8

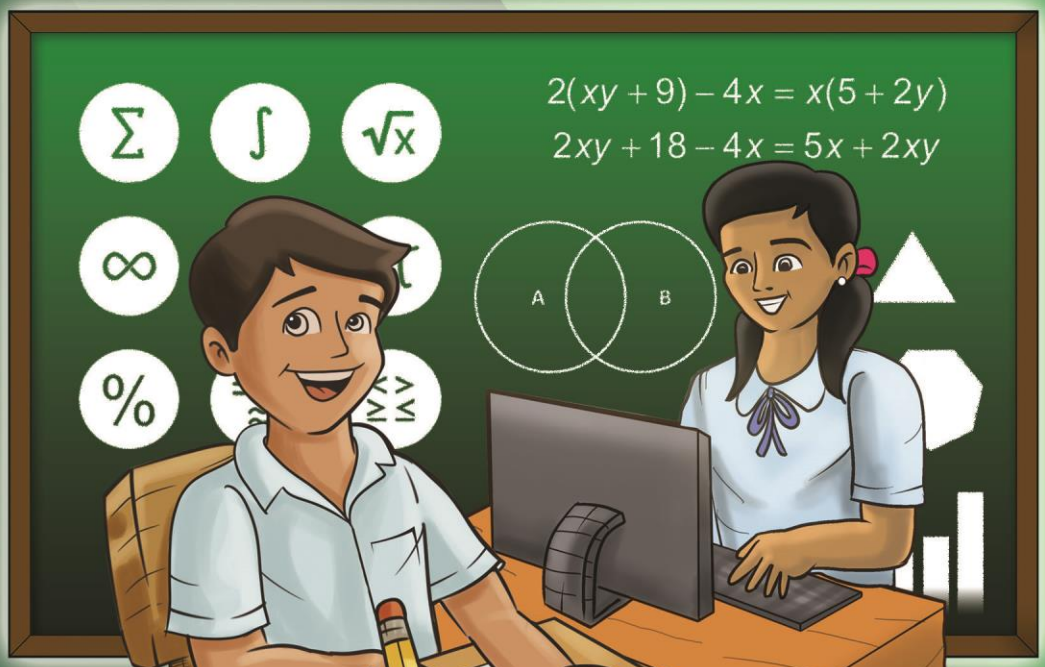
NATIONAL LEARNING CAMP

DepED DEPARTMENT OF EDUCATION

Mathematics

Enhancement Learning Camp

Notes to Teachers



Enhancement Learning Camp

Notes to Teachers

Mathematics Grade 8

Weeks 1 to 3

Contents

Notes to Teachers 1

Part A: Introduction to Mathematics in the 2023 Learning Camp 1

Part B: Commentary on Lesson Components in All Lessons 2

Part C: Commentary on Lesson Components in All Lessons 6

Lesson Component 1 (Lesson Short Review) 6

Lesson Component 2 (Lesson Purpose/Intention) 6

Lesson Component 3 (Lesson Language Practice) 7

Lesson Component 4 (Lesson Activity) 7

Lesson Component 5 9

Part D: Syllabus Codes, Matters for Students to Observe, and Worked Answers for the Individual Lessons 10

Lesson 1: Factors of Trinomials 10

Lesson 2: Problems with Polynomials 12

Lesson 3: Simplifies Rational Algebraic Expressions 14

Lesson 4: Solves Problems with Rational Expressions 16

Lesson 5: Co-Ordinate Geometry 19

Lesson 6 Consolidation: Factorizing Trinomials, Problems with Polynomials, Simplifying Ration al Algebraic Expressions, Solving Problems with Rational Expressions and Co-Ordinate Geometry 22

Lesson 7: Applications of Co-Ordinate Geometry 24

Lesson 8: Solving Problems using Co-Ordinate Geometry 26

Lesson 9: Simultaneous Equations 28

Lesson 10: Inequalities 31

Lesson 11: Relations and Functions 33

Lesson 12 Consolidation: Applications of Co-Ordinate Geometry, Solving Problems using Co-Ordinate Geometry, Simultaneous Equations, Inequalities, Relations and Functions 35

Lesson 13: 'If-Then' Statements 39

Lesson 14: Congruent Triangles 41

Lesson 15: Proof 43

Lesson 16: Probability 44

Lesson 17: Illustrates experimental and theoretical probability 45

Lesson 18 Consolidation: Congruence, If=then statements, Parallel Lines and Probability 47

Dear Reader

Every care has been taken to ensure the accuracy of the information provided in this Booklet. Nevertheless, if you identify a mistake, error or issue, or wish to provide a comment we would appreciate you informing the **Office of the Director of the Bureau of Learning Delivery** via telephone numbers (02) 8637-4346 and 8637-4347 or by email at bld.od@deped.gov.ph

Thank you for your support.

Notes to Teachers

Part A: Introduction to Mathematics in the 2023 Learning Camp

Overview

There are 30 lessons in Mathematics for the Consolidation Camp. This is best considered in two broad parts. The first part involves 18 lessons, which are designed to be provided over the first three weeks of the Camp, see detailed information below. These 18 lessons are the same as the lessons being used in the Grade 8 Enhancement Camp.

In the second part of the Consolidation Camp, the teachers and students proceed further in Week 4 and week 5. This involves a further 12 lessons Lesson 19 to Lesson 30. The same structure occurs in these 12 lessons as in the first 18 lessons. Each of these lessons is different to what was provided in the first 18, however, significantly they address 12 of the same Key ideas. The purpose here is allow students to have a second chance at working on a Key idea but with a different stimulus. This is an example of deliberate practice in education where students are offered the opportunity to further refine their skills on the same idea.

Links between Enhancement and Consolidation Camps

The link between the two Mathematics camps for the first three weeks is done to facilitate deeper discussions among Mathematics teachers in their Planning and Review days. As described elsewhere; see write up on Consolidation Camp information in the Consolidation Grade 8 Lesson Plans Booklet for the information.

It is important to restate here that the materials for the first 3 weeks of the Mathematics Consolidation Camp including the lesson plans are the same as offered to students attending the Mathematics Enhancement Camp. There are important reasons for both Camps sharing the same content. However, there are differences between the Camps associated with the teaching focus which is related to the breadth and depth of conceptual knowledge and understanding of the students.

When teachers come together to discuss Mathematics they can talk about the exact same content and potentially identify differences in approach and reach depending on the Camp in which they are working. This opens the discussion among teachers on how the identified needs of students of different abilities, skills and understandings can be addressed on the same key ideas of Mathematics.

In both the Consolidation and Enhancement Camps important lower-order content skills, knowledge and understandings are re-visited at the beginning of each lesson. This helps ensure that potential learning obstacles are made visible to the student and the teacher. It also means that errors in understanding or misconceptions are identified. This information is important in helping all students move forward regardless of their achievement levels.

Significantly, students in the Consolidation Camp may not be able to achieve at the same level as students in the Enhancement Camp. However, at a minimum, students in the Consolidation Camp should become aware of the more difficult questions as they allow them to consider links or connections between concepts previously taught. There is value in students, having first attempted the earlier questions, also proceeding to the latter questions in a problem set. There is learning development for low-achieving students to practice reading the question (fluently) and being able to say, using their own words, what the question is about in terms of the problem asked.

Structure of this Notes to Teacher Booklet

There are five Parts in this Booklet. In view of the similarities with the lessons discussed in Part D and Part E, no discussion is provided in the latter, although the link to the equivalent earlier lesson is identified and should be referenced by the teacher. Any additional teacher advice can be found in the Lesson Plans Booklet under the Lesson Number.

Part B: Commentary on Lesson Components in All Lessons

Overview

The NLC lessons emphasizes consolidating and, where possible, extending student knowledge in previously covered topics. Lesson sets are designed to strengthen students' current foundational knowledge ready for future learning. The review lessons have been designed to be interactive among teachers and their students, and with students and their peers.

At the same time, the expectation is that teachers will enhance their own pedagogical practices and subject knowledge as well as refine further their teaching methods. The thinking behind the Camp lessons is grounded in the 'Science of Learning' framework, creating a dynamic, learning environment employing the findings of cognitive research and evidence-informed approaches.

Lesson Component 1 (Lesson Short Review)

Component 1 offers teachers the chance to:

- settle the class quickly;
- review or preview previously encountered information;
- address previous content in the form of a few targeted questions that are *relevant to the current lesson;*
- note what students already know;
- elicit answers from the class to reinforce the important content needed for the lesson; and
- briefly address issues that may arise.

Overall, Component 1 acts as a partial advance organizer designed to remind students of previous work that has relevance to activities to be undertaken in the current lesson. When done carefully, this replay of previous information directed at what is to come in the lesson helps students prepare for future memory recall and decision-making.

Reminding students of *relevant* information at the beginning of a lesson, prior to encountering the main lesson learning focus, can enhance the brain's ability to:

- access information to be used in problems/questions/information to come;
- prepare, and have some oversight, for the direction of future learning; and
- further consolidate ideas in long-term memory.

The teacher should note any issues that may arise in student answers. This may be addressed later in the lesson or in later lessons, if relevant.

Lesson Component 2 (Lesson Intention)

This component offers teachers the opportunity to explain to the class the intention or purpose of the lesson. The explanation should link with student prior knowledge or experience. This may mean connecting the purpose to the responses and levels of understandings in Component 1. The words and phrases used by the teacher should be familiar to, and understandable by, students. Information could include ideas personal to students that could facilitate student engagement in the lesson such as:

- the provision of a relevant context;
- asking a question that sounds interesting to that age group; and/or
- addressing an aspect that has a special interest to the class.

In addition, this component is an appropriate time to address what students might expect/aim to achieve, i.e., the lesson goal(s). Teachers should clarify, in clear language, the learning intention and what success looks like.

(Note**:** Evaluation of the degree of success or partial success of student learning intention should occur as part of Component 5.)

Lesson Component 2 is about activating, in the student brain, ideas already relevant to the students. The purpose is to help students contextualize their new learning experiences and to help them make sense of any new information.

