

# Mathematics

NATIONAL

# **Consolidation Learning Camp**

# **Lesson Plans**



# Consolidation Learning Camp Lesson Plans Booklet

**Mathematics Grade 9** 

Weeks 1 to 3

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Dear Reader,

Every care has been taken to ensure the accuracy of the information provided in this Booklet. Nevertheless, if you identify a mistake, error, or issue, or wish to provide a comment, we would appreciate you informing the **Office of the Director of the Bureau of Learning Delivery** via telephone numbers (02) 8637-4346 and 8637-4347 or by email at <u>bld.od@deped.gov.ph</u>

Thank you for your support.

#### **National Learning Camp Overview**

The Learning Camp offers grade-level review lessons that are directed by the teacher and designed to be highly interactive among:

- (i) students with their teacher and
- (ii) students with their peers.

A strength of the design is the focus on both student and teacher learning. The intentions and expectations of the Learning Camp are for:

- students to consolidate and enhance their thinking in topics covered;
- teachers to enhance their pedagogical practices by focusing on selected skills that include 21<sup>st</sup> century skills; and
- teachers to update, strengthen, and expand their subject knowledge in ways that encourage students to be involved in learning activities at different levels including those considered higher order.

Teachers are afforded resources, time, and opportunity to use deliberate practice to further extend their skills, knowledge, and understanding of teaching and how students learn. As the lessons are designed to apply subject content already encountered by students, they do not contain repetitive and routine questions of a particular subject aspect.

One important purpose of the subject content review by teachers at the beginning of each lesson is to determine learners' subject background knowledge and skills and to enable teachers to build on this. This activity also reminds students of what they have already encountered. This approach is different to 'teaching' students anew as if they have not been taught previously.

Under an integrated banner of 'Science of Learning', research-evidence advances in applying disciplines to education practice such as the learning brain, cognition, teacher and teaching growth, and learning development is adding to our knowledge of teaching and learning. This Project draws on information in this field that has highlighted the need to go beyond what might be considered current practice in the Philippines and incorporate ideas and approaches, including the 21<sup>st</sup> Century skills, to enhance teaching and learning for all.

#### **Review lessons**

The review lessons are based on content related to the students' current grades. All lessons **involve the exploration of ideas, concepts, and content** already encountered by students. The purpose of the lessons is two-fold:

- (i) to establish in students a stronger basis for future learning development (prior to enrolling in a new grade after the summer break) and
- (ii) to enable teachers to strengthen their teaching practice in a research-based professional development program (prior to a new academic year).

The primary focus of the lessons is on students revising and then applying previously taught subject content to support problem-solving and/or comprehension activities focused on applying knowledge, skills, and understanding to real-world problems.

#### **Lesson Overview**

All lessons in each of the three subjects, English, Mathematics, and Science, contain five components. Approximate time for the components is indicated as timing advice to guide the teacher in pacing the lessons. Time management involves moving through components at a pace that is appropriate to the learners while still ensuring that the components are completed in a timely, efficient, and constructive manner. Ultimately, however, the pace will be determined by the learners' needs and strengths.

Nevertheless, there needs to be practical limits on the duration of the components to prevent major disruption of lessons. When times are allocated appropriately, and students become familiar with the approach and teacher expectations, concept development and student skill levels are improved.

Research findings related to student learning quality and 'time' are related to student 'time-on-task'. Timeon-task refers to the time students are actively involved (engaged) in some aspects of the learning process in class. The suggested timing for each component is intended to maximize the time available for student involvement. This will encourage the students to work efficiently, timewise, through the lesson without jeopardizing the importance of such activities as students:

- answering routine and non-routine questions
- responding to verbal questions and explanations
- interpreting and using appropriate terminology
- discussing aspects with their peers
- explaining or justifying their approaches and thinking
- working productively on their own
- listening carefully to their teacher or peers.

Care needs to be exercised in determining what engagement means. Engagement is evident when students are **doing the learning** through answering questions, writing, discussing, and reading.

Establishing on-task time is more problematic when the teacher talks and students passively listen, such as in didactic teaching. With such an approach it is difficult to determine whether students are listening or paying attention. Often in lessons requiring problem-solving or intense reading and comprehension, student activity is more subtle and cerebral as students work.

