞

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

arithmetic mean/geometric mean, increase, initial, net, period, sequence, term.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Henrietta managed a plantation of pine trees which were grown and sold for their timber. The plantation was developed from an initial planting of 45 000 trees, with a net increase, after sales and plantings, of 2500 trees each year over a period of 15 years.

At the beginning of the 16th year of the plantation, it was decided to sell-off 40% of the trees remaining on the plantation each year for timber, without any further plantings, so that the land could ultimately be used for another purpose.

Part 4B

Item 1

Questions

1. Use a formula for the nth term of a sequence to determine:
   (i) how many trees were on the plantation at the beginning of the 5th year (i.e., $n = 5$) of the plantation.

   (ii) at the beginning of which year of the plantation there were $65\ 000$ trees on the plantation.

2. (i) Show that there were $82\ 500$ trees on the plantation at the beginning of the 16th year of the plantation.

   (ii) Find the arithmetic mean of the initial ($45\ 000$) and final number of trees ($82\ 500$) on the plantation.

3. (Optional) Each year the plantation received a government subsidy of ₱100 for each tree that it had on the plantation at the beginning of that year. Calculate the total subsidy that the plantation had received by the beginning of the 16th year of the plantation.

Part 4C

Item 2

Questions

1. Calculate how many trees remained on the plantation at the beginning of the 17th year of the plantation (i.e., one year after beginning to sell-off trees).

2. (i) Use a formula for the nth term of a sequence (with $n = 5$) to determine how many trees remained on the plantation at the beginning of the 21st year of the plantation. (Note that a_1 will correspond to the answer obtained in Part 4C Question 1.)

(ii) Show that there were less than 500 trees left on the plantation at the beginning of the tenth year after beginning to sell-off trees.

3. (Optional) Show that the limiting amount (S_∞) of government subsidy that the plantation could receive after beginning to sell-off trees is ₱$12\ 375\ 000$.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 4

Solving Problems involving Polynomials and Polynomial Equations

Lesson Component 1 (Lesson Short Review)

Questions

1. What is the volume of a rectangular prism with length $(x+5)$.meters, breadth $(x+2)$ meters, and height x meters?

2. (i) The boxes in a set of boxes each have a height of $(x+6)$ meters and a base area of $(2x^2+x-1)$ square meters. Find the height and base area of the box for which $x=2$.

   (ii) What is the volume of the box for which $x=-4$?

3. Another box in the set of boxes in Question 2 (i), has a base area of 5 square meters. Solve the polynomial equation $2x^2+x-1=5$ to find the possible values of x for this box.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

binomial, long division, polynomial, polynomial equation, quadratic, rectangular prism, trinomial.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

A set of storage spaces (Set A), in the shape of a rectangular prism, have a height of $(2x-1)$ meters and a volume of $(6x^3 - 13x^2 + x + 2)$ cubic meters.

Another set of storage spaces (Set B), again in the shape of a rectangular prism, also have a height of $(2x-1)$ meters, with a base area of $(3x^2 - 7x - 6)$ square meters.

A third set of storage spaces (Set C), also in the shape of a rectangular prism, have a height of $(3x-2)$ meters and a base area of $(2x^2+x-3)$ square meters.

Part 4B

Item 1

Questions

1. (i) For Set A, show that the base area of the storage spaces in square meters is $(3x^2 - 5x - 2)$.

   (ii) Hence, show that the volume in cubic meters of the storage spaces in Set A may be written as $(6x^3 - 13x^2 + x + 2) = (2x - 1)(3x + 1)(x - 2)$.

2. (i) Find the height, base area, and volume of the storage space in Set A for which $x = 3$.

   (ii) By considering the height of the storage spaces in Set A, explain why the value of x for the storage spaces cannot be equal to or less than $\frac{1}{2}$.

3. (Optional)

   (i) One of the storage spaces in Set B has a base area of 14 square meters. Solve the polynomial equation $3x^2 - 7x - 6 = 14$ to show that the value of x for this storage space is $x = 4$.

   (ii) Hence, calculate the height and volume of this storage space.

Part 4C

Item 2

Questions

1. For Set C, write down a polynomial that expresses the volume in cubic meters of each storage space in the set.

2. (i) Write the base area, $(2x^2 + x - 3)$ square meters, of the storage spaces in Set C in factored form.

(ii) One of the storage spaces in Set C has a height of 16 meters. Show that the value of x for this storage space is $x = 6$, and hence find the base area and volume of the storage space.

3. (Optional)

(i) A new Set B storage space is to be built that has a base area 6 square meters more than that of the Set C storage space with the same value of x. Solve a polynomial equation to show that this value of x is $x = 9$.

(ii) Calculate the height, base area, and volume of the new storage space.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 5

Solving Problems involving Polynomial Functions

Lesson Component 1 (Lesson Short Review)

Questions

1. (i) $U(x) = -10 + 7x - \frac{1}{3}x^2$ and $V(x) = 4x + 25$.

   If $W(x) = U(x) - V(x)$, show that $W(x) = -35 + 3x - \frac{1}{3}x^2$.

   (ii) Solve the polynomial ('quadratic') equation $-8 + 5x - \frac{1}{2}x^2 = 0$.

2. The equation of the axis of symmetry of the graph (a parabola) of the polynomial function $P(x) = ax^2 + bx + c$ is given by $x = -\frac{b}{2a}$.

   What is the equation of the axis of symmetry of the graph of $A(x) = -12 + 4x - \frac{1}{4}x^2$?

3. The graph of the polynomial function $Q(x) = -10 + 3x - \frac{1}{5}x^2$ is an inverted parabola that cuts the $x -$ axis at $(5, 0)$ and $(10, 0)$.

   Write down two inequalities in terms of x to represent the values of x for which $Q(x) < 0$.

   Write down the equation of the axis of symmetry of $Q(x)$ and the maximum value of $Q(x)$.

Lesson Component 3 (Lesson Language Practice)

axis of symmetry, break-even, inverted, polynomial function, profit, revenue.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

GizmoCo is an exporting company that produces a product, the gizmo. The monthly profit that the company makes is determined by subtracting the cost of producing its gizmos from the revenue received from the sale of the gizmos.

In 2022, the cost (in thousands of U.S. dollars) of producing the company's gizmos each month was given by $C_1(x) = 2x + 30$, where x is the number of thousands of gizmos produced in a month, while the revenue received (in thousands of U.S. dollars) was given by $R_1(x) = 6x - \frac{1}{10}x^2$.

In 2023, GizmoCo found that it could reduce the cost of producing its gizmos each month, for $x > 30$, to $C_2(x) = x + 60$, while maintaining the same mathematical model for revenue i.e., $R_2(x) = 6x - \frac{1}{10}x^2$.

Part 4B

Item 1

Questions

1. (i) Show that GizmoCo's monthly profit (in thousands of U.S. dollars) in 2022 was given by the polynomial function $P_1(x) = -30 + 4x - \frac{1}{10}x^2$.