Design considerations in statements of the lesson intention are about promoting student engagement and enthusiasm. This is best done by stating things in ways that make sense to as many students as possible in the class. In terms of timing, this component is relatively brief. Its presence, as one-of-five components, lies in *its importance* to the student brain and learning. Finally, it is important *not to* overwhelm students with excessive and unnecessary detail that could disengage them at this early point in the lesson.

Lesson Component 3 (Lesson Language Practice)
Component 3 concerns language use – speaking, hearing, listening, and comprehending. The focus is on words or phrases that are important to this lesson. It maybe language that has the potential to cause difficulties for the students through speech, interpretation, or understanding, or simply a reminder that these words are important to, or will be used in, this lesson. Typically, the language identified is restricted to about 6 words/phrases so that there is enough time for students to use them in practice.

Deliberate practice concerns repeating aspects of learning that the teacher has deliberately identified/selected because it is where students are making an error that needs to be corrected, or because of its important role in learning. In the case of unfamiliar or unknown textual or symbolic language, deliberate practice can help students reduce cognitive load (reduce working memory) by making some aspect more familiar, enabling students to re-allocate resources to a problem solution, comprehending a passage, answering a question, explaining a concept, or describing some event or story, etc.

Overall, Component 3 can help achieve language familiarity by saying the word/phrase, being able to spell it, or using it in a specific context. This may also involve helping students to understand or unpack a visual text, diagram or graph, e.g., for a graph, the teacher may need to point out such things as the graph heading, the axes, units, data points, or trend lines.

Lesson Component 4 (Lesson Activity)
Addressing the key idea for the lesson is the focus of Component 4. It involves students applying known content to solve non-routine problems or interpreting new texts. This requires students to interpret/understand the meaning of the stem of the problem correctly and then answer a few questions of varying degrees of complexity related to the stem. The stem holds the needed information that will be the basis for the questions. Following the stem is a small number of questions that can be answered by utilizing students' background content knowledge and understanding, together with information in the stem.

From a learning perspective, the lessons are intended to help students consolidate their understanding at different levels of difficulty, e.g., the early questions are at an elementary level allowing the students to get started, then the next level is directed at the majority of students and usually requires a number of steps to reach a conclusion, and, finally, the third question attempts to offer all students the opportunity to be challenged and experience enhancements of their learning through seeing how ideas are connected or applied.

(Note: The level of difficulty of the questions should not stop any student from being given the opportunity to experience, with support, questions at higher levels, including the more challenging questions, and to hear about, and be involved in, discussions about the answers.

Most students should be able to make some progress and be acknowledged for that. The point of question levels is to at least have students experience these more demanding questions and their answers as the start of the

process for their learning journey. It is also designed to offer teachers a more realistic view of potential expectations of students in their class.)

Component 4 has three aspects, 4A, 4B, and 4C. Students are first presented in 4A with the stem. This can be a stimulus or passage/text or diagram or … and are given the time/opportunity to understand the stem.

Then, in 4B and 4C, two separate set of questions related to the same stem are presented. This process involves a set of three questions based on the same stem, which is then repeated, resulting in one set of questions in each of 4B and another set of questions in 4C.

Note: The early components, Components 1, 2 and 3, can be seen as bringing together the pre-requisite information that will place the student in the best possible position to be successful in Component 4. Component 4 begins with 4A.

4A Reading and Understanding the Stem

4A involves understanding the language of the stem. The purposes here are for the teacher:

- to model fluent reading of the stem (first);
- to identify any unfamiliar language the student (possibly addressed in Component 3);
- to read the passage or describe the figure; etc
- to hear and experience fluency in reading the stem.

Other activities here could include students:

- reading to each other;
- reading silently to themselves; and
- exploring the meaning of the vocabulary.

4B Solving the First Set of Questions

4B involves students answering questions associated with the stem. The students will recognize that they have a stem (previously met in **4A**) and that this is followed by a small set of questions. Students find their own way to a response for each question in the set. The students write down responses or attempts at each question. It is important that every student in the class is expected to have a response. To achieve this desired result, it is important for teachers to ensure all students start on time at the same time.

When the students are finished, or sufficient time has been allocated, students provide answers to the questions and the teacher marks the questions. Discussion takes place about:

- the quality of the answers;
- the implications of errors; and
- what this information tells the class about the content.

The time allocated for 4B provides teachers with an opportunity to observe the quality and levels of student response, which they can build on as a base of what the student knows.

Note: It is important that students start the questions promptly. This involves student self-regulation concerning focus and attitude to work, and may need to be consistently encouraged or reinforced by the teacher.

Teachers can seek out different responses or approaches or thinking exhibited. Errors made by students should be *acknowledged and valued* for their contribution to the class discussion and student learning. Those who achieve correct answers on different questions should also be acknowledged. **Note:** The questions are usually arranged in increasing difficulty from basic to more challenging.

4C Solving the Second Set of Questions

4C uses the same Stem as **4B** and repeats the same process as **4B** but offers students a second (different) batch of questions, again in order of increasing difficulty. When all questions are completed, as was the case in **4B**, students provide answers to all questions, i.e., the students write down responses to, or attempts at, each question. When they are finished, the questions are marked (either using teacher or student answers) and discussion takes place about the quality of correct answers and the implications of errors and what this tells the class about the content.

Note: 4C offers a new start for students regardless of how they performed in **4B**. It allows all students to see **4C** as a new starting point and the class focus for all students should now be around the content and answers in **4C**.

For teachers this approach serves two purposes. *First,* it is a practical way to ensure all students have experiences and are able to contribute perspectives with all questions asked. *Second,* the teacher will have the opportunity to practice further problem-solving questions where different sets of questions can be used with a familiar Stem. This approach is efficient as students obtain more problem-solving practice on the same underlying content.

Reducing cognitive load (working memory demands) is important in writing a stem. Stems in the lessons are designed to facilitate students reading and interpretation. This is achieved by restricting materials to several sentences and a few paragraphs in length, with no more than one diagram for each item. The teacher could have students read the stems together or individually to assist the development of their fluency with the language used.

In Component 4 students are expected to provide answers using:

- factual knowledge
- application of skills and procedures (fluency)
- understanding
- communicating skills
- reasoning and justification.

Clear feedback to students is very important. Teachers should assist students at a level that they can understand in addressing issues, misconceptions or errors that have arisen.

Lesson Component 5

Component 5 offers a student-focused summary of the lesson intention. **Students** reflect on their progress, achievement, or partial achievement of goals (lesson intention) and their performance and understandings. It takes up comments from Component 2 about teacher expectations. Here teachers can confirm student progress. Honesty is needed, as positive as circumstances permit, including the long-term impact of student effort and persistence.

Component 5 has a high metacognitive aspect for students – thinking about their own thinking – which can be further enhanced by teacher modelling.

Part C: Commentary on Lesson Components in All Lessons

Lesson Component 1 (Lesson Short Review)

Component 1 offers teachers the chance to:

- settle the class quickly,
- review or preview previously encountered information,
- address previous content in the form of a few targeted questions that are **relevant to the current lesson,**
- note what students already know,
- elicit answers from the class to reinforce the important content needed for the lesson, and
- address briefly issues that may arise.

Overall, Component 1 acts as a partial advance organizer for the brain. When done carefully, this replay of previous information directed at what is to come in the lesson helps future decision making and memory recall.

Reminding students of *relevant* information at the beginning of a lesson, prior to encountering the main lesson learning focus can enhance the brain's ability to:

- access information to be used in problems/questions/information to come;
- prepare, and have some oversight, for the direction of future learning; and
- further consolidate ideas in long-term memory.

Purpose of Component 1 set questions

The questions set for the Short Review section of a lesson are designed to remind students of knowledge, skills and understanding developed when first studying the topic area addressed, and that is relevant to the activities to be undertaken in the lesson.

Further assistance to students

The teacher could provide some further assistance to students by simply providing additional information in giving answers to the set questions. For example, when providing the answer 'pie chart' to a set of question, the teacher could remind students that a pie chart is also referred to as a 'sector graph'.

Other issues that may have arisen in student answers

The teacher should inform the students that any issues that may arise in their answers and not addressed in Component 1 will be noted at the end of the lesson.

Lesson Component 2 (Lesson Purpose/Intention)

This component offers teachers a chance to acquaint students with the purpose and/or intention of the lesson. The explanation should be directed to the comprehension level of the students and the language used should be familiar to students. It is valuable if students see a link here with their prior knowledge or experience, especially if the teacher can connect it to the responses and levels of understanding evident from students in Component 1.

The information presented to the class can include ideas personal to the students that could facilitate student engagement in the lesson such as:

- the provision of a relevant context
- asking a question that sounds interesting, and/or
- addressing an aspect that has intrinsic interest to the class.

In addition, this component is an appropriate time to address what students might expect/aim to achieve, i.e., their lesson goal(s). Teachers should clarify, in straightforward language, the learning intention for the students as well as what success will look like. (Note**:** The degree of success or partial success of student learning intention should occur as part of Component 5.)

Overall, this component is about activating mental 'structures' or ideas already relevant to the student that will help them ground their new learning experiences and assist them to make sense of any new information by incorporating it into their already established neural networks.

Design considerations in statements of purpose/intention of the lessons

The teacher needs to be aware of the importance of promoting student engagement and enthusiasm, as well as stating things in a way that can be understood by as many students in the class as possible. The statements provided for the teacher to use as the purpose/intention of a lesson are designed to do this, while not providing the students with excessive and unnecessary detail which could overwhelm and disengage them at this early point in the lesson.

Accuracy of purpose/intention statements

The teacher should always make accurate statements, while being conscious of not 'losing' students by being unnecessarily detailed and technical.

Lesson Component 3 (Lesson Language Practice)

Component 3 concerns language use – speaking, hearing, listening and comprehension. The focus is on words or phrases that are seldom used, or that the student has forgotten or has had difficulty with in the past.

These words or phrases have been identified by considering the whole lesson and identifying those words/phrases that have the potential to cause difficulties for the students through speech, or interpretation, or understanding. Typically, the language identified is restricted to about 6 words/phrases so that there is enough time to use a variety of approaches in the practice.