Critical aspects of the National Learning Camp for the teacher include questions related to learning areas, based around a key idea. The questions are offered at different levels of difficulty involving lower- to higher-order thinking, starting with questions of modest complexity up to those that require more developed reasoning. Students are provided with opportunities to practice solving non-routine questions to help improve their conceptual understanding by applying known content to subject-related problems.

Teacher reflection on the lessons presented can offer important insights to stimulate teachers to enhance their own practice and the learning of their students. This includes:

- new teaching approaches encouraged by lesson components that can contribute in different ways to student learning and lesson success;
- the use of review lessons that help review learnt material and extend student abilities in problemsolving by utilizing known information;
- a focus on student concept and skill acquisition, pedagogical approaches, student errors, time-ontask, deliberate practice, and working memory demands.

#### **Consolidation Camp**

**The Consolidation Camp** offers students the chance to review their subject background knowledge by consolidating previously taught material. The intention is for students to have opportunities to practice applying their knowledge of concepts and ideas through grade-related sets of questions of developing difficulty.

In the case of lessons for students in the Consolidation Camp, the materials including the lesson plans are the same as offered to students attending the Enhancement Camp. There are important reasons for both Camps sharing the same content. However, there are differences between the Camps associated with the teaching focus elated to the breadth and depth of conceptual knowledge of the students.

In all learning, lower-order thinking is a prerequisite for higher-order skill and knowledge development. Many students are disadvantaged in their attempts to move forward in their learning through a lack of practice and conceptual development of needed lower-order skills, knowledge, and understanding. Hence, all students benefit from a stock-take on relevant lower-order skills with previously addressed content. This helps establish a basis upon which student learning should build.

In both the Consolidation and Enhancement Camps important lower-order content skills, knowledge, and understanding are re-visited at the beginning of each lesson. This helps ensure that potential learning obstacles are made visible to the student and the teacher. It also means that errors in understanding or misconceptions are identified. This information is important in helping all students move forward regardless of their achievement levels.

Small sets of questions asked in the lessons are also the same. These questions range from those of modest difficulty to those that require more insight and knowledge. Exploring and answering these question sets has value to all students, albeit in different ways. It enables students to work through a range of ideas on their own before hearing from their peers and teacher concerning the same questions – a very rich learning environment.

As the questions posed are about applying content already encountered to a new problem, students have the opportunity to use or apply their current knowledge, skills, and understanding in a practical way at their level, further developing their conceptualization and understanding of the content.

This Consolidation Camp offers students the opportunity to improve their learning and conceptual development by a stepped approach:

- (i) reminding students of relevant lower-order skills through practice,
- (ii) having students apply and discuss their knowledge in sets of graded questions with an emphasis on earlier questions,
- (iii) expecting students to apply their knowledge leading to more breadth in learning,
- (iv) beginning an initial focused practice on higher-order skill development.

The approach advocated to solve problems or comprehend passages extends student learning beyond simple repetitive exercises sets. For these students the teaching part of the lesson requires teachers reviewing closely student solution attempts through student explanation, discussion, and questioning of fundamental aspects of topics that are typically found in the earlier questions. Teachers should be sensitive to students' self-perceptions here as the students may meet the ideas presented in the lessons, maybe after many failures with these concepts in the past.

Nevertheless, these students should become aware of the more difficult questions as they allow them to consider links or connections between concepts previously taught. There is value in a problem for students to read the question and be able to say what it is about in their own words.

It is important that students in the Intervention Camp become aware of where their learning is at and where it is progressing. Teachers need to be nurturing and supportive of this development and continually look for evidence of success and growth. Teachers also need to encourage students to persist, continue to practice individual aspects, and use any mistakes they make as an opportunity to learn more. These are important features of a successful learning journey.

#### **Lesson Components: Short Overview**

#### Lesson Component 1 (Lesson Short Review)

Component 1 offers teachers the chance to:

- settle the class quickly;
- review or preview previously encountered information;
- address previous content in the form of a few targeted questions that are relevant to the current lesson;
- note what students already know;
- elicit answers from the class to reinforce the important content needed for the lesson; and
- address briefly issues that may arise.

The questions set for the Short Review section of a lesson are designed to remind students of knowledge and skills developed when first studying the topic area, which is relevant to the lesson.