   (ii) Show, by solving the polynomial equation $P_1(x) = 0$, that in 2022 GizmoCo would only have 'broken even' (i.e., made a profit each month of 0 U.S. dollars) if it produced 10 thousand or 30 thousand gizmos.

2. (i) The graph of the polynomial function $P_1(x)$ is an inverted parabola with axis of symmetry $x = 20$. Use this information to find the maximum possible profit that GizmoCo could have made each month in 2022 from producing gizmos.

   (ii) Write down two inequalities in terms of x to represent the values of x for which the number of gizmos would have resulted in a loss each month for GizmoCo in 2022.

3. (i) Show that the polynomial function $P_2(x)$ that represented GizmoCo's profit each month in 2023 is given by $P_2(x) = -60 + 5x - \frac{1}{10}x^2$.

   (ii) (Optional) Find the amount of profit that GizmoCo would have made each month in 2023 if it produced the same number of gizmos (20 thousand) that would have maximized its profits in 2022.

Part 4C

Item 2

Questions

1. (i) Use the formulae for $P_1(x)$ and $P_2(x)$ to determine in which year, 2022 or 2023, GizmoCo would have made the greater loss by producing 5000 gizmos only.

   (ii) By considering the information in Part 4B Question 1. (ii) and the symmetry of the polynomial function $P_1(x)$, write down the number of gizmos that, if produced in 2022, would have produced the same loss as for producing 5000 gizmos.

2. (i) Show, by solving the polynomial equation $P_2(x) = 0$, that GizmoCo would only have 'broken even' (i.e., made a profit of 0 U.S. dollars) if it produced 20 thousand or 30 thousand Gizmos in 2023.

   (ii) Write down an inequality in terms of x to represent the values of x for which the number of gizmos would have resulted in a profit for GizmoCo in 2023.

3. (Optional)

   (i) Show that the value of x that would have maximised GizmoCo's profit in 2023 was $x = 25$.

   (ii) Find how much smaller GizmoCo's maximum possible profit each month was in 2023 compared to in 2022.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 6 Deliberate Practice

Solving Problems involving Sequences
Solving Problems involving Polynomials and Polynomial Equations
Solving Problems involving Polynomial Functions

Lesson Component 1 (Lesson Short Review)

Questions

1. Consider the following sequences: Sequence A: $-1, 3, 7, \ldots$ and Sequence B: $3, -1, \frac{1}{3}, \ldots$

   (i) For Sequence A, write down a_n (the nth term of the sequence), and calculate S_6.

   (ii) For Sequence B, calculate S_4 and S_∞.

2. If $P(x)$ is a polynomial function such that $P(x) = -9 + 4x - \frac{1}{3}x^2$, write $P(x) = 0$ in the form $ax^2 + bx + c = 0$, where $a = 1$, and solve $P(x) = 0$.

3. The graph of the polynomial function $Q(x)$ is an inverted parabola that cuts the x-axis at $(4, 0)$ and $(10, 0)$, and has vertex $(7, 9)$.

   (i) Write down two inequalities in terms of x that represent the values of x for which $Q(x) \leq 0$.

   (ii) Find the maximum value of $Q(x)$.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

arithmetic sequence, euro, geometric sequence, polynomial equation, polynomial function, vertex.

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

<u>Stem for Items 1 and 2</u>

Prism, a new type of modular multi-storey building, was constructed recently in Europe. The cost of constructing the first storey was $60\ 000$ euros, the second storey $70\ 000$ euros, and the third storey $80\ 000$ euros, with the cost continuing to increase by $10\ 000$ euros for each subsequent storey.

Jake works for The Whatsit Company in the new building and drops a ball from the top of the tenth storey to a hard flat surface on ground level, 40 meters below. The ball bounces to $\frac{3}{5}$ of its original height. Until it comes to rest, Jakes's ball continues to bounce to $\frac{3}{5}$ of each previous height.

The Whatsit Company produces a product called the doovalacky. Its monthly profit is determined by subtracting the cost of producing the doovalackies from the revenue received from the sale of the doovalackies. In 2023, this cost (in thousands of euros) was given by $C(x) = 2x + 45$, where x is the number of thousands of doovalackies produced in a month, while the revenue received (in thousands of euros) was given by
$R(x) = 11x - \frac{1}{4}x^2$.

Part 4B

<u>Item 1</u>

<u>Questions</u>

1. Write down a formula for finding the cost of constructing a particular storey of *Prism*.

   Use the formula to find the cost of constructing the 15th storey of the building.

2. (i) Use the formula $a_n = a_1 r^{n-1}$ to write down a formula for the sequence of heights (including the original height) of Jake's ball.

   (ii) Use your formula for the nth term of the sequence to write down the 5th term of the sequence (the height Jakes's ball attains on its 4th bounce).

3. (i) Show that The Whatsit Company's monthly profit (in thousands of euros) in 2023 was given by the polynomial function $P(x) = -45 + 9x - \frac{1}{4}x^2$.

   (ii) (Optional) Show, by solving the polynomial equation $P(x) = 0$, that in 2023 The Whatsit Company would only have 'broken even' (i.e., made a profit each month of 0 euros) if it produced 6 thousand or 30 thousand doovalackies.

Part 4C

Item 2

Questions

1. Write down a formula for finding the cost of constructing *Prism*.

   Use the formula to find the cost of constructing the building if it was built to a height of 20 storeys.

2. (i) Use the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ to find the sum of the heights (including its original height) attained by Jake's ball after its first four bounces.

   (ii) Find the limiting sum (or sum to infinity S_∞) of the sequence of heights of Jake's ball.

3. (i) The graph of the polynomial function $P(x)$ is an inverted parabola with axis of symmetry $x = 18$. Use this information to find the maximum possible profit that The Whatsit Company could have made each month in 2023 from producing doovalackies.

   (ii) (Optional) Write down two inequalities in terms of x to represent the values of x for which the number of doovalackies would have resulted in a loss each month for The Whatsit Company in 2023.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 7

Solving Problems involving Circles

Lesson Component 1 (Lesson Short Review)

<u>Questions</u>

1. Complete: An angle of 170 degrees forms a *complete revolution* with an angle of ………….. degrees.

   Complete: The angle at the center of a circle is ………….. the angle at the circumference standing on the same arc.

2. (i) Find, in terms of π, the arc length and area of a quadrant of the circle with radius 20 meters.

   (ii) Complete: When two chords intersect within a circle, the products of the intercepts are ………….. .

3. (i) Complete: The product of the intercepts on a secant from an external point is equal to the ………….. of the tangent from that point.

   Complete: Tangents to a circle from an external point have ………….. length.