Deliberate practice of unfamiliar or unknown textual or symbolic language can help students reduce cognitive load (reduce working memory) enabling them to allocate greater resources to solving a problem, comprehending a passage, answering a question, explaining a concept; or describing some event or story.

Overall, Component 3 can help achieve language familiarity by saying the word/phrase, being able to spell it, or use it in a specific context. This may also involve helping students to understand or unpack a visual text, diagram or graph, e.g., for a graph, the teacher may need to point out such things such as the graph heading, the axes, units, data points or trend lines.

Lesson Component 4 (Lesson Activity)

Addressing the 'key idea' for the lesson is the focus of Component 4. In the case of the Learning Camp Activity, Component 4 is about students applying known content to solve non-routine problems. This requires students to interpret/understand the meaning of the stem of the problem correctly and then answer a few questions of differing degrees of complexity related to the stem. The stem holds the needed information that will be the basis for the questions. Following the stem is a small number of questions that can be answered by utilizing, the students' background content knowledge, skills and understanding, together with the information in the stem.

From an assessment perspective, the lessons are about helping students consolidate their understanding at three broad levels of difficulty, e.g., the early questions are at an elementary level allowing the students to get started, then next level is directed at the majority of students and usually requires a number of steps to reach a conclusion, and finally the third question attempts to offer all students the opportunity to be challenged and experience enhancements of their learning through seeing how ideas are connected.

(Note: The level of difficulty of the questions should not stop any students from being given the opportunity to experience, with support, questions at higher levels, including the more challenging questions, and to hear about, and be involved in, discussions about the answers. Most students should be able to make some progress and be able to be acknowledged for that. The point of question levels is to at least have students experience these more demanding questions and their answers as the start of the process for their learning journey. It is also designed to offer teachers a more realistic view of potential expectations of students in their class.

Overall, Component 4 has three aspects, 4A, 4B, and 4C. Here in 4A the students are first presented with the stem (stimulus or passage/text or diagram or …) and are given the time/chance to understand the stem of the problem. Then in 4B and 4C two separate set of questions related to the same stem are asked. There are approximately three questions related to the stem in both 4B and 4C. This process of a set of three questions based on the same stem is then repeated, resulting in one set of questions in each of 4B and another in 4C.

(Note: The early components, Components 1, 2 and 3, can be seen as bringing together the required pre-requisite information that will place the student in the best possible position to be successful in Component 4.

To begin Component 4, the initial focus is on Component 4A.

Component 4A Reading and Understanding the stem

4A involves understanding the language of the stem. The purpose here is for:

- the teacher to model fluent reading of the stem (first)
- students to read the passage or describe the figure, etc.
- any unfamiliar language (possibly addressed in Component 3) to be identified, and
- students to hear and experience fluency in reading the stem.

Other activities here could include students:

- reading to each other
- reading silently to themselves, as well as
- exploring the meaning of the vocabulary.

Component 4B Solving the First Set of Questions

Students are asked to address the questions associated with the stem (4A). The students will recognize that they have a stem (previously met in **4A**) and that this is followed by a small set of questions. Teachers have students read the stem and then find their own way to respond to each set of questions. The students write down responses or attempts at each question. It is important that every student in the class is expected to have a response. An implication here for teachers is the importance of all students starting on time at the same time.

When the students are finished, or sufficient time has been allocated, students provide answers to the questions and the teacher marks the questions. Discussion takes place about:

- the quality of the correct answers
- the implications of errors, and
- what this information tells the class about the content.

This time, the teachers are provided with an excellent opportunity to observe the quality and levels of student responses that they can build on as a basis for what the student knows.

(**Note:** It is important that students start with the questions promptly. This involves student self-regulation concerning focus and attitude to work and may need to be consistently encouraged by the teacher.)

The teachers can seek out different responses or approaches, or thinking exhibited. Errors made by students should be **acknowledged and appreciated** for their contribution to the class discussion and student learning. Those who achieve correct answers on different questions should be acknowledged. (**Note:** The 3 or 4 questions are usually arranged in increasing difficulty from basic to more challenging.)

Component 4C Solving the Second Set of Questions

Using the same Stem from **4A**, **4C** repeats the same process as **4B** by asking a second batch of questions, again in order of increasing difficulty, similar to **4B**. When all questions are completed, as was the case in **4B**, students provide answers to all questions, i.e., the students write down responses or attempts at each question. When they are finished, the teacher marks the questions and discussion takes place about the quality of correct answers and the implications of errors and what this tells the class about the content.

(**Note: 4C** offers a new start for students regardless of how they performed in **4B**. It allows a refresh for student brain processing as a new starting point. It also allows the class to become centered around a new activity.)

For teachers this approach serves two purposes. **First,** it is a practical way to bring all students back together to proceed as a group. This way the issues discussed can be considered by every student at the same time. **Second,** the teacher will understand and practice activities where different sets of questions can usually be used with a single Stem. This approach is efficient as students obtain more problem-solving practice on the specific content.

Facilitating students' reading and interpretation of the stems of items (Part 4A)

To help facilitate students' reading and interpretation of the stem, they are designed to be less than six sentences and three paragraphs in length, with no more than one diagram for each item. The teacher could have students read the stems together or individually to assist the development of their fluency with the language used.

What students employ in answering the questions in Component 4

In answering the questions, students provide their answers using:

- factual knowledge
- application of skills and procedures (fluency)
- understanding
- communicating skills
- reasoning and justification.

Importance of clear feedback to students

In giving the answers and marking the questions, the teacher needs to provide clear feedback to students to assist them in addressing any issues that have arisen in undertaking and answering the questions. (Guidance to teachers regarding appropriate feedback is provided in resources).

Lesson Component 5

Component 5 is designed to offer a student-focused wrap-up to the main objectives of the lesson. The focus for Component 5 is on the whole lesson. In particular, the focus is about helping students reflect on their progress, achievement, or partial achievement of goals (lesson intention) and their performance and understanding during the lesson. It picks up comments from Component 2 about teacher expectations. There is the chance here to confirm student progress during the lesson. Teachers need to be honest and as positive as circumstances permit, including the long-term impact of effort and persistence.

Component 5 has a high metacognitive aspect for students – thinking about their own thinking – which can be further enhanced by teacher modelling. A teacher may use a diagram or picture to facilitate a discussion about Component 4 as a catalyst to stimulate student discussion and reflection.

Part D: Syllabus Codes, Matters for Students to Observe, and Worked Answers for the Individual Lessons

Information provided in Part C includes matters that are important for students to observe, in the 15 main lessons and three consolidation lessons. This information, together with the 'Syllabus Codes' from the **K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016** are listed as well as worked answers to the Component 1 and Component 4 questions. This is provided for assistance, as needed, in working through the respective lessons.

Lesson 1: Factors of Trinomials

Syllabus Codes: M8AL-1a-b-1

K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016

Quarter: Grade 8 – First Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of factors and polynomials**.**

Performance Standard

The learner can formulate real-life problems involving factors and polynomials.

Most Essential Learning Competency

Factors completely different types of polynomials

Key Idea

Solve problems using factors and polynomials.

Matters for Students to Observe

Students need to:

- know well how to manipulate algebraic expressions;
- know the highest common factor of an algebraic expression;
- be confident to factorize an expression completely;
- be sure to follow the order of operations indicated by the brackets;
- be careful with + and – signs when they are expanding brackets;
- know well how to interpret a written statement into an algebraic expression;
- be able to recognize immediately what operations are needed to solve the problem; and
- be able to recognize immediately that the answer they have come to is a reasonable answer

Worked Answers to Component 1 and Component 4 Questions

Component 1

(Full answers are already provided in Lesson Booklet)

Component 4

Part 4B Item 1

Let x be Alan's present age.

Alan's age 6 years ago = x - 6.

Three times Alan's age 6 years ago is 3(x - 6).

Twice his present age = 2x.

If three times Alan's age 6 years ago is subtracted from twice his present age, the result would be equal to his present age.

$2x - 3(x - 6) = x$

$2x - 3x + 18 = x$

$-x + 18 = x$

$18 = 2x$

$x = 9$A

Alan's present age is 9 years old.

Students may have difficulty in interpreting the question. Make sure that the students can understand the information in the question and can translate that information into a mathematical expression.

.

Part 4C Item (Optional)

Let x be the number.
If 18 is taken away from 8 times a number and the result is 30
Then, we have
$8x - 18 = 30$
Add 18 to both sides.
$8x = 48$
Divide both sides by 8.
$x = 6$
The number is 6.

As in 4B, the intent is to encourage students to be able to translate the information in a question into a mathematical expression.

Lesson 2: Problems with Polynomials

Syllabus Codes: M8AL-Ia-b-2

Quarter: Grade 8 – First Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of polynomials, rational algebraic expressions, linear equations.

Performance Standard

The learner can formulate real-life problems involving factors of polynomials, rational algebraic expressions, linear equations.

Most Essential Learning Competencies

Solves problems involving factors of polynomials.

Key Idea

Solve problems involving factors and polynomials.

Matters for Students to Observe

Students need to:

- be readily aware of how to use the four operations with algebraic expressions;
- know how to simplify an algebraic expression;
- know how to interpret a word problem to form an algebraic expression;
- know the difference between area and perimeter; and
- be readily aware of the reasonableness of their answers

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. $15x - 36$

It is important here that students recognize the continuity of the '-' sign in the expanded answer.

Q2. $15a(a + 2)$

Similarly, it is important here that students recognize the continuity of the '+' sign in the expanded answer.

Q3. $-15t + 4$

Students are required to perform two operations: expansion and collection of like terms. This may cause students to struggle, and it may be a sign that further practice will be necessary.

Q4. $(x + 4)(x + 5)$

This is a straightforward factorization. If students are not confident using their factorization techniques, then some further practice will be required.

Q5. $x^2 - 6x + 9$

Component 4

Part 4B Item 1

Q1. $60x^2 + 62x + 10$

Students will need to be aware of the formula for area and make the connection to the area of the TV using multiplication of algebraic expressions.