#### Lesson Component 2 (Lesson Purpose/Intention)

This component offers teachers a chance to acquaint students with the purpose/intention of the lesson. It is valuable if students see a link here with their prior knowledge or experience, especially if the teacher can connect it to the responses and levels of student understanding evident in Component 1.

In addition, this component is an appropriate time to address what students might expect/aim to achieve, i.e., their lesson goal(s). Teachers should clarify, in clear language, the learning intention for the students as well as what success will look like. (Note: The degree of success or partial success of student learning in the lesson should occur as part of Component 5.)

#### Lesson Component 3 (Lesson Language Practice)

Component 3 concerns language use – speaking, hearing, listening and comprehending. The focus is on words or phrases that are to be used.

The language practice suggested has been identified by considering the whole lesson and identifying those words/phrases that have the potential to cause difficulties for students through speech, listening, or understanding. Typically, the language identified is restricted to less than 6 words/phrases so that there is enough time to use a variety of approaches in practice within the time available.

#### Lesson Component 4 (Lesson Activity)

Component 4 has three aspects, 4A, 4B, and 4C.

In the case of the Learning Camp activity, Component 4 addresses the key idea for the lesson. It is about students applying known content to solve real-world problems. This requires students to correctly interpret/understand the meaning of the 'stem', a stimulus, (passage/text or diagram or the first part of the problem or story) correctly before answering questions of varying degrees of complexity related to the stem.

Students are first presented with the stem in 4A and are given the time/chance to understand it. Then in 4B and 4C, two separate sets of questions related to the same stem are asked.

#### 4A Reading and Understanding the Stem

4A involves understanding the language of the stem.

The purposes here are for the teacher to model fluent reading of the stem (first) and to identify any unfamiliar language (possibly addressed in Component 3). On the other hand, the students' purposes are to read the passage or describe the figure, etc. and to hear and experience fluency in reading the stem.

#### 4B Solving the First Set of Questions

**4B** involves a set of questions associated with the stem. Students will need to refer to the stem as they prepare to answer the set of questions. Students write down responses or attempts at each question. It is important that every student in the class is expected to have a response for each question. It is expected and acceptable that students would make errors, which provide teachers with important information about students' learning needs. A critical procedural action here for teachers is the importance of all students starting on the same set of questions at the same time.

When the students have finished, or sufficient time has passed, the teacher marks the questions. This can be achieved by students providing answers to the questions and by explaining or justifying their reasons. Time should be allocated to student discussion, explanation, reasoning, etc., about the answers.

#### 4C Solving the Second Set of Questions

**4C** offers a new start for students regardless of how they performed in Component 4B. The structure is similar to Component 4B, i.e., undertaking a new set of questions related to the same stem. The lesson structure allows a refresh for the student's brain processing as 4C represents, for the student, a new starting point. This structure also allows all students in the class to become centered around starting a new activity.

This approach serves two purposes for teachers. First, it enables the teacher to bring all students back together to proceed as a group with issues able to be directed to and considered by every student at the same time. Second, it offers the teacher to practice offering their students extended problems where different sets of questions can be used with a single Stem, enabling an efficient way to incorporate more problem-solving or comprehension practice on specific content into a lesson.

#### Lesson Component 5 (Lesson Conclusion)

**Component 5** has a high metacognitive aspect for students – students thinking about their own thinking – which can be further enhanced by teacher modeling. Component 5 is designed to offer a student-focused overview of the main intentions of the lesson. In particular, the focus is about helping students reflect on their progress and achievements (or partial achievements) of the lesson intention as well as their understanding development during the lesson.

It builds on comments from Component 2 about teacher expectations. There is the chance here to confirm student progress during the lesson. A teacher may use a diagram, picture or some aspect of the lesson as a catalyst to stimulate student discussion and reflection.

**NOTE:** A fuller description of the Components and features of the lessons is provided in the Learning Camp – Notes to Teachers Booklet. It is recommended that these notes are read and discussed by teachers as they provide a further basis to understanding the structure of lessons and the pedagogy recommended.