   (ii) A tangent of length 15 meters from an external point T meets a circle with center O and radius 8 meters at the point P on its circumference. Calculate the length of OT.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

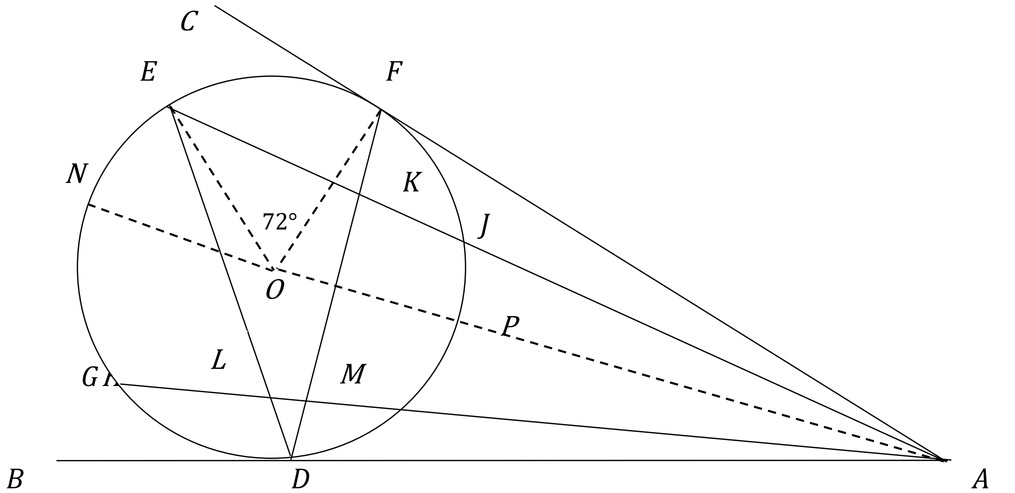
circumference, inscribed angle, intercept, intersecting, major/minor sector, secant, tangent, theorem.

Lesson Component 4 (Lesson Activity)

Time: 25 minutes

Part 4A

Stem for Items 1 and 2



A large park includes a circular area of radius 50 meters and a number of pathways that cross the area, as well as two pathways that are tangent to the area. Sebastian works for the council that manages the park and needs to check various measurements in relation to the circular park area for an upcoming project.

Sebastian knows that the circular park area, having a 50-meter radius, has an area of 2500π square meters and a circumference of 100π meters.

As part of the upcoming project, it is proposed that a new pathway AN (shown as a broken line) be constructed.

Part 4B

Item 1

Questions

1. (i) Show that the central reflex angle $\angle EOF$ measures 288°.

   (ii) In the diagram, the central angle $\angle EOF$ stands on the same arc EF as the inscribed angle $\angle EDF$. Sebastian has measured $\angle EOF$ to be 72°. What measurement should he obtain for $\angle EDF$, the angle between pathways DE and DF?

2. (i) Given that the circular park area has a circumference of 100π meters and an area of 2500π square meters, calculate the length of the arc EF and the area of the minor sector EOF.

(ii) The intersecting pathways DF and EJ are represented by intersecting chords in the diagram. Sebastian has obtained the measurements $EK = 12$ meters, $KJ = 8$ meters and $FK = 6$ meters. What length in meters should he obtain for the length of pathway section DK?

3. (Optional) For pathway AE and section AJ of pathway AE, Sebastian has obtained the measurements $AE = 170$ meters and $AJ = 150$ meters. Use this information to show that the length of pathway section AF, which is tangent to the circular park area, is 160 meters (to the nearest meter).

Part 4C

Item 2

Questions

1. (i) Sebastian measures $\angle DEO$ and $\angle DFO$ and finds that they are equal. Show that Sebastian would have found that the two angles each measure $18°$.

   (ii) Given that the circular park area has a circumference of 100π meters and an area of 2500π square meters, use the result in Part 4B Question 1. (i) to calculate the length in meters of the arc EDF and the area in square meters of the major sector EOF.

2. (i) Explain why pathway section AD is equal in length to pathway section AF and hence state the type of triangle represented by triangle ADF.

   (ii) Given that $\angle DFO = 18°$, find the sizes of $\angle AFD$ and $\angle DAF$.

3.	(Optional) In relation to the construction of the new pathway AN, Sebastian first measures OA. Use Pythagoras' theorem in triangle AFO to find the length of OA, and hence the length of AN (to the nearest meter).
Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)	

Student Workbook Mathematics Grade 10 Lesson 8

Determining the Center and Radius of a Circle given its Equation, and vice versa

Lesson Component 1 (Lesson Short Review)

Questions

1. (i) Write down the center and radius of the circle $x^2 + y^2 = 4$.

   (ii) Write down the equation of the circle with twice the radius of $x^2 + y^2 = 4$.

2. Determine the points where:

   (i) the circle $x^2 + y^2 = 9$ crosses the x- and y-axes.

   (ii) the circle $(x + 1)^2 + y^2 = \frac{9}{4}$ crosses the x-axis.

3. (i) Write the equation of the circle in Question 2 (ii) in general form.

   (ii) Use the equation to determine whether the point $(-1, 1\frac{1}{2})$ lies on the circle.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

center-radius form, concentric, construct, general form, radii, radius.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Eliza has been studying circles, including finding their centers, radii, and equations, whether particular points lie on the circles being considered, as well as constructing a number of circles on graph paper.

She has also become interested recently in studying concentric circles (i.e., circles with the same center, but of different radius) and is constructing such circles for a range of artistic designs.

Part 4B

Item 1

Questions

1. Eliza has constructed on graph paper a circle (Circle A) with equation (in center-radius form) $x^2 + y^2 = 6\frac{1}{4}$.

   (i) What is the center and radius of Circle A?

   (ii) Write down the equation of the circle (Circle B) with the same center as Circle A and four times the radius.

2. Eliza constructs on graph paper a circle (Circle C) with center $(-\frac{3}{4}, 0)$ and with radius half of that of Circle A.

   (i) Write down the equation of Circle C in center-radius form.

   (ii) Determine the points where Circle C crosses the y-axis.

3. (i) Eliza constructs two concentric circles (Circle D and Circle E) with center $(-2, 5)$. Circle D has radius $(\sqrt{7} + 1)$ cm, while Circle E is nine times the area of Circle D.

   Find the equation of Circle D (in center-radius form), and the equation of Circle E (in center-radius form) after first finding the radius of Circle E.

   (ii) (Optional) To graph the circle, Circle F, with equation (in general form) $x^2 + y^2 - 6x + 2y - 15 = 0$, Eliza needs first to write the equation in center-radius form. By completing squares, write the equation in center-radius form and use it to state the center and radius of the circle.

Part 4C

Item 2

Questions

1. Write down the center and radius of the circles:

   (i) Circle $G: 5x^2 + 5y^2 - 45 = 0$

   (ii) Circle $H: 16(x+1)^2 + 16(y-5)^2 = 81$.

2. (i) Write down the equation (in center-radius form) of the circle (Circle I) concentric with the circle (Circle J)$(x-7)^2 + (y+24)^2 = 625$ and one twenty-fifth its area.

   (ii) Determine the points where Circle J crosses the x-axis.