Q2. $32x + 14$

This will require students to simplify an algebraic expression to its simplest terms. They may need further practice in this operation.

Q3. 6630 sq.cm

Q4. 334 cm

Part 4C Item 2

Q1. $area$ *=1820 sq. cm*

If students are finding difficulty with substitution into algebraic expressions it may be necessary for some further revision and practice.

Q2. $perimeter = 174\ cm$

Q3. (Optional) *The area is less than half the area of the first TV but the perimeter is half the perimeter of the first TV.*

This question may be challenging for some students and may require some discussion in small groups where their conclusions have to be justified by some calculations.

Lesson 3: Simplifies Rational Algebraic Expressions

Syllabus Code: M8AL-Ic-1

Quarter: Grade 8 – First Quarter

Content Section: Numbers and Number Sense

Content Standard

The learner demonstrates understanding of rational algebraic expressions.

Performance Standard

The learner is able to illustrate how to simplify rational algebraic expressions.

Most Essential Learning Competency

Solves problems involving rational algebraic expressions

Key Idea

Uses knowledge of algebra and fractions to simplify rational algebraic expressions.

Matters for Students to Observe

Students need to:

- have clear knowledge that multiplication of fractions involves, where possible, cancelling out factors that are common in numerators and denominators, whereas addition and subtraction of fractions requires the use of common denominators;
- know that manipulating rational algebraic fractions uses the same mathematical manipulation as fractions with numbers; and
- be aware of index laws to be able to simplify algebraic fractions that contain exponents.

Worked Answers to Component 1 and Component 4 Questions

Component 1

This component will identify where students have difficulty in their cancellation techniques. It may require some further questions or examples for them the be confident in their cancellation strategies.

Q1. $\frac{3}{4}$

Q2. x

Q3. $\frac{5}{6}$

Q4. $\frac{2}{3m}$

Component 4

Part 4B Item 1

Peter's Answers:

Q1. $\frac{a}{2}$

Q2. $\frac{3b}{5}$

Q3. $\frac{x}{2}$

Q4. $\frac{7}{5}$

Q5. 2xy

John's Answers:

Q1. $\frac{7a}{5}$

Q2. $\frac{5x}{7}$

Q3. $\frac{3xy}{2}$

Q4. 14m

Q5. $\frac{21y}{4a}$

Part 4C Item 2 (Optional)

These questions involve the students looking for numbers as well as algebraic terms to cancel down:

Q1. $\frac{2}{b}$

Q2. $\frac{4x^2y^2}{5}$

Q3. $\frac{3}{8x^3yz^3}$

It may be useful for students who are confident with their cancelling techniques to create their own questions to swap with a partner and then complete the problems.

Lesson 4: Solves Problems with Rational Expressions

Syllabus Code: M8AL-Id-2

Quarter: Grade 8 – Second Quarter

Content Section: Patterns and Algebra

Content Standard

The learners demonstrate understanding of key concepts of factors of polynomials and rational algebraic expressions.

Performance Standard

The learner is able to formulate real-life problems involving factors of polynomials, rational algebraic expressions.

Most Essential Learning Competency

Performs operations on rational algebraic expressions.

Key Idea

Use algebraic skills to solve problems.

Matters for Students to Observe

Students need to:

- have confidence in the manipulation of numerical fractions and this can be extended into the manipulation of algebraic fractions;
- be able to find common factors to simplify algebraic fractions;
- be able to interpret the language in a question to determine which mathematical procedures are required; and
- Check the reasonableness of the answer to a problem.

Worked Answers to Component 1 and Component 4 Questions

Q1. For $\frac{6x}{12xy}$*, there is a common factor of 6 and x in the numerator and the denominator. When cancelling has taken place, the answer is* $\frac{1}{2y}$.

Q2. $\frac{x}{3} + \frac{2x}{5}$ *should be treated like a numerical fraction addition problem, where students find a common denominator and then calculate the answer. The common denominator is 15. So, the fraction becomes*

$\frac{5\times x+3\times 2x}{15}$.

Simplify the numerator $\frac{5x+6x}{15} = \frac{11x}{15}$.

Q3. $\frac{3x-6}{5} - \frac{2x+1}{3}$ should also be treated similarly to a numerical fraction. The common denominator is 15.

$\frac{3(3x-6)-5(2x+1)}{15}$*. The numerator is expanded and like terms collected.*

$\frac{9x-18-10x-5}{15}$*. Care must be taken when multiplying with a negative sign.*

After collecting like terms in the numerator, the answer is $\frac{-x-23}{15}$.

Q4. $\frac{3}{2x} - \frac{y+1}{3xy}$ *is a more complex question as there are algebraic expressions in the denominator. Use the cross-multiplication technique: multiply the 2 algebraic denominators to get a common denominator and then cross- multiply the numerators.*

$$\frac{3 \times 3xy - 2x(y+1)}{2x \times 3xy} = \frac{9xy - 2xy - 2x}{6x^2y}$$

$$= \frac{7xy-2x}{6x^2y}$$

$$= \frac{x(7y-2)}{6x^2y}$$ *Many students may finish their simplification at this point and miss the common factor of x in the numerator.*

$$= \frac{7y-2}{6xy}$$

Component 4
Part 4B Item 1
Q1.
$\frac{y+1}{2} + \frac{y-1}{3}$ *can be simplified by using a common denominator of 6.*

$\frac{3(y+1)+2(y-1)}{6} = \frac{3y+3+2y-2}{6}$

$= \frac{5y+1}{6}$

Q2. To calculate the number of people, let the equation be equal 51 and then solve for y.

$$\frac{5y+1}{6} = 51$$

$$5y + 1 = 51 \times 6$$

$$5y + 1 = 306$$

$$5y = 305$$

$$y = 61$$

Therefore, Joe will require 61 boxes of carrots.

Q3. Let y=19 and substitute into the expression.

$\frac{5\times19+1}{6} = \frac{96}{6}$

=16

There were 16 people at the restaurant.

Part 4C Item 2

Q1.

$\frac{2x+1}{5} + \frac{x-1}{2}$ Using a common denominator of 10, the expression can be simplified to:

$$\frac{2(2x+1)+5(x-1)}{10} = \frac{4x+2+5x-5}{10}$$

$$= \frac{9x-3}{10}$$

Q2. The expression is made equal to 60.

$\frac{9x-3}{10} = 60$ *Solve for* x.

$9x - 3 = 60 \times 10$

$9x = 600 + 3$

$9x = 603$

$x = 67$

Therefore, Joe will need 67 boxes.

Q3. (Optional)

Substitute $x = 40$ into the expression to find the number of people.

$$\frac{9x-3}{10}$$

$$= \frac{9 \times 40 - 3}{10}$$

$$= \frac{357}{10}$$

$$= 35\frac{7}{10}$$

The number of people is $35\frac{7}{10}$, therefore only 35 people can have potato soup. Students need to understand the reason of their answers.

Lesson 5: Coordinate Geometry

Syllabus Codes: M8AL-Ie-1

Quarter: Grade 8 – Second Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of co-ordinate geometry and systems of linear equations in two variables.

Performance Standard

The learner is able to formulate real-life problems involving linear equations.

Most Essential Learning Competency

Illustrates the rectangular coordinate system and its uses

Key Idea

Understand the Number Plane and Coordinate Geometry

Matters for Students to Observe

Students need to:

- know the terms 'co-ordinate', 'ordered pair', 'x-axis', 'y-axis';
- be able to substitute x-values into an equation to find the y-value;
- plot points on a number plane;
- know how to find the gradient of a line;
- know how to find the x and y intercepts; and
- know how to find the midpoint of an interval.

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. a) Gradient = $\frac{7-1}{-1-2}$ *Student need to be confident the find the difference in the y-values and the difference in the x-values*

$$=\frac{2}{-3}$$

b) Students should be aware of the formula to calculate the distance between 2 points. They must be careful of the minus signs in the formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_{1)}{}^2}$$

$$d = \sqrt{(2 - -1)^2 + (5 - 7)^2}$$

$$= \sqrt{(3^2 + (-2)^2)}$$

$$=\sqrt{(9 + 4)}$$

$$= \sqrt{13}$$

c) Midpoint = $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ *It is important that students know the formula and insert values correctly*

Midpoint = $(\frac{2+-1}{2}, \frac{5+7}{2})$

Midpoint = $(\frac{1}{2}, \frac{12}{2})$

Midpoint = $(\frac{1}{2}, 6)$

Q2. Each value for x is substituted into $y = 3x + 2$ *to find the value for y*

x	*-2*	*-1*	*0*	*1*	*2*	*3*	*4*
y	*-4*	*-1*	*2*	*5*	*8*	*11*	*14*

a) $(\frac{-2}{3}, 0)$ *Substitute y=0 into* $y = 3x + 2$ *to find the x-value*

b) *(0,2) Similarly, substitute x=0 into* $y = 3x + 2$ *to find the y-value*

c) *Gradient =* $\frac{Difference\ in\ the\ y-values}{Difference\ in\ the\ x-values}$*. Choose any two points from the table and substitute them into the formula.*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{8 - 5}{2 - 1}$$
$$m = \frac{3}{1}$$

gradient=3

d) *Equation of the line is* $y = 3x + 2$ *where 3 is the gradient or slope.*

Component 4
Part 4B Item 1

Q1. C is the dependent variable and h is the independent variable.

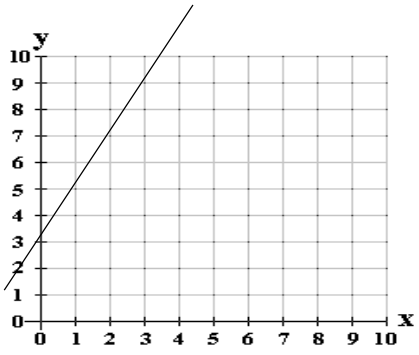
Q2.

x	0	1	2	3	4	5
y	3	5	7	9	11	13

Q3. Continue the pattern to get 17

Q4. 16 hours

Q5. See graph



Q6. Yes, as the graph is a straight line.