#### **Quadratic Equations**

#### Key Idea

Solving quadratic equations

#### **Component 1: Short Review**

Time: 10 minutes

- Ask students to answer the following questions on their worksheet.
- 1. If  $x^2 = 16$ , is x = 4 the only answer? Why?
  - 2. Solve  $6y^2 = 0$
  - 3. Solve x(x + 3) = 0
  - 4. Solve (x 1)(x 2) = 0
  - 5. Solve  $4x^2 20 = 0$
  - 6. Solve  $2x^2 + 16 = 34$
  - 7. Solve  $h^2 = -9$
- Ask students to volunteer answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. (This may come from one of the students or from the sample answers below).

#### Sample answers:

- 1. There is another answer x = -4 because  $(-4)^2 = 16$
- 2. y = 0
- 3. x = -3 or 0
- 4. x = 1 or 2
- 5.  $x = \pm \sqrt{5}$
- 6.  $x = \pm 3$
- 7. There is no solution. The answer is imaginary since the square root of -9 is 3*i*.

#### Component 2: Lesson Purpose/Intention

#### Time: 3 minutes

This lesson is about factorizing to solve quadratic equations. This is an important process that helps us understand more about equations which will eventually assist in solving problems that rely on the solution to a quadratic equation.

**Component 3: Lesson Language Practice** Time: 5 minutes

Product - the result of two or more numbers when multiplied together
Factor - numbers or terms that divide exactly into another number or expression.
Squares – a number that is multiplied by itself
square root - is a number that when multiplied by itself gives the number required

*Expand* – to expand brackets, or multiply out, involves multiplying every term inside the bracket by the term on the outside and then collecting like terms with the aim of removing the set of brackets.
 *Factorizing* – find what can be multiplied to get a given expression.

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence.

#### **Component 4: Lesson Activity**

Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.

Factorize (if necessary) and then solve the following equations:

 Ask the students if there are any words that they are not familiar with (or suggest and give descriptions of any words that may be problematic.

#### **Component 4B**

- i) Factorize if necessary
- ii) Find the answer
- 1.  $x^{2} + 4x = 0$ 2.  $3x^{2} - 15 = 0$ 3. (x + 3)(x - 7) = 04. (3x + 5)(x - 8) = 05. -3(x + 4)(x - 1) = 06.  $y^{2} - 4y - 21 = 0$ 7.  $m^{2} - m - 30 = 0$ 8.  $x^{2} + 5x = 6$ 9.  $2n^{2} + 8n - 64 = 0$ 10.  $(x + 2)^{2} = 5$

Sample answers:

```
1. x = 0, -4

2. x = \pm \sqrt{5}

3. x = -3, 7

4. x = \frac{-5}{3}, 8

5. x = -4, 1

6. y = 7, -3

7. m = 6, -5

8. x = -6, 1

9. n = -8, 4

10. x = -2 + \sqrt{5}
```

#### Component 4C

Read out the following questions

Sandra is twice as old as Russell. Eight years ago, the product of their ages was 10. Find their current ages.

Q1. Write expressions to show the information in the problem.

Q2. Form a trinomial using your expressions from Q1.

Q3. Factorize to solve the problem.

Q4. Why is one of the solutions not useable?

Q5. What are the ages of Sandra and Russell?

• Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.

#### Sample answers:

Russell is 9 years old and Sandra is 18 years old.

#### **Component 5: Lesson Conclusion** Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

**REMINDER:** Collect student worksheets to review and analyze student's learning.

#### The Discriminant

#### Key Idea

Using the discriminant to determine the nature of the roots of a quadratic equation

**Component 1: Short Review** Time: 5 minutes Ask students to answer the following questions. For the quadratic  $ax^2 + bx + c = 0$ , the formula to solve for x is  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 1. Use the formula to solve: Q1.  $x^2 + 2x - 5 = 0$ Q2.  $x^2 - 3x - 18 = 0$ Q3.  $x^2 - 9x + 14 = 0$ Q4.  $4x^2 - 9 = 0$ 2. Solve  $7x^2 + 42x - 112 = 0$  by: a) Factoring first and then solve b) Using the formula. c) Which one was easier? Why? 3. Determine the discriminant  $\Delta = b^2 - 4ac$  for Q1.-Q4. Ask students to volunteer answers, giving positive feedback. Read out answers for all students to listen to and write down. (This may come from one of the students or the answers below). Sample answers: 1. *Q1.*  $x = -1 \pm \sqrt{6}$ *Q2.* x = 6 or - 3*Q3.* x = 2 or 7Q4.  $x = \pm \frac{3}{2}$ 2. a)  $7(x^2 + 6x - 16) = 0$ 7(x+8)(x-2) = 0x = -8 or 2b)  $x = \frac{-42 \pm \sqrt{42^2 - 4 \times 7 \times (-112)}}{2 \times 7}$  x = -8 or 2Factorizing is much easier because the numbers are cumbersome in the formula. 3.  $Q1 \Delta = 24$  $Q2.\Delta = 81$ *Q3.*  $\Delta = 25$ *Q4.*  $\Delta = 144$ 