3. (Optional) Write the equation in general form of the circle, Circle K, with center $(-\frac{1}{2}, 3\frac{1}{2})$ and radius $5\frac{1}{2}$units and use the equation to determine whether the point $(-1,-2)$ lies on the circle.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 9

Graphing and Solving Problems involving Circles and other Geometric Figures on the Coordinate Plane

Lesson Component 1 (Lesson Short Review)

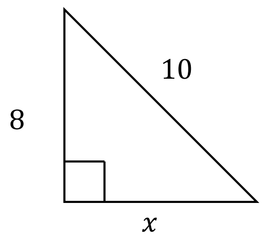
Questions

1. Complete: The midpoint (x, y) of the point (x_1, y_1) and (x_2, y_2) can be found using $M(x, y) = (\ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots\ldots)$.

   Complete: The distance between the points (x_1, y_1) and (x_2, y_2) can be found using the formula $d = \sqrt{\qquad\qquad\qquad\qquad}$.

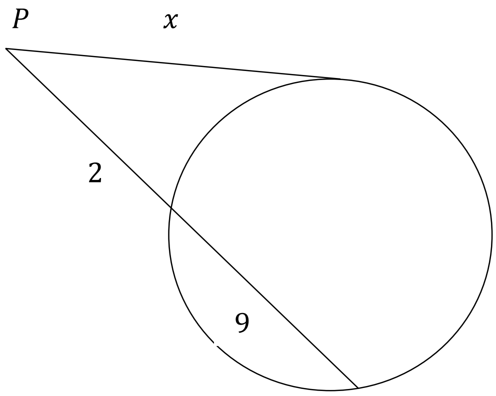
2. (i) Without using the distance formula, write down the length of the vertical interval joining $(3, 1\frac{1}{2})$ and $(3, 6\frac{1}{2})$.

   (ii)



   Find the value of x and the area of the triangle.

3.



   The diagram shows a tangent and a secant to a circle from the point P.

   Find the value of x.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

coordinate plane, geometric, interval, perpendicular, point of contact, Pythagoras, vertical.

Lesson Component 4 (Lesson Activity)

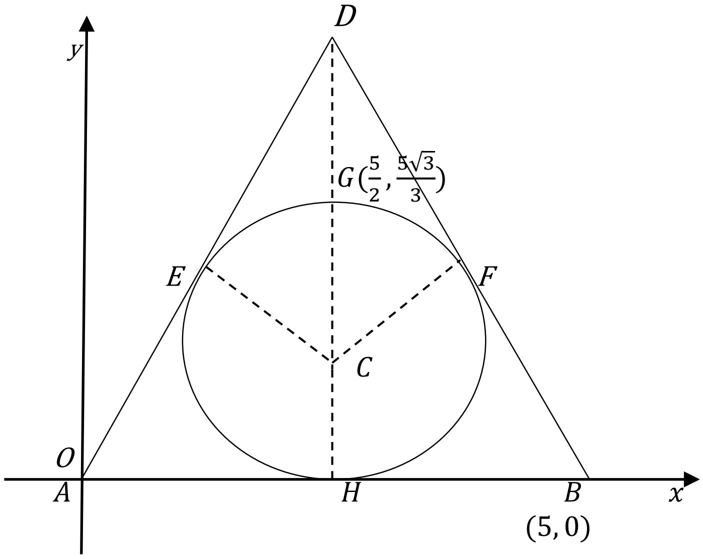
Time: 25 minutes

Part 4A

Stem for Items 1 and 2

Jason is designing a company logo. He has inscribed a circle in an equilateral triangle of side length 5 units. Vertex A of the triangle is at the origin $(0, 0)$ and vertex B is at $(5, 0)$ on the x-axis. Jason has drawn the straight line DH which divides the triangle ABD in half and passes through the center of the circle C and the point G on the circumference of the circle. He knows that the coordinates of G are $(\frac{5}{2}, \frac{5\sqrt{3}}{3})$.

The diagram below shows Jason's design on the coordinate plane.



Part 4B

Item 1

Questions

1. Write down the coordinates of the point H on the x-axis.

   Use the coordinates of G and H to show that the coordinates of C, the center of the circle, are $(\frac{5}{2}, \frac{5\sqrt{3}}{6})$.

2. (i) Show that the length of CH is $\frac{5\sqrt{3}}{6}$ i.e., the radius of the circle.

   (ii) Use Pythagoras' theorem in triangle BDH ($\angle BHD$ is a right angle) to find the length of DH. Hence, write down the coordinates of point D.

3. (Optional) Use the coordinates of C and D to write down the length of CD, and use the distance formula to find the length of AC and BC. Hence show that $AC = BC = CD$. (This means that the point C, the center of the circle, is equidistant from the vertices of the triangle ABD.)

Part 4C

Item 2

Questions

1. (i) Use the coordinates of D and G to find the length of DG and hence show that this length is equal to the radius of the circle.

   (ii) Write down the geometric reason that $\angle CFD$ is a right angle.

   Use Pythagoras' theorem in triangle CFD to find the length of DF.

2. Let the radius of the circle be equal to R units. Write CF and CD in terms of R and use Pythagoras' theorem to express DF in terms of R.

   State what multiple of the radius of the circle is the side length of the triangle.

3. (Optional)

   Jason knows that $DF^2 = DH.DG$ from his knowledge of geometry theorems involving tangents and secants. Complete the relevant theorem: The product of the intercepts on a secant from an external point equals the ……………………….. from that point.

   Use $DF^2 = DH.DG$ to find the length of DF and check that it is consistent with your answer in Part 4C Question 1. (ii).

   Find the area of triangle CFD and what fraction it is of the area of triangle ABD.

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 10

Differentiating Permutation from Combination of Objects taken r at a time

Lesson Component 1 (Lesson Short Review)

Questions

1. How many different pairs of symbols can be taken from the set {▲,⬟,⬣} if:

    (i) order within a pair is important?

    (ii) order within a pair is not important?

2. Complete the formulae:

    (i) $^{n}P_{r} = \frac{\quad}{(n-r)!}$

    (ii) $^{n}C_{r} = \frac{\quad}{r!\,(\ldots\ldots)}$

3. Calculate:

    (i) $^{5}P_{2}$

    (ii) $^{7}C_{3}$

Lesson Component 3 (Lesson Language Practice)

combination, digit, element, ordered selection, permutation, repetition, unordered selection.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Ralph is learning about permutations (ordered selections of items) and combinations (unordered selections of items) and the different results obtained when making ordered and unordered selections.

He is considering:

(i) the set (Set A) of the first four letters of the English alphabet i.e., Set $A = \{a, b, c, d\}$.

(ii) the set (Set B) of the whole numbers from 1 to 6 i.e., Set $B = \{1, 2, 3, 4, 5, 6\}$.

Part 4B

Item 1

Questions

1. (i) List the possible permutations of the elements of Set A if Ralph takes the elements two at a time (Hint: ab is one of the possible permutations).

   (ii) Of the possible permutations in Part B Question 1. (i), find how many are the different possible combinations of the elements of Set A when the elements are taken two at a time, using the formula ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$

2. If Ralph takes the elements of Set A three at a time, how many different permutations and how many different combinations are possible?