Q7. **(Optional)** *The graph is only in the first quadrant because you cannot have negative time or negative number of pieces of chocolate.*

Part 4C Item 2

Q1. Substitute the values to show that the point lies on the line.

Substitute $x = 1$ *and* $y = 3$ *to confirm that the point lies on the line.*

3=(2×1)+1

=3. Therefore, (2,1) lies on the line.

Q2. m=2 as the equation of a line is $y = mx + b$*. Therefore, the number in front of x is 2.*

Q3. (0,1) as the 'b' part of the equation $y = mx + b$ *is the y-intercept.*

Q4. There are many correct answers, but they must have $y = 2x$ *+ or – a number.*

Q5. There are many answers, but the equation must be of the form $y = mx + 1$.

Q6. Substitute the values into the equation.

$3 = -2 \times 1 + 5$ *which is true. So (1,3) lies on the line* $y = -2x + 5$.

Q7. It is the point of intersection of the two lines on the graph. Students should be able to draw a number plane and sketch the two graphs to show the point of intersection accurately.

Lesson 6 Consolidation: Factorizing Trinomials, Problems with Polynomials, Simplifying Ration al Algebraic Expressions, Solving Problems with Rational Expressions and Coordinate Geometry

Syllabus Codes: M8AL-1a-b-1, M8Al -1a-b-2mM8AL-1c-1, M8AK-1d-2,8AL-1e-1

Quarter: Grade 8 – First Quarter

Content Section: Patterns and Algebra

Content Standards

The learner demonstrates understanding of key concepts of factorization**,** simplifying rational expressions, solving problems, and using co-ordinate geometry.

Performance Standard

The learner is able to formulate challenging situations involving problems with factorization, simplification of rational expressions, and solving problems with co-ordinate geometry.

Most Essential Learning Competencies

The learner solves problems involving sets with the use of Venn Diagrams.

The learner represents real-life situations and solves problems involving real numbers.

The learner solves problems involving conversion of units of measurement.

Key Ideas

Solve problems by simplifying algebraic expressions.

Solve problems involving co-ordinate geometry.

Matters for Students to Observe

(See above for notes for Lessons 1–5 and relevant to this lesson.)

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. Simple factorizing $5(2a + 1)$

Factorizing a trinomial

Q2. $(x + 5)(x + 2)$

Q3. $(3m - 1)(2m + 5)$

Q4. Recognizing m *as a factor* $4m(3m - 4n)$

Q5. Factorizing a trinomial with negatives $(y + 4)(y - 5)$

Component 4

Part 4B Item 1

These questions are challenging but reinforce the need to be confident to manipulate algebraic expressions so that problems can be solved.

Q1. $\frac{3ab}{18a^2b^3}$ *Take out the common factor of* $3, a \text{ and } b$ *from the numerator and the denominator*

$$\frac{1}{6ab^2}$$

Q2. $\frac{25-x^2}{x+5}$ *The numerator is the difference of 2 squares.*

$=\frac{(5-x)(5+x)}{x+5}$. *There is now a common factor of (x+5)*

$= (5 - x)$

Q3. $\frac{25x^2 12y^3}{15x^3 16y}$ *Take out common factors of 5,* x^2*, 4 and y*

$= \frac{5\times 3y^2}{3x\times 4}$

$=\frac{15y^2}{12x}$ *Now take out a common factor of 3 and cancel down*

$=\frac{5y^2}{4x}$

Part 4C Item 2

Q1. Transform the word of the problem into an equation.
Let the number be x *.*

$x + x^2 = 20$
$x^2 + x - 20 = 0$

Factorizing the quadratic.

$(x + 5)(x - 4) = 0$

$x = 4$ *or* -5

Q2. The equation that was formed was a quadratic equation and therefore there is more than one answer.

Q1. $m = 3$

Q2. $y = -5$

Q3. There are multiple answers here. The teacher should check that the line that the student has drawn has a gradient of 3.

Lesson 7: Applications of Coordinate Geometry

Syllabus Codes: M8AL-If-3, M8AL-Ig-1

Syllabus: K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016

Quarter: Grade 8 – Second Quarter

Content Section: Patterns and Algebra

Content Standard: The learner is able to find the slope of a line given two points, determine the equation and able to draw the graph

Performance Standard: The learner should be able to formulate real-life problems involving linear equations and coordinate geometry to solve problems

Most Essential Learning Competency

- Describes the graph of a linear equation in terms of its intercepts and slope.

- Finds the equation of a line given:
  (a) two points;
  (b) the slope and a point; and
  (c) the slope and its intercepts.

Key Idea

Applies knowledge of co-ordinate geometry.to solve real life problems.

Matters for Students to Observe

Students need to:

- develop a good understanding of how to translate expressions in words into mathematical expressions;
- have a good understanding of what it means to graph a line; and
- know the importance of reading information carefully to identify the correct values to substitute into the algebraic equations.

Worked Answers to Component 1 and Component 4 Questions

Component 1

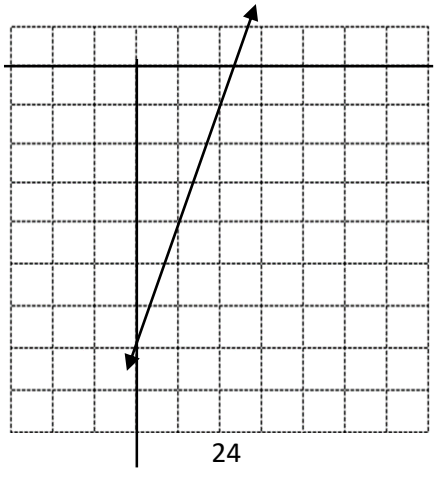
Q1. $m = 3$

Q2. $y = -7$

Q3. $3x - y - 7 = 0$

Q4. There will be many answers, but a correct answer has to have a 3 in front of the x

Q5.



Q6. They have the same gradient.

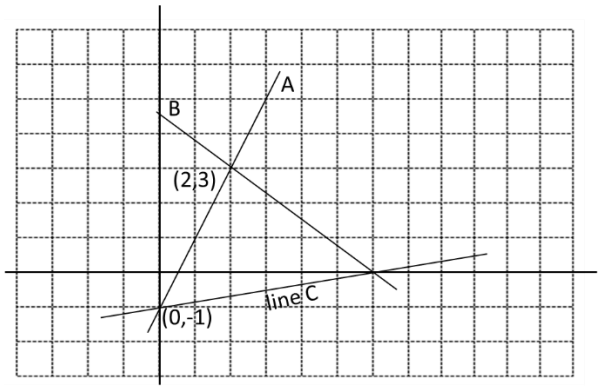
Component 4

Q1. A: $2x - y + 1 = 0$

B: $3x + 4y - 18 = 0$

C: $x - 6y - 6 = 0$

Q2.



Q3. (Optional) Scalene triangle

Component 4c

Q1. (0,2) and (2,2) or (0,-2) and (2,-2) Students should realize there is more than one answer depending on which direction they choose to position their square.

Q2. (1,√3) or (1,-√3). *Similarly, as in question 1, there is more than one possible answer.*

Q3. (-1,-1)

Lesson 8: Solving Problems using Coordinate Geometry

Syllabus Codes: M8AL-Ie-5, M8AL-If-1, M8AL-If-2

Syllabus: K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016

Quarter: Grade 8 – Second Quarter

Content Section: Patterns and Algebra

Content Standard: The learner applies knowledge of coordinate geometry.

Performance Standard: The learner should be able to formulate real-life problems involving linear equations and co-ordinate geometry to solve problems.

Most Essential Learning Competency

Graphs a system of linear equations in two variables.

Key Idea

Solves problems using co-ordinate geometry.

Matters for Students to Observe

Students need to:

- develop a good understanding of how to translate expressions in words into mathematical expressions;
- have a good understanding of what it means to graph a line; and
- know the importance of reading information carefully to identify the correct values to substitute into the algebraic equations.

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. From the equation, m=7, y-intercept is -5.

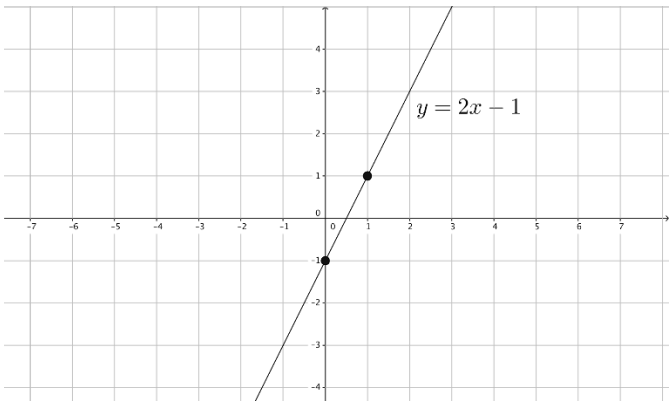
Q2. $m = \frac{9-5}{-1-3}$

$$m = \frac{4}{-4}$$

$$m = -1$$

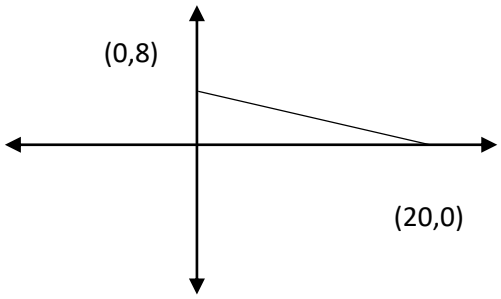
Q3. $y = \frac{-2}{3}x + 2$. *This is an algebraic manipulation that some students may have some difficulty.*

Q4.



Component 4

Q1.



Q2. The y-intercept is $y = 8$

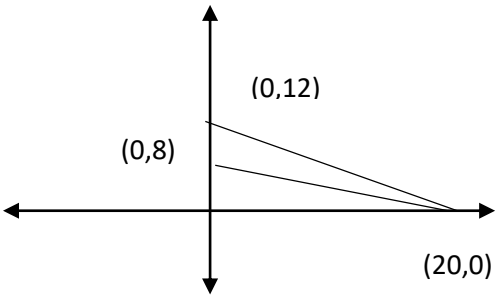
Q3. The x-intercept is $x = 20$

Q4 Gradient of the ramp is $\frac{8-0}{0-20} = \frac{2}{-5}$

Q5. $y = \frac{-2}{5}x + 8$ *or* $2x - 5y + 40 = 0$

Part 4C Item 2

Q1.