#### Component 2: Lesson Purpose/Intention

Time: 3 minutes

The lesson is about using the discriminant to determine the nature of the roots of quadratic equations. The discriminant allows us to determine if there are no solutions to a quadratic equation, only one solution, two solutions and whether those two solutions are unequal and rational or not.

#### **Component 3: Lesson Language Practice** Time: 5 minutes

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

**Discriminant** - the expression under the square root symbol,  $b^2 - 4ac$  **Rational** - is a number that can be expressed as the fraction of two integers, a numerator p and a non-zero denominator q **Roots** - the values of the unknowns that satisfy the equation **Quadratic** - is an equation that contains at least one squared unknown. **Radical** - The  $\sqrt{}$  symbol that is used to denote square root or nth roots

- What is the difference between the words 'square' and 'square root'?
- Ask the students to practice saying the words in a sentence.

#### **Component 4: Lesson Activity**

Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with and give descriptions of any words that may be problematic.

Q1. Determine the nature of the roots by investigating the discriminant for each of the equations below:

- 1.  $2x^2 + x 7 = 0$
- 2.  $x^2 + 10x + 25 = 0$
- 3.  $2x^2 + 3x + 3 = 0$

#### Component 4B

- Read out the following questions and ask students to answer Q2. Find the values of k so that  $3x^2 6x + k = 0$  has:
  - a) Equal roots
  - b) No real roots
  - c) 2 real and unequal roots

Q3. If  $mx^2 - 5x = -6$  has equal roots, find the value of *m*.

Q4. Use the discriminant to determine the nature of the roots for

a)  $3x^2 + 7x + 2 = 0$ b)  $x^2 - 3x + 11 = 0$ 

- Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.
- Select a good answer for all students to write down. This may come from one of the students or from the following answer.

#### Sample answers:

```
Q1.
```

- 1.  $\Delta > 0$ , roots are real and unequal
- 2.  $\Delta = 0$ , roots are real and equal
- 3.  $\Delta < 0$ , no real roots

#### Q2.

- a)  $\Delta = 0, k=3$ b)  $\Delta < 0, k > 3$ c)  $\Delta > 0, k < 3$ Q3.  $\Delta = 0, m = \frac{25}{24}$ Q4.
- a)  $\Delta = 25, 2 \text{ real rational roots}$ b)  $\Delta < 0, \text{ no real roots}$

#### Component 4C (O)

Read out the following questions and ask students to answer

Q1. Use the discriminant to prove  $rx^2 + 2rx + r = 0$  has equal roots for any value of r

- Observe students' answers
- Ask the students to volunteer their answers, giving positive feedback.
- Select a good sample answer for all students to write down. This may come from one of the students or from the following sample answer.

#### Sample answers:

Q1.  $\Delta = (2r)^2 - 4(r)(r) = 0$ The roots are real and equal, since  $\Delta = 0$ .