3. Ralph wishes to create three-digit numbers from the digits in Set B, using no digit more than once.

   (i) How many three-digit numbers can he create?

   (ii) (Optional) How many three-digit numbers that are less than 500 can he create?

Part 4C

Item 2

Questions

1. (i) How many permutations of the elements of Set A are possible if Ralph takes the elements four at a time?

   (ii) Of the possible permutations in Part 4C Question 1. (i), find how many are the different possible combinations of the elements of Set A where the elements are taken four at a time.

2. How many different numbers are possible if Ralph uses no digit more than once and takes the elements of Set B:

   (i) two at a time to create two-digit numbers?

   (ii) four at a time to create four-digit numbers?

3. (Optional) Using the digits in Set B, Ralph wishes to:

   (i) create four-digit numbers, with repetition of digits allowed. How many numbers can he create?

   (ii) select two digits, place them on separate cards, and place the two cards in a separate envelope. How many envelopes will he need for the possible different pairs?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 11

Solving Problems involving Permutations and Combinations

Lesson Component 1 (Lesson Short Review)

Questions

1. Show that ${}^5P_3 = 6 \times {}^5C_3$

2. In how many ways can 5 objects $\{V, W, X, Y, Z\}$ be arranged in a line if objects W, X and Y are to be together at the front of the line?

3. Groups of 5 people are to be selected from 6 men and 4 women.

   How many different groups can be selected if:

   (i) there are to be 3 men and 2 women in each group?

   (ii) there are to be 2 men and 3 women in each group?

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

arrange, combination, permutation, restriction, selection.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Suellen is a teacher who needs to arrange groups of her students in different ways for different class activities.

On a recent day she needed to:

(i) arrange six of her students $\{A, B, C, D, E, F\}$ in a line for a morning activity.

(ii) select groups of four of her students from five boys and three girls for an afternoon activity.

Part 4B

Item 1

Questions

1. (i) How many permutations of the six students could Suellen have obtained for the morning activity if she took only three students at a time?

   (ii) In how many ways could Suellen have arranged all six students for the morning activity if there were no restrictions on how they were to be arranged?

2. In how many ways could Suellen have arranged all six students for the morning activity if A and B were to be together in the line?

3. How many groups of four of her students could Suellen have selected for the afternoon activity if there were the following restrictions:

   (i) all members of the group were to be boys?

   (ii) (Optional) there were to be all three girls and one boy in the group?

Part 4C

Item 2

Questions

1. (i) How many permutations of the six students could Suellen have obtained for the morning activity if she took only five students at a time?

   (ii) How many groups of four of her students could Suellen have selected for the afternoon activity if there were no restrictions on how they were to be selected?

2. In how many ways could Suellen have arranged all six students for the morning activity if D was to be second in the line and F was to be fifth in the line?

3. How many groups of four of her students could Suellen have selected for the afternoon activity if there were the following restrictions:

(i) there were to be two boys and two girls in the group? (ii) (Optional) a particular girl X was to be included in the group?
Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 12 Deliberate Practice

Solving Problems involving Circles
Graphing and Solving Problems involving Circles and Other Geometric Figures on the Coordinate Plane
Solving Problems involving Permutations and Combinations

Lesson Component 1 (Lesson Short Review)

Questions

1. (i) Without using the distance formula, write down the length of the horizontal interval joining $(-4\frac{1}{2}, 1)$ and $(7, 1)$.

   (ii) Find the equation of the circle with center $(0, 0)$ and radius 12 units.

2. (i) Use the distance formula to find the length of the interval joining the points $(5, -2\sqrt{3})$ and $(2, 2\sqrt{3})$.

   (ii) Complete: The area of a rhombus is given by the formula $Area\ of\ rhombus = \frac{1}{2}xy$, where x and y are the lengths of the …………… of the rhombus.

3. Groups of 7 people are to be selected from 5 men and 6 women.

   How many different groups can be selected if there are to be 3 men and 4 women in each group?

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

arc, coordinate plane, diagonal, rhombus, sector, vertex/vertices, vertical.

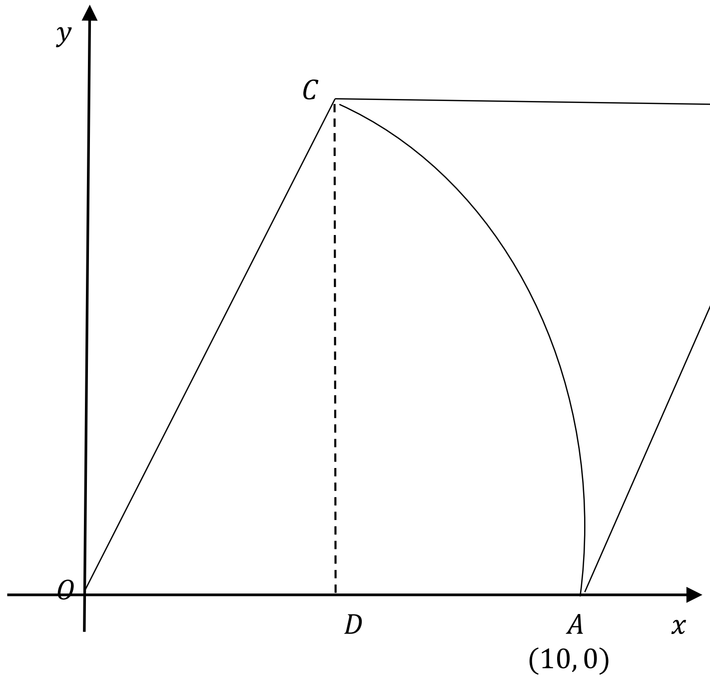
Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Sarah is a member of a professional association of graphic designers. The first stage of her latest design involves a rhombus of side length 10 units and a sector of a circle with center at the vertex O of the rhombus, which corresponds with the origin of the coordinate plane. The vertices A and C of the rhombus lie at the ends of the arc of the sector. The point D on the x-axis is the midpoint of the side OA of the rhombus and lies at one end of the vertical line CD.

The diagram below shows the first stage of Sarah’s design.



Sarah has also recently become responsible for establishing a number of committees for her professional association.

Part 4B

Item 1

Questions

1. (i) Find the coordinates of the point D on the x-axis.

   (ii) Sector AOC is part of the circle center O and radius OA.

   Write down the equation of this circle.

2. (i) Use the equation of the circle found in Part 4B Question 1. (ii) and the fact that CD is a vertical line to find the coordinates of the point C.

   (ii) Use the distance formula to find the length of AC, the shorter diagonal of the rhombus $OABC$.

3. Sarah is forming Committee A, a committee of six graphic designers from a group of three men and five women.

   (i) How many different committees can she form if there are no restrictions?

   (ii) (Optional) How many different committees can she form if there are to be 2 men and 4 women on the committee?

Part 4C

Item 2

Questions

1. (i) Find the coordinates of the vertex B of the rhombus.