Q2. $m = \frac{12-0}{0-20}$

$m = \frac{12}{-20}$

$m = -\frac{3}{5}$

Q3. $y = \frac{-3}{5}x + 12$ or $5y - 3x + 60 = 0$

Q4. **(Optional)** *Joshua would have to move the horizontal section of the ramp by 30 meters to be longer.*

Lesson 9: Simultaneous Equations

Syllabus Code: M8AL-1g-2, M8AL-1h-1, M8AL-1h-2, M8AL 1i-j-1

Quarter: Grade 8– Second Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of systems of linear equations and inequalities in two variables and linear functions.

Performance Standard

The learner is able to solve problems accurately using a variety of strategies.

Most Essential Learning Competency

Solves problems involving linear equations in two variables.

Key Idea

Solves problems using a systematic representation of a pair of linear equations.

Matters for Students to Observe

Students need to:

- have a good understanding of:
  - how to translate expressions in words into mathematical expressions.
  - Drawing lines on a number plane from a given linear equation; and
- have a good understanding of what it means to 'set up and solve an equation'.

Worked Answers to Component 1 and Component 4 Questions

Component 1

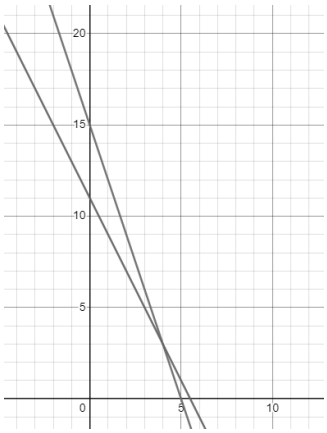
Sample Answers:

1. $2x + y = 11$….A
   $3x + y = 15$….B
   B-A
   $3x - 2x + y - y = 15 - 11$
   $x = 4$
   Substitute $x = 4$ *into* $2x + y = 11$

   $2 \times 4 + y = 11$
   $y=3$

Solution is $x = 4$ *and* $y = 3$

1.



2. From the graph, the lines intersect at the point determined by the algebra. (4,3)

Component 4

Part 4B Item 1

Let s be the number of skirts and d be the number of dresses.
From the information:
$6s + 8d = 6700$ and
$12s + 10d = 9500$

Solving simultaneously:
$6s + 8d = 6700$….A
$12s + 10d = 9500$…B
$A \times 2$ is $12s + 16d = 13400$…C
$C - B\ 6d = 3900$
$d = 650$
Insert $d = 650$ into B
$12s = 3000$
$s = 250$

A dress cost ₱650 and a skirt costs ₱250.

Part 4C Item 2

$2x + 5y = 9$….A
$2x + 3y = 7$….B
A-B

$2y = 2$

$y = 1$

Substitute $y = 1$ *into B*

$2x + 3 = 7$

$2x = 4$

$x = 2$

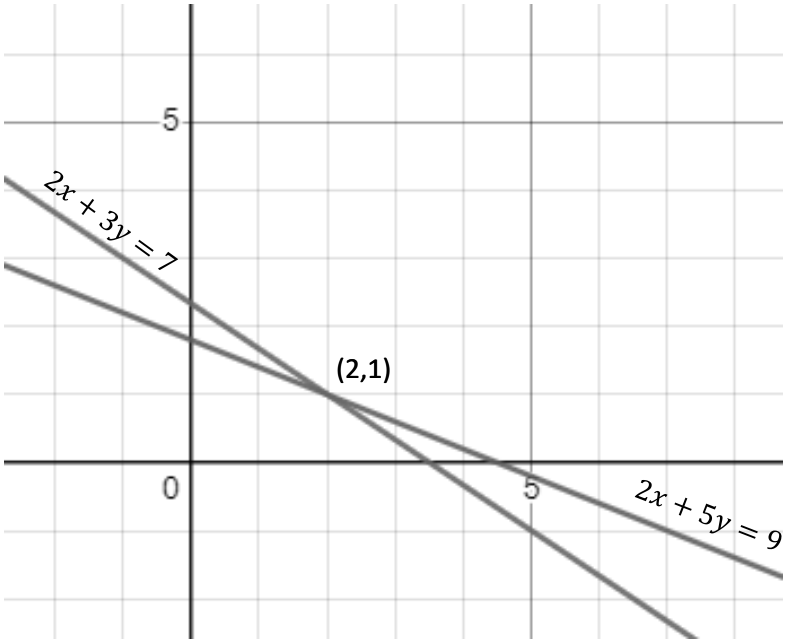
Substitute into A

$4 + 5y = 9$

$5y = 5$

$y = 1$

Solution is $x = 2, y = 1$


5
0
5
$2x + 3y = 7$
(2,1)
$2x + 5y = 9$

Lesson 10: Inequalities

Syllabus Codes: M8AL-IIa-1, M8AL-IIa-3, M8AL-IIa-4

Quarter: Grade 8 –Second Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of inequalities.

Performance Standard

The learner is able to use linear inequalities and can solve real life problems.

Most Essential Learning Competencies

Solves problems involving systems of linear inequalities in two variables.

Key Idea

To understand where and why mathematical inequalities are used.

Matters for Students to Observe

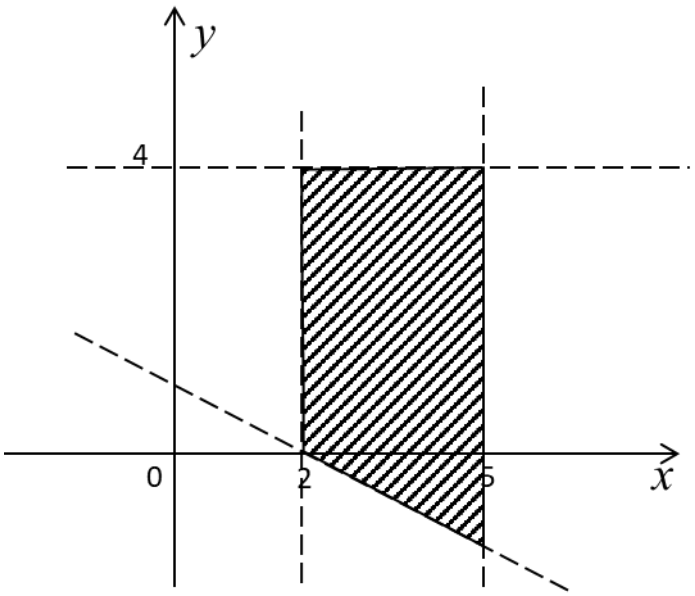
Students need to:

- have a good understanding of how to write inequalities;
- know what the < and > signs mean; and
- be able to translate the words of a problem into a mathematical expression.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1.



Component 4

Part 4B Item 1

1. $700 + 550 = 1250$

The mass of the truck is 1250kg. The bridge can handle 25 000kg. The truck can carry $25000 - 1250 = 24750$kg.

2. $24750 \div 1365 = 18.13.$

Thus, the tuck can carry 18 cars as you cannot have a fraction of a car.

3. (Optional) Mass of 1 silo = $150 + 500 \times 2.5$

= 1400 kg

Mass of 20 silos = 20×1400

= 28000 kg > 23750kg

This is too heavy. He can only carry 16 full silos (23750 ÷ 1400 = 16.96).

Part 4C Item 1

1. In this example, we will state the systems of inequalities that satisfy the conditions for a carpenter who wants to purchase two types of nails.

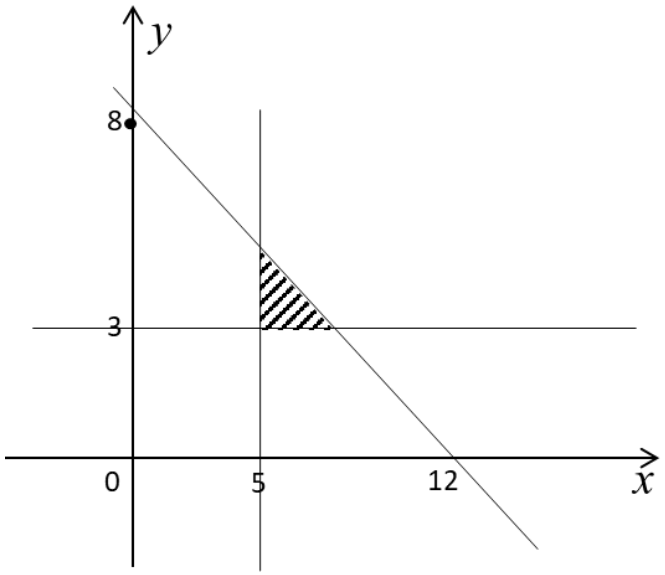
Since x and y are the amounts of nails (in kilograms) from the first and second type, respectively, and the carpenter needs at least 5 kg of the first type and 3 kg of the second, we have the condition $x \geq 5$ and $y \geq 3$.

As the first type costs \$6 per kilogram and the second type costs \$9 per kilogram, the total price for each type would be $6x$ and $9y$, respectively.

The sum of these must be less than \$75, and thus we have $6x + 9y < 75$.

To summarize, the system of inequalities for each condition for the given situation is $x \geq 5$, $y \geq 3$, $6x + 9y < 75$.

2. And 3.



Lesson 11: Relations and Functions

Syllabus Code: M8AL-IIc-1, M8AL-IIc-2

Quarter: Grade 8 – Second Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates understanding of key concepts of linear functions.

Performance Standard

The learner is able to communicate domain and range of a function and can solve real life problems.

Most Essential Learning Competency

The learner is able to communicate domain and range of a function and can solve real life problems.

Key Idea

To determine the difference between a relation and a function.