#### **Component 5: Lesson Conclusion**

Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

**REMINDER:** Collect student worksheets to review and analyze student's learning.

#### Solves quadratic inequalities

#### Key Idea

Using knowledge of quadratic inequalities to illustrate solutions graphically

Component 1: Short Review
Time: 5 minutes
<ul> <li>Ask students to answer the following questions.</li> </ul>
Solve
$Q1. x^2 = 64$
Q2. $x^2 > 64$
Q3. $x^2 < 64$
Q4. $x^2 \ge 64$
Q5. $x^2 < -64$
Q6. $2x^2 > 18$
Q7. $x^2 - 2x - 3 < 0$
<ul> <li>Ask students to volunteer answers, giving positive feedback.</li> </ul>
Sample answers:
Q1. $x = \pm 8$
<i>Q2.</i> $x > 8, x < -8$
<i>Q3.</i> $x > -8, x < 8, or - 8 < x < 8$
<i>Q4.</i> $x \ge 8, x \le -8$
Q5. No solution. No real number can be satisfied by the inequality.
Q6.x > 3, x < -3
Q71 < x < 3
Component 2: Lesson Purpose/Intention
Time: 3 minutes

The lesson is about using quadratic inequalities to illustrate solutions to quadratic inequalities both algebraically and graphically. This skill will assist in solving problems with quadratic inequalities.

#### **Component 3: Lesson Language Practice**

Time: 5 minutes

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

**Discriminant** - the expression under the square root symbol,  $b^2 - 4ac$ 

*Rational* - is a number that can be expressed as the fraction of two integers, a numerator p and a non-zero denominator q

Roots - the values of the unknowns that satisfy the equation

*Quadratic-* is an equation that contains at least one squared unknown.

**Radical** - the  $\sqrt{}$  symbol that is used to denote square root or nth roots

**Inequalities** - the mathematical expressions in which both sides are not equal. In inequality, unlike in equations, we compare two values. The equal sign in between is replaced by less than (or less than or equal to), greater than (or greater than or equal to), or not equal to sign.

- Explain the difference between the roots of an equation and the solution of an equation.
- Ask the students to practice saying the words in a sentence.

# Component 4: Lesson Activity

Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with (and give descriptions of any words that may be problematic.

Solve the inequality  $x^2 + 5x \ge -4$ 

#### Component 4B

- Q1. Rearrange the inequality to have 0 on the right-hand side of the inequality sign
- Q2. Factorize the quadratic expression
- Q3. Sketch the parabola
- Q4. Test the values inside and outside the parabola
- Q5. Write the solution
- Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.
- Select an answer for all students to write down. This may come from one of the students or from the following sample answer.



Q2. 2 < x < 8 (Optional)

# Component 5: Lesson Conclusion

*Time:* 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

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- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

#### **REMINDER:** Collect student worksheets to review and analyze student's learning.

#### **Solving Problems with Quadratic Inequalities**

#### Key Idea

Students use their skills with quadratic inequalities to solve problems

**Component 1: Short Review** Time: 5 minutes Ask students to answer the following questions. Solve the following inequalities. *Q1.*  $x^2 + 2x - 15 > 0$ Q2.  $x^2 - 2x - 3 \le 0$ Q3.  $x^2 - 8x + 12 < 0$ Q4.  $x^2 - 5x \ge 0$  $05.9x > 2x^2 - 18$ Ask students to volunteer answers, giving positive feedback. Answers: *Q1.* x < -5, x > 3*Q2.*  $-1 \le x \le 3$ *Q3.* 2 < x < 6*Q4.* x < 0, x > 5Q5.  $x < 6, x > -\frac{3}{2}$  or  $-\frac{3}{2} < x < 6$ **Component 2: Lesson Purpose/Intention** Time: 3 minutes The lesson is about solving quadratic inequalities in order to find the answer to real problems. **Component 3: Lesson Language Practice** Time: 5 minutes Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class. Ask the students to practice saying the words in a sentence. Ask students to volunteer their answers, giving positive feedback and discussing. **Discriminant** - the expression under the square root symbol,  $b^2 - 4ac$ **Rational** - is a number that can be expressed as the fraction of two integers, a numerator p and a nonzero denominator q

**Roots** - the values of the unknowns that satisfy the equation **Quadratic** - is an equation that contains at least one squared unknown **Radical** - the  $\lor$  symbol that is used to denote square root or nth roots **Inequalities** - the mathematical expressions in which both sides are not equal. In inequality, unlike in equations, we compare two values. The equal sign in between is replaced by less than (or less than or equal to), greater than (or greater than or equal to), or not equal to sign.

• Ask the students to practice saying the words in a sentence.

#### **Component 4: Lesson Activity** Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with and give descriptions of any words that may be problematic.

A rocket is launched from ground level directly upward at a rate of 48m/sec. The equation for the rocket's height is  $y = -16x^2 + 48x$ .