   (ii) Use the distance formula to find the length of OB, the longer diagonal of the rhombus.

2. (i) Sarah knows that the area of a rhombus is given by the formula $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the rhombus.

   Use the formula to find the area of the rhombus $OABC$.

   (ii) (Optional) Given that $\angle AOC = 60°$, find the area of the sector AOC, and the area of the section of the rhombus outside the sector (in square units correct to 1 decimal place).

3. Sarah is forming Committee B, a committee of five graphic designers from a group of four men and seven women.

   (i) How many different committees can she form if all members of the committee are to be women?

   (ii) (Optional) How many different committees can she form if a particular man and a particular woman are to be included on the committee?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 13

Illustrating and finding the Probability of a Union of Two Events $(\boldsymbol{A} \cup \boldsymbol{B})$, including for when the events are mutually exclusive

Lesson Component 1 (Lesson Short Review)

Questions

1. For two events X and Y, $P(X) = \frac{1}{2}$, $P(Y) = \frac{1}{4}$ and $P(X \cap Y) = \frac{2}{5}$.

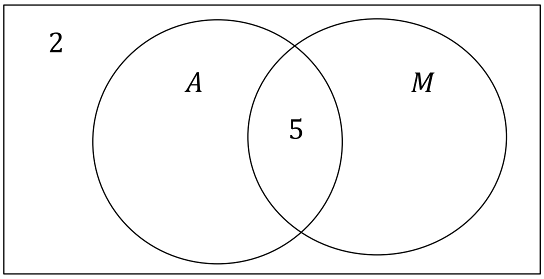
   Find $P(\bar{Y})$; $P(X \cup Y)$.

2. For two mutually exclusive events V and W , $P(V) = \frac{2}{3}$, $P(W) = \frac{1}{10}$.

   Find $P(V \cap W)$; $P(V \cup W)$; $P(\overline{V \cup W})$.

3. In a class of 20 students, 11 students study Art (A), 12 study Music (M), 5 study both subjects, and 2 study neither subject.

   (i) Complete the diagram to show this information.



   (ii) One student is to be selected from the class. A is the event of selecting a student who studies Art and M is the event of selecting a student who studies Music.

   Find $P(A \cup M)$.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

at random, event, mutually exclusive, probability, respectively, union.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Mary-Jane attends Southwood Girls' High School. In her Mathematics class she has been illustrating and finding the probability of a union of two events $(A \cup B)$, including for when the events are mutually exclusive.

Mary-Jane is in a class of 30 students for Mathematics and Science. In a recent vote, 15 students voted Mathematics as their preferred subject, 13 students voted Science as their preferred subject, while 2 students said that they had no preference and liked both subjects the same. In Mary-Jane's class of 27 students for Geography and History, 10 students voted Geography as their preferred subject, 13 students voted History as their preferred subject, while 4 students said that they had no preference and liked both subjects the same.

There are 150 students in Mary-Jane's Grade at Southwood Girls' High School. For sport, 71 play tennis, 57 play netball, 53 play hockey, 21 play hockey and netball, 17 play tennis and netball, 25 play tennis and hockey, while 5 students play all three sports.

Part 4B

Item 1

Questions

1. (i) Mary-Jane is considering events A and B which are not mutually exclusive. $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$.and $P(A \cap B) = \frac{1}{10}$.

   What should she obtain for: $P(\bar{A})$, $P(A \cup B)$?

   (ii) Mary-Jane then considers events C and D which are mutually exclusive, where:

   $P(C) = \frac{1}{2}$, $P(D) = \frac{2}{5}$.

   What should she obtain for: $P(C \cup D)$, $P(\overline{C \cup D})$?

2. If M and S denote the events that Mathematics and Science (respectively) are preferred subjects, find $P(M \cap S)$, $P(M \cup S)$.

3. If one of the girls in Mary-Janes' Grade is selected at random, what is the probability that she plays:

   (i) one sport only?

   (ii) at least two sports?

   (Hint: First complete a Venn diagram for the sports information for Mary-Jane's Grade.)

Part 4C

Item 2

Questions

1. (i) Mary-Jane is considering events Q, R and S which are not mutually exclusive. She knows that $P(Q) = \frac{2}{5}$, $P(R) = \frac{3}{10}$, $P(S) = \frac{2}{5}$, $P(Q \cap R) = \frac{1}{5}$, $P(Q \cap S) = \frac{1}{10}$, $P(R \cap S) = \frac{1}{4}$.

   What should she obtain for: $P(R \cup S)$; $P(\overline{Q \cup S})$?

   (ii) Mary-Jane then considers the mutually exclusive events T, U and W, where:

   $P(T) = \frac{2}{5}$, $P(U) = \frac{1}{3}$, $P(W) = \frac{1}{5}$.

   What should she obtain for: $P(\overline{U})$; $P(\overline{T \cup U})$; $P(T \cup U \cup W)$?

2. If G and H denote the events that Geography and History (respectively) are preferred subjects, find $P(\overline{G \cap H})$; $P(G \cup H)$.

3. (Optional) If one of the girls in Mary-Janes' Grade is selected at random, what is the probability that she plays:

   (i) none of the three sports?

   (ii) at most one sport?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 14

Solving Problems involving Probability

Lesson Component 1 (Lesson Short Review)

Questions

1. Bag 1 contains 3 blue and 2 red marbles. Two marbles are drawn one at a time from the bag, without replacement. What is the probability that:

   (i) the first marble drawn is blue?

   (ii) both marbles drawn are blue?

2. Bag 2 contains 5 blue marbles and 1 red marble, and Bag 3 contains 2 blue marbles and 3 red marbles. If one of Bags $1, 2, \text{and } 3$ is selected at random and then two marbles are drawn in succession without replacement, what is the probability that both marbles are red?

3. Bag 4 contains 3 white (W), 4 yellow (Y) and 2 green (G) marbles.

   (i) In how many ways can three different colors be drawn in three draws in succession from the bag? (As examples, two of these ways are the sequences W, Y, G and $W, G, Y.$)

   (ii) If, after each draw, the marble drawn is replaced in the bag, what is the probability that one white, one yellow, and one green marble, are drawn in any order?

Lesson Component 3 (Lesson Language Practice)

at random, experiment, in succession, marble, with/without replacement.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Don and Phil are in Grade 10 at Everly High School. They are conducting different experiments in their study of probability in their Mathematics class.

For Experiment A, they have two bags of marbles: Bag 1 contains 2 red (R) marbles and 4 yellow(Y) marbles, while Bag 2 contains 5 red marbles and 3 yellow marbles.

For Experiment B, they also have two bags of marbles: Bag 1 contains 4 blue (B) marbles and 2 white(W) marbles, while Bag 2 contains 6 blue marbles and 3 white marbles.

For Experiment C, they have one bag of marbles that contains 5 black (B) marbles, 4 purple (P) marbles, and 3 green (G) marbles.