Matters for Students to Observe

Students need to:

- have a good understanding of domain and range;
- have a good understanding of substituting numbers into functions; and
- have a good understanding of function notation.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $-4 \leq x \leq 4$*……domain*
   $4 \leq y \leq 4$*……..range*
2. *a)* $f(x) = 3x + 8$ *when x=5*
   $f(5) = 3 \times 5 + 8$

   $f(5) = 23$
   b) $f(x) = 2x^2 + 5x - 9$
   $f(-3) = 2(-3)^2 - 15 - 9$
   $= 18 - 15 - 9$
   $= 3 - 9$
   $= -6$
   c) $f(x) = \frac{3x-1}{4x+5}$
   $f(6) = \frac{3 \times 6 - 1}{4 \times 6 + 5}$

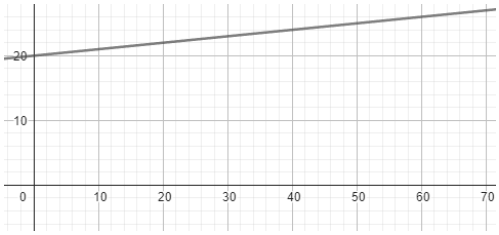
   $f(6) = \frac{18 - 1}{24 + 5}$

   $f(6) = \frac{17}{29}$

3. *a) function*
   b) function
   c) function
   d)relation

Component 4

Part 4B Item 1

1. $P = 20 + 0.1t$
2. $20 \leq P \leq 27, t \geq 0$
3. $range\ is\ the\ same\ as\ question\ 2, domain\ is\ 0 \leq t \leq 70.$
4. 

Check the graphs of the students to ensure that they can draw the graph of the function accurately.

Part 4C Item 2

1. The independent variable is time.
2. The dependent variable is the amount of water in the tank.
3. $V = 2000 - 20h$
4. Domain is $0 \leq h \leq 100$
   Range is $0 \leq V \leq 2000$

5. **(Optional)** The graph only needs to be a sketch and not necessarily developed from a table of values.

Lesson 12 Consolidation: Applications of Coordinate Geometry, Solving Problems using Coordinate Geometry, Simultaneous Equations, Inequalities, Relations and Functions

Syllabus Code: M8Al-1f-3, M8AL-1g-1, M8AL-1e-5, M8AL-1g-1, M8AL-1f-1, M8AL-1f-2,M8AL-1g-2, M8AL-1h-1,M8AL-1i-j-1, M8AL-11a-1, M8AL-11a-3, M8AL-11a-4,M8AL-11c-1, M8AL-11c-2

Quarter: Grade 8 – Third Quarter

Content Section: Patterns and Algebra

Content Standard

The learner demonstrates key concepts of linear inequalities in two variables, systems of linear inequalities in two variables and linear functions.

Performance Standard

The learners able to formulate and solve accurately real-life problems involving linear inequalities in two variables, systems of linear inequalities in two variables, and linear functions.

Most Essential Learning Competency

Refer to this section in lessons 7-11

Key Ideas

- Applies knowledge of coordinate geometry to solve real life problems.
- Solves problems using a systematics representation of a pair of linear equations.
- Understand where and why mathematical inequalities are used.
- Determine the difference between a relation and a function.

Matters for Students to Observe

Students need to:

- be confident using algebraic techniques to simplify algebraic expressions;
- solve linear equations;
- translate word problems into mathematical expressions; and
- decide what processes they need to undertake to solve a problem.

Worked Answers to Component 1 and Component 4 Questions

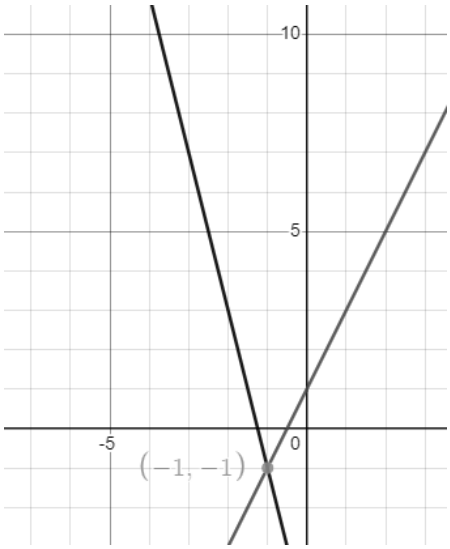
Component 1

Q1. $m = 3$

Q2. $y = -2$

Q3. $3x - y - 2 = 0$

2.



Q1. $(-1,-1)$

Q2. $x = -\frac{1}{2}$

Q3. $y = 2x + 1$ *and* $y = -4x - 5$

Equate both sides.

$2x + 1 = -4x - 5$

$6x = -6$

$x = -1$

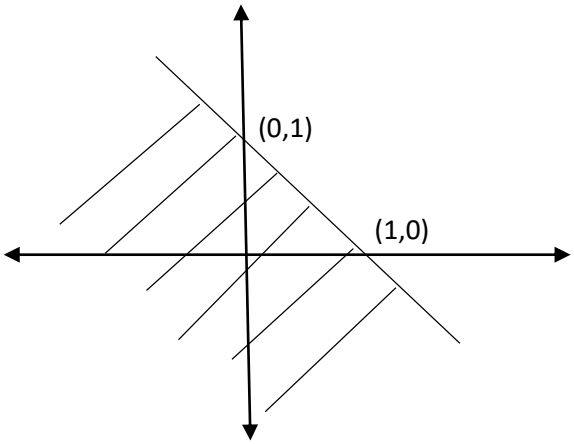
Substitute x=-1 into the first equation.

$y = 2 \times -1 + 1$

$y = -2 + 1$

$y = -1$

3.



4. $x^2 + y^2 = 1$is a relation because for every x-value there is more than one y-value. Or if a vertical line test was drawn on the graph of $x^2 + y^2 = 1$, then the vertical line would cross at 2 points.

Component 4

Part 4B Item 1

1. Let x be the number of tops and y be the number of shirts.

The equations are:

$2x + 2y = 18$ and $3x + 2y = 22$

Solve the equations simultaneously.

$2x + 2y = 18$ .....a

$3x + 2y = 22$.....b

b-a

$x = 4$

Substitute into a

$8 + 2y = 18$

$2y = 10$

$y = 5$

The solution is each top costs ₱4 and each shirt cost ₱5.

Part 4C Item 2

$J + M = 26.30$

$J = M - 10$

$M - 10 + M = 26.30$

$2M - 10 = 26.30$

$2M = 36.30$

$M = 18.15$

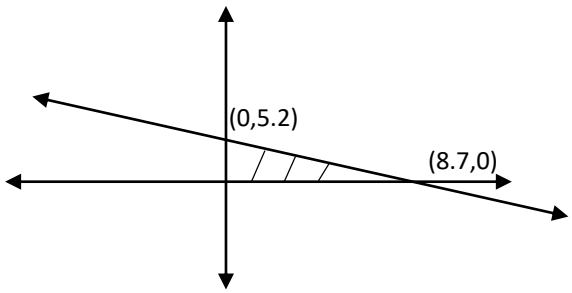
$J = 18.15 - 10$

$J = 8.15$

Jason has ₱8.15 and Melanie has ₱18.15.

Q2.

$$3x + 5y \leq 26$$



The line cannot go below the x-axis or to the left of y-axis because there can be no negative values for the tokens.

Lesson 13: 'If-Then' Statements

Syllabus Codes: M8GE-IIf-1, M8GE-IIf-2, M8GE-IIg-1

Quarter: Grade 8 – Third Quarter

Content Section: Geometry

Content Standard

The learner demonstrates understanding of key concepts of logic and reasoning.

Performance Standard

The learner is able to communicate mathematical thinking with coherence and clarity in formulating and analyzing arguments.

Most Essential Learning Competency

The learner determines the relationship between hypotheses and conclusions.

Key Ideas

Investigates If-the statements.

Matters for Students to Observe

Students need to:

- interpret the meaning of written statements;
- understand the inverse of a statement; and
- draw conclusions from statements made.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1.
   a) *Hypothesis →Today is Tuesday.*
      Conclusion → Tomorrow is Wednesday.
   b) *Hypothesis→ A truck weighs 200kg.*
      Conclusion → The truck weighs 200 000g.

2. a) *If an animal monkey, then it loves bananas.*
   c) *If points lie on the same line, then they are collinear.*
3. *a) It is false because other animals can swim, e.g., dog.*
   b) It is false because two angles can be equal and not necessarily equal 90°.

Component 4
Part 4B Item 1
Sample answers:

i) If Shirley hurries, then she will arrive on time (However, she still may be late even if she hurries).
ii) If it is raining, then the picnic will be cancelled (This may depend on the nature of the rain. It may just be a light shower and then the picnic could go ahead).
iii) If lines have many points in common, then they intersect (This is not a true statement).
iv) *If angles are along a straight line, then they are supplementary.*

Part 4C Item 2

Assume x is a positive integer and it is a solution to the inequality $x + 3 > 4$.

x must solve both inequalities of $x > 0$ and $x > 12$.

If $x > 12, then\ it\ is\ also\ greater\ than\ 0$.

From $x + 3 > 4$, $x > 1$. However, only values greater than 12 solve the second part of the statement. Thus, the statement is partially true.

Lesson 14: Congruent Triangles

Syllabus Codes: M8GE-IIId-1

Quarter: Grade 8 – Fourth Quarter

Content Section: Geometry

Content Standard

The learner demonstrates understanding of key concepts of logic and reasoning.

Performance Standard

The learner is able to communicate mathematical thinking with coherence and clarity in formulating and analyzing arguments.

Most Essential Learning Competency

Proves two triangles are congruent.

Key Idea

Investigates properties of congruent triangles.

Matters for Students to Observe

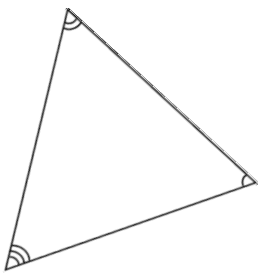
Students need to:

- be aware of the different types of triangles;
- understand the tests that are used to identify congruent triangles; and
- be able to apply the tests to triangles to determine congruence.