#### **Component 4B**

• Read out the following questions and ask students to answer in the space on their worksheet.

Q1. What values of x is the rocket at or above 32m?

Q2 How long is the rocket at or above this height?

- Observe students' answers.
- Select an answer for all students to write down. This may come from one of the students or from the following sample answer.

Sample answers

Q1.  $1 \le x \le 2$ 

Q2.1 second

#### **Component 4C (Optional)**



#### **Graphs of Quadratic Functions**

#### Key Idea

Uses the key features of a parabola to sketch its graph

Component 1: Short Review
Time: 10 minutes
Follow the steps below to graph $f(x) = x^2 - 6x + 8$
a) Decide whether the parabola is concave up or concave down. Give reasons
b) Find the axis of symmetry
c) Find the vertex
d) Find the <i>y</i> -intercept
e) Find the x-intercept
f) Use these points to sketch the graph of the function
<ul> <li>Ask students to volunteer answers, giving positive feedback.</li> </ul>
Sample answers:
a) $a > 0$ , the parabola is concave up
b) The axis of symmetry is x=3
c) The vertex is (3, -1)
d) The y-intercept is y=8
e) The x-intercept/s: $x = 4$ , 2
f) Sketch of the parabola including the above points
Component 2: Lesson Purpose/Intention
Time: 25 minutes
The lesson is about using the number plane to determine the position or location of a parabola on a
number plane. Being able to draw accurate sketches of parabola is a tool that is used to solve
Mathematical problems. Confidence in using a Number Plane to sketch graphs will assist in many other
aspects of Mathematics.
Component 3: Lesson Language Practice

Time: 5 minutes

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence

Coordinates – two numbers which locate a specific point on a coordinate plane.
Variable – is a quantity that can be changed and is not fixed
Dependent – is the variable that depends on the value of some other number
Independent – is the variable that refers to the cause or reason for an outcome
Vertex – is a point at which the parabola makes its sharpest turn
Concave – A parabola is concave upward if the curve opens upward. A parabola is concave downward if the curve opens downward.

**Axis of symmetry** – the line that will divide a parabola into two equal halves which are mirror images of each other

*Intercepts* – the points at which the parabola crosses the x -axis and the y-axis

#### **Component 4: Lesson Activity** Time: 25 minutes

nine. 25 minutes

#### Component 4A

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with and give descriptions of any words that may be problematic.

Graph the function  $f(x) = 3(x-1)^2 + 2$ .

#### Component 4B

- Read out the following questions and ask students to answer
  - 1) Expand the equation
  - 2) Determine whether the parabola faces up or down
  - 3) Find the axis of symmetry
  - 4) Determine the vertex
  - 5) Determine *x* and *y* intercepts (there may not be any *x* intercepts)
  - 6) Draw your sketch
- Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.
- Select an answer for all students to write down. This may come from one of the students or from the following sample answer.

#### Sample answers:

```
Q1 f(x) = 3x^2 - 6x + 5

Q2. Concave up

Q3. axis of symmetry is x = 1

Q4. vertex is (1,2)

Q5. y-intercept is y=5, there is no x-intercept

Q6.
```

0 1 X

#### Component 4C (O)

Read out the following questions and ask students to answer

Graph the parabola

 $f(x) = -2(x-2)^2 + 1$ 

Use the steps from the previous question as a guide

- Observe students' answers.
- Ask the students to volunteer their answers, giving positive feedback.
- Select an answer for all students to write down. This may come from one of the students or from the following sample answer.

X

#### Sample answers:

- Concave down
- Axis of symmetry x = 2
- Vertex (2,1)
- *y*-intercept *y*=-7, *x* -intercept *x*=1.29 and *x*=2.71

### Component 5: Lesson Conclusion

Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

#### **REMINDER:** Collect student worksheets to review and analyze student's learning.

#### **Mathematics Grade 9 Lesson Plan 6 Deliberate Practice**

#### **Review Patterns and Algebra**

#### **Key Ideas**

**Review Patterns and Algebra** 

<b>Component 1. Short Re</b>	view
Solve:	
$01.x^2 - x = 30$	

Q2.  $x^2 + 7x + 10=0$ 

Q3.  $6x^2 + 8x - 5 = 0$ 

Q4.  $16x^2 - 25 < 0$ 

Q5.  $y^2 - y - 20 \ge 0$ 

#### Sample Answers

Q1. x = 6, -5Q2. x = -2, -5Q3.  $x = \frac{-4 \pm \sqrt{46}}{6}$ Q4.  $x > \frac{-5}{4}, x < \frac{5}{4}$ Q5.  $y \ge 5, y \le -4$ 