Part 4B

Item 1

Questions

1. Don conducts Experiment A first and for this experiment draws a marble from each bag. What is the probability that the marbles he draws are both red?

2. Phil conducts Experiment B first and for this experiment chooses a bag at random and then draws:
   (i) one marble from that bag. What is the probability that the marble is white?

   (ii) two marbles in succession without replacing the first marble drawn in the bag. What is the probability that both marbles are blue?

3. Don conducts Experiment C first and for this experiment draws three marbles at random from the bag. What is the probability of drawing one black, one purple, and one green marble (in any order) if the marbles are drawn without replacement?

Part 4C

Item 2

Questions

1. When Phil conducts Experiment A, he also draws a marble from each bag. What is the probability that the marbles he draws are:
   (i) both yellow?

(ii) the same color?

2. When Don conducts Experiment B, he also chooses a bag at random and then draws:

   (i) one marble from that bag. What is the probability that the marble is blue?

   (ii) two marbles in succession, replacing the first marble drawn before drawing the second marble. What is the probability that the marbles are of different colors?

3. (Optional) When Phil conducts Experiment C, he also draws three marbles at random from the bag. What is the probability of drawing one black, one purple and one green marble (in any order) if the marbles are drawn with replacement? (i.e., after each draw, the marble selected is replaced in the bag.)

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 15

Calculating and Interpreting Measures of Position (quartiles, deciles, and percentiles) of a Set of Data

Lesson Component 1 (Lesson Short Review)

Questions

1. (i) Complete: The median is equivalent to the ………….. quartile and the ………….. percentile.

   (ii) What value or expression do A and B represent in the following formulae:

   $Position\ of\ D_i = \frac{i}{A}(n+1);$ $\qquad Position\ of\ P_i = \frac{i}{100}(B)$

2. For the set of scores $14, 15, 16, 16, 17, 18, 18, 19, 20$:

   (i) Find the median.

   (ii) Use Tukey's Method to find Q_1.(the lower quartile).

3. Using the second formula in Question 1. (ii), what will be the position of the fifty-fifth percentile in a set of 60 scores?

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

cumulative frequency, decile, frequency, measure of position, median, percentile, quartile, Tukey's Method.

Lesson Component 4 (Lesson Activity)

Part 4A

<u>Stem for Items 1 and 2</u>

Magdelena has collected a small set of data, Set A, and has arranged the data in ascending order:

Set A: $\{2, 3, 3, 5, 5, 6, 7, 7, 8, 9, 9, 9, 10, 10, 11, 12\}$.

She has also collected a larger set of data, the set of scores Set B, and has tabulated the data as follows:

Set B scores x	Frequency f	Cumulative Frequency
20	6	6
21	8	14
22	9	23
23	14	37
24	15	52
25	19	71
26	20	91
27	24	115
28	19	134
29	13	147
30	11	158
31	10	168
32	9	177
33	8	185
34	7	192
35	6	198
36	2	200
	$\Sigma f = 200$	

Part 4B

<u>Item 1</u>

<u>Questions</u>

1. Magdalena wishes to calculate the median for the data in Set A.

   (i) What value should she obtain for the median?

   (ii) What does this value represent for the data set and to which quartile is it equivalent?

2. Magdalena wishes to use Tukey's Method to calculate the other quartiles for the data in set A.

   (i) What do the first quartile and the third quartile represent?

(ii) What value should she obtain for the first quartile (Q_1)?

3. Magdalena wishes to use the formula $Position\ of\ P_i = \frac{i}{100}(n+1)$ to calculate different percentiles for the data in Set B.

   (i) What values should she obtain for the 60th percentile and 90th percentile?

   (ii) What does the percentile midway between the 60th percentile and 90th percentile represent?

Part 4C

Item 2

Questions

1. In order to find the median of the set of scores in Set B, Magdalena knows that she can use the Cumulative Frequency column of the table to help her locate its value. What value should she obtain for the median?

2. Magdalena wishes to use the formula $Position\ of\ D_i = \frac{i}{10}(n+1)$ to calculate different deciles for the data in Set B.

   (i) What values should she obtain for the third decile and seventh decile?

   (ii) What does the decile midway between the third decile and seventh decile represent?

3. (Optional) Magdalena uses Tukey's Method and the formula $Position\ of\ P_i = \frac{i}{100}(n+1)$ to calculate the first quartile (25th percentile) for the data in Set B.

   (i) What two results should she obtain?

   (ii) How do the results compare? Why won't this always be true?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Worksheet Mathematics Grade 10 Lesson 16

Solving Problems involving Measures of Position

Lesson Component 1 (Lesson Short Review)

Questions

Score x	Frequency f	Cumulative Frequency
5	2	
6	3	
7	4	
8	4	
9	5	
10	3	
	$\sum f=21$	

For the data in the table above:

1. (i) Complete the Cumulative Frequency column.

   (ii) Use the Cumulative Frequency column to find the median.

2. Use Tukey's Method to find the third quartile.

3. Use the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ and $Position\ of\ P_i = \frac{i}{100}(n+1)$ to find the position of the fourth decile and the forty-fifth percentile.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

decile, median, percentile, quartile, survey, tabulate, Tukey's Method.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Alessandro has surveyed 50 locomotive drivers (Set L) and 50 drivers of large haulage trucks (Set T) that are based in his local area, regarding the average number of hours (to the nearest hour) that they drive on each of their workdays. He has then tabulated the data obtained in the tables below:

Data for Locomotive Drivers (Set L)

Average number of hours driving locomotive per workday x	Frequency f	Cumulative Frequency
1	2	2
2	5	7
3	6	13
4	8	21
5	9	30
6	9	39
7	7	46
8	4	50
	$\sum f$=50	

Data for Truck Drivers (Set T)

Average number of hours driving truck per workday x	Frequency f	Cumulative Frequency
3	3	3
4	5	8
5	7	15
6	10	25
7	9	34
8	6	40
9	6	46
10	4	50
	$\sum f$=50	

Part 4B

Item 1

Questions

1. Alessandro has calculated the median for the set of average hours that the locomotive drivers drive their vehicles each workday. What should he have obtained for the median for Set L?

2. Alessandro has used Tukey's Method to find the first quartile for Set L. What value should he have obtained?

3. Alessandro has used the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) and $Position\ of\ P_i = \frac{i}{100}(n+1)$ (for calculating position of percentiles) to calculate the:
   (i) ninth decile for set L. What value should he have obtained?
   (ii) thirty-fifth percentile for Set L. What value should he have obtained?

Part 4C

Item 2

Questions

1. Alessandro has calculated the median for the average hours that the truck drivers drive their vehicles each workday. What should he have obtained for the median for:

   (i) Set T?

   (ii) How much greater in percentage terms is the median for Set T than that for Set L?

2. Alessandro has used Tukey's Method to find the third quartile for Set T. What value should he have obtained?

3. (Optional) Alessandro has used the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) and $Position\ of\ P_i = \frac{i}{100}(n+1)$ (for calculating position of percentiles) to calculate the:

   (i) second decile for Set T. What value should he have obtained?