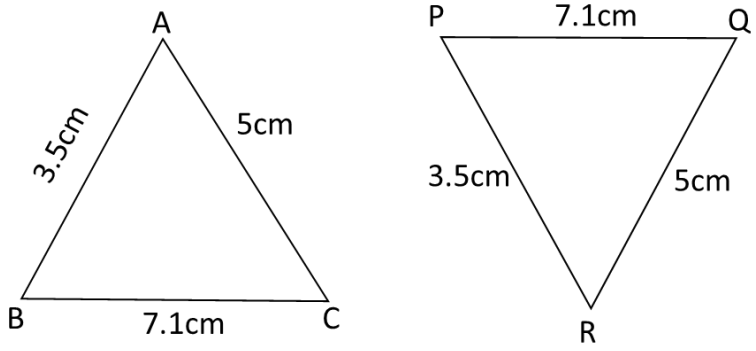
Worked Answers to Component 1 and Component 4 Questions

Component 1

Sample answers:

1. $\angle A = \angle F, \angle B = \angle E, \angle C = \angle G$
2. AB=EF, AC=FG, BC=EG
3. 
4. SSS, SAS, AAS, RHS
5. Because triangles with the same angles can be of any size. They are similar triangles.

Component 4
Part 4B Item 1



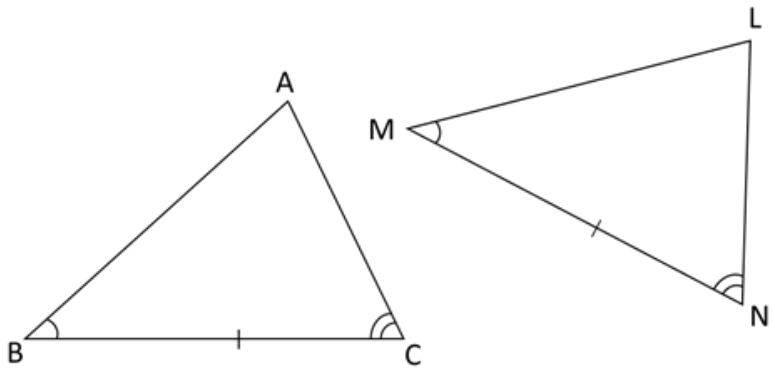
Sample answers:

AB=PR

BC=PQ

AC=QR

The three sides are equal, so the triangles are congruent. SSS Test



Answer: AAS Test

Component 4C

(Optional) AC=CE (C is the midpoint of AE)……..S

BC=CD (C is the midpoint of BD)……..S

$\angle ACB = \angle DCE$ (Vertically opposite angles)……..A

$\therefore \ \triangle ABC \equiv \triangle DEC$ from the SAS Test

Lesson 15: Proof

Syllabus Codes: M8GE-1vd-1

Quarter: Grade 8 – Fourth Quarter

Content Section: Geometry

Content Standard

The learner demonstrates understanding of key concepts of inequalities in a triangle, and parallel and perpendicular lines.

Performance Standard

The learner is able to communicate mathematical thinking with coherence and clarity in formulating, investigating, analyzing, and solving real life problems parallelism and perpendicularity of lines using appropriate and accurate representations.

Most Essential Learning Competencies

Proves properties of parallel lines cut by a transversal.

Key Idea

Proves properties of parallel lines cut by a transversal.

Matters for Students to Observe

Students need to:

- know the difference between supplementary and complementary angles;
- know angle properties of vertically opposite angles, angles on a straight line, co-interior angles, alternate angles and corresponding angles; and
- know the properties of different triangles.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. Collinear
2. 180°
3. 90°
4. 360°
5. 180°
6. 360°
   Opposite sides are parallel; Opposite sides are equal length; diagonally opposite angles are equal.

Component 4

Part 4B Item 1

1. $\angle E$= 108° because $\angle G$ and $\angle E$ lie on a straight line and are supplementary (add up to 180°)
2. $\angle C$, $\angle F$, $\angle B$
3. $\angle E + \angle C = 180°$ $\angle C$=$\angle G$ and because they are supplementary then $\angle E + \angle C = 180°$
4. (Optional) $\angle E$, $\angle D$, $\angle H$

Part 4C Item 2

If the lines are parallel then $(3x - 15)° + 150° = 180°$, because the angles would be supplementary.

$3x + 135 = 180$

$3x = 45$

$x = 15$

Lesson 16: Probability

Syllabus Code: M8GE-1vi-j-1

Quarter: Grade 8 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of probability.

Performance Standard

The learner is able to solve practical problems involving probability of simple events.

Most Essential Learning Competency

The learner solves problems involving probabilities of simple events.

Key Idea

Applies knowledge of probability to solve problems.

Matters for Students to Observe

Students need to:

- develop a good understanding of how and why experiments are undertaken;
- understand the difference between experimental and theoretical probability;
- have a good understanding of the expected outcomes oof events; and
- Understand the difference between 'with replacement' and without replacement'.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $\frac{2}{5}$
2. *The answer will depend on how many girls and boys there are in the class.*
3. $\frac{3}{7}$

Component 4

Part 4B Item 1

Q1. 4 of each color

Q2. Many diagrams could be used here as long as there is an indication of 4 reds, 4 whites and 4 greens.

Q3. $\frac{4}{12}$ *this should be simplified to* $\frac{1}{3}$

Q4. Once he takes out 1 green ball, there are only 3 green balls left and only 11 balls left in the bag. $\frac{3}{11}$

Q5.(Optional) *4*

Part 4C Item 2

There will be a variety of different answers. Ask students to check with each other to see if their answers are correct. Similarly, the teacher could ask students to share with the class to see if their answers are correct, especially note the different answers for 'with replacement' and 'without replacement'.

Lesson 17: Illustrates experimental and theoretical probability

Syllabus Code: M8GE-!vi-j-1

Quarter: Grade 7 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates understanding of key concepts of probability.

Performance Standard

The learner is able to solve practical problems involving probability of simple events.

Most Essential Learning Competency

The learner solves problems involving probabilities of simple events.

Key Idea

Demonstrates the difference between experimental and theoretical probability**.**

Matters for Students to Observe

Students need to:

- develop a good understanding of how and why experiments are undertaken;
- understand the difference between experimental and theoretical probability;
- have a good understanding of the expected outcomes oof events; and
- understand the difference between 'with replacement' and without replacement'.

Worked Answers to Component 1 and Component 4 Questions

Component 1

a) $\frac{5}{16}$

b) The expected outcome would be $\frac{1}{6}$ *which is much lower than* $\frac{5}{16}$ *in the experiment.*

c) The table provides information about experimental probability.

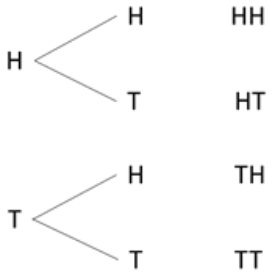
d) We know this because the result is different from what is expected, and the result are different for each roll of the dice.

Component 4

Part 4B Item 1

Q1. $\frac{4}{10}$*, but we would simplify the fraction to* $\frac{2}{5}$*. Encourage students to reduce the fraction to its lowest terms.*

Q2.



Q3. $\frac{1}{4}$

Q4. The experimental and theoretical probabilities are different. The number of times he rolled 2 heads is greater than what would be expected.

Q5. **(Optional)** *The more times she rolls the dice, the closer the experimental probability and theoretical probability will be.*

Part 4C Item 1

Q1. 1H,1T, 2H,2T, 3H,3T, 4H,4T, 5H,5T, 6H,6T

Q2. $\frac{1}{12}$

Q3. 24

Q4. $\frac{2}{24}$ which would be simplified to $\frac{1}{12}$.

Lesson 18: Congruence, If-then statements, Parallel Lines and Probability

Syllabus Code: *M8GE-IIIf-1, M8GE-IIIg-1, M8GE-IIIh-1, M8GE-Ivd-1, M8GE-Ivi-j-1*

Quarter: Grade 8 – Fourth Quarter

Content Section: Statistics and Probability

Content Standard

The learner demonstrates key concepts of geometric proof, congruent triangles, parallel lines and probability.

Performance Standard

The learner is able to formulate and solve problems involving geometric proof, investigate if-then statements, investigate a transversal crossing parallel line, apply congruent triangles to prove geometric axioms, and solve problems with probability.

Most Essential Learning Competency

Refer to the information in lessons 13-17

Key Idea

Review the learning on If then statements, congruence, parallel lines, and probability.

Matters for Students to Observe

Refer to the matters in lessons from 13-17

Worked Answers to Component 1 and Component 4 Questions

Component 1

1.

a) If you are a student, then you like geometry.

b) If the triangle is equilateral, then the angles are all equal.

2. SSS, SAS, AAS, and RHS

3. Alternate

4. 5 even numbers <=10 so $\frac{5}{10}$ *or* $\frac{1}{2}$

Component 4

Part 4B Item 1

1. The angle on the same horizontal line as 60°is 120°as the angles are supplementary.
   Thus, $4x + 20 = 120$.

   $4x = 100$

   $x = 25$

   1. AC=DE (given)
      Angle ACB=Angle DFE (from diagram)
      Angle CAB= Angle FDE (from diagram)
      Therefore ΔABC≡ΔDEF (AAS test)

Part 4C Item 2

1. There are 5 different choices of her first scoop, 4 different choices for her second scoop, and 3 different choices for her third scoop. Therefore, she has 5 X 4 X 3 different choices. So, she has 60 different combinations of ice cream.

2.

a) There are 1000 cases and 20 for less than 4000.

$P=\frac{20}{1000}=\frac{1}{50}$

b) More than 9000 = 325 + 445

=770

$P=\frac{770}{1000}=\frac{77}{100}$

More than 4000 and less than 14000 is 210+325.

210+325=535

$P=\frac{535}{1000}=\frac{107}{200}$

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Delivery (DepEd-BLD)
Contact Numbers: 8637-4366; 8637-4347; 8633-9347

Department of Education - Bureau of Learning Resources (DepEd-BLR)
Contact Numbers: 8634-1072; 8631-6922

Email Address: blr.od@deped.gov.ph

WITH SUPPORT FROM Australian Aid