   (ii) sixty-fifth percentile for Set T. What value should he have obtained?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Worksheet Mathematics Grade 10 Lesson 17

Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data

Lesson Component 1 (Lesson Short Review)

Questions

A class of Mathematics students is analyzing and interpreting the following set of scores:

$$23, 24, 25, 26, 26, 27, 27, 28, 28, 29$$

1. For the set of scores above, calculate the:

   (i) Range

   (ii) median

2. (i) For the set of scores above, calculate the mean.

   (ii) Complete: $\sqrt{\frac{\Sigma(fd^2)}{\Sigma f}}$, where for each score in a set of scores f is the frequency and d is the deviation from the mean, is used to calculate the ………….. for a set of scores.

3. For the set of scores above:

   (i) use Tukey's Method to calculate the lower quartile Q_1.

   (ii) use the formula $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) to calculate the seventh decile.

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

decile, modal, percentile, quartile, standard deviation, statistical.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Buddy and Holly are in the same Grade 10 Mathematics class. In their last six Mathematics class tests for the year (results of each reported as a mark out of 10), Buddy scored $6, 7, 7, 7, 8, 7$ while Holly scored $9, 1, 8, 9, 7, 5$.

Their teacher, Mr Statz, has calculated the mean, median, lower (first) quartile, upper (third) quartile, and standard deviation for Buddy's and for Holly's test scores.

At the end of the year, the 24 students in the class sat for their final examination in Mathematics (results reported as a mark out of 100). Mr Statz has listed below their results in increasing order:

$$41, 43, 47, 51, 52, 55, 55, 57, 58, 60, 64, 65, 67, 67, 68, 71, 74, 78, 80, 83, 86, 88, 92, 96$$

Part 4B

Item 1

Questions

1. (i) Calculate the mean score and median score that Mr Statz calculated for Buddy and for Holly.

   (ii) Explain why one of the measures Is the better measure of their Mathematics ability.

2. For the last six Mathematics class tests, find the lower quartile that Mr Statz calculated for:

   (i) Buddy's scores

   (ii) Holly's scores.

3. (i) If Buddy wished to score higher than the sixth decile in the final examination, what is the lowest of the marks that Mr Statz listed that Buddy would have had to achieve?

   (ii) (Optional) If Holly wished to score higher than the eightieth percentile in the final examination, what is the lowest of the marks that Mr Statz listed that Holly would have had to achieve?

Part 4C

Item 2

Questions

1. (i) Calculate the median score and the range of the scores for Mr Statz' class in the final examination.

(ii) What are the modal scores for the final examination?

2. For the last six Mathematics class tests, find the:

   (i) upper quartile for Holly's scores.

   (ii) standard deviation for Buddy's scores.

3. (Optional)

   (i) If Mr Statz tells Buddy that he scored the first mark higher than the seventh decile in the final examination, what mark did Buddy achieve?

   (ii) If Mr Statz tells Holly that she scored the first mark higher than the eighty-fifth percentile in the final examination, what mark did Holly achieve?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

Student Workbook Mathematics Grade 10 Lesson 18 Deliberate Practice

Solving Problems involving Probability
Solving Problems involving Measures of Position
Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data

Lesson Component 1 (Lesson Short Review)

<u>Questions</u>

1. Bag A contains 3 yellow marbles and 2 black marbles and Bag B contains 4 yellow and 2 black marbles. If one of the bags is selected at random and then two marbles are drawn in succession without replacement, what is the probability that both marbles are black?

2. For the set of scores: $1, 2, 2, 1, 4, 2$

   (i) Show that the mean for the set of scores is $\bar{x} = 2$.

   (ii) Complete the columns for the frequency, deviation from the mean ($\bar{x} = 2$), the square of the deviation, and the frequency × the square of the deviation.

Score x	Frequency f	d	d^2	fd^2
1				
2				
4				
	$\Sigma f=$			$\Sigma(f\,d^2)=$

Use the formula, Standard Deviation $= \sqrt{\frac{\Sigma(fd^2)}{\Sigma f}}$, to find the standard deviation for the scores.

3. Use Tukey's Method to find the Interquartile Range (IQR) for the set of scores. (The Interquartile Range is the difference between the Upper Quartile (Q_3) and the Lower Quartile (Q_1) for a set of scores.)

Lesson Component 3 (Lesson Language Practice)

Key words/terms are:

at random, disc, in succession, interquartile range, standard deviation, survey, with/without replacement.

Lesson Component 4 (Lesson Activity)

Part 4A

Stem for Items 1 and 2

Melinda is the Population Data Officer for a city's Municipal Council. The city is divided into two large sections called the Northern Division, which has 4 Eastern Suburbs and 3 Western Suburbs, and the Southern Division, which has 3 Eastern Suburbs and 5 Western Suburbs.

One of Melinda's recent tasks was to select one of the city's suburbs at random in order to conduct a survey of 100 families in the suburb to collect data on the number of children per household.

Melinda wants to analyze the survey data and has partly completed the table below:

Number of children per household in survey suburb x	Frequency f	$f \times x$	Cumulative Frequency	d	d^2	fd^2
0	25					
1	18					
2	21					
3	17					
4	9					
5	7					
6	3					
	$\sum f=$	$\sum(f \times x) =$				$\sum(f\,d^2)=$

Part 4B

Item 1

Questions

1. To select the suburb for the survey, Melinda placed the names of the suburbs on small discs in one of two bags, with the Northen Division suburbs being placed in Bag 1 and the Southern Division suburbs in Bag 2. She then selected one bag at random and then one disc from the bag selected.

   (i) What was the probability of Melinda selecting a Western Suburb from the Southern Division?

   (ii) What was the probability of selecting any of the city's Eastern Suburbs?

2. (i) Write down the mode and range for the survey data.

   (ii) Complete the $f \times x$ column and Cumulative Frequency column of the table and use them to find the mean (using Mean $= \frac{\Sigma(f \times x)}{\Sigma f}$) and median number of children per household in the surveyed suburb.

3. (Optional) Melinda used Tukey's Method to find the first and third quartiles for the survey data. What values should she have obtained?

Part 4C

Item 2

Questions

1. For the suburb selection process described in Part 4B Question 1:

   (i) What was the probability of Melinda selecting an Eastern Suburb from the Northern Division?

   (ii) What was the probability of selecting any of the city's Western Suburbs?

2. (i) Complete Columns 5 (using $\bar{x} = 2$), 6 and 7 of the table for the survey.

   (ii) Show that the standard deviation for the data is approximately 1.68.

3. (Optional) Melinda used the formulae $Position\ of\ D_i = \frac{i}{10}(n+1)$ (for calculating position of deciles) and $Position\ of\ P_i = \frac{i}{100}(n+1)$ (for calculating position of percentiles) to calculate the fourth decile and ninety-fifth percentile for the survey data. What values should she have obtained?

Lesson Component 5 (Lesson Conclusion – Reflection/Metacognition on Student Goals)

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