#### **Component 2**: Lesson Purpose/Intention Time: 5 minutes

The lesson is about ensuring the skills learned during the week will be consolidated in this review lesson

#### **Component 3: Language Practice**

Time: 5 minutes

Select 6 to 8 of the most important words encountered during the week.

product, factor, squares, square root, expand, factorizing, discriminant, rational, roots, quadratic, radical, inequalities, co-ordinate, variable, dependent, independent, vertex, concave, axis of symmetry, intercepts

#### **Component 4: Lesson Activity** Time: 30 minutes

#### **Component 4A**

- Have the students read out the question aloud. Ask the students what the word equation and the equal sign mean.
- Ask the students what features stand out in  $2x^2 18 = 16x$

#### **Component 4B**

For the equation  $2x^2 - 18 = 16x$ :

- 1. Rearrange the equation into the form  $ax^2 + bx + c = 0$
- 2. Use the discriminant to determine the nature of the roots
- 3. Using the equation  $y = 2x^2 16x 18$ , determine the axis of symmetry
- 4. Determine the vertex of the equation in (3)
- 5. What are the *x* and *y* intercepts of the equation in (3)
- 6. Sketch the graph of the equation in (3).

Sample answers:

- 1.  $2x^2 16x 18 = 0$
- 2. The nature of the roots is real since the computed discriminant is greater than 0 which is 400.
- 3.  $\Delta = 400$
- 4. x = 4, y = -50
- 5. *x*-intercepts:

```
x = 9, -1
```

```
y-intercept: y = -18
```

```
6.
```

#### **Component 4C**

Read out the following questions and ask students to answer.
 Here is a rectangle and a right angles triangle.



3*x*-2

All measurements are in cm. The area of the rectangle is greater than the area of the triangle. What are all the possible values of x?

#### Sample answers:

x > 2

### **Component 5: Lesson Conclusion**

Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### Determines the Equation of a quadratic Function

#### Key Idea

Uses the graph of a quadratic function to determine its equation

**Component 1: Short Review** Time: 5 minutes Ask students to answer the following questions. Q1. Complete the square for the expressions below: a).  $x^2 + 4x + 1$ b).  $x^2 - 6x + 16$ c).  $-3x^2 - 6x + 18$ Q2 Solve for x in the following: a).  $(x + 2)^2 = 5$ b).  $3(x-7)^2 = 8$ Q3. Complete the square and then solve for *x*. a).  $x^2 + 6x + 2 = 0$ b).  $x^2 - 10x + 15 = 0$ Ask students to volunteer answers, giving positive feedback. Sample answers: Q1. a).  $(x-2)^2 - 3$  $b)(x-3)^2 + 7$ c). $-3[(x-1)^2+5]$ 02. a). $x = -2 + \sqrt{5}$ ,  $x = -2 - \sqrt{5}$ b).  $x = 7 + \sqrt{\frac{8}{3}} \quad x = 7 - \sqrt{\frac{8}{3}}$ *Q3.* a).  $x = \pm\sqrt{7} - 3$ *b*).*x* =  $+\sqrt{10} + 5$ **Component 2: Lesson Purpose/Intention** 

Time: 5 minutes

The lesson is about using a graph and knowledge of completing the square to find the equation of a parabola.

#### **Component 3: Lesson Language Practice**

Time: 5 minutes

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence.
- Ask students to volunteer their answers, giving positive feedback and discussing.

*Intercept* - is the point where the line crosses the x-axis, called the x-intercept, and the point where the line crosses the y-axis, or the y-intercept.

Vertex - is a point at which the parabola makes its sharpest turn

*Concave* - A parabola is concave upward if the curve opens upward. A parabola is concave downward if the curve opens downward.

**Axis of symmetry** -the line that will divide a parabola into two equal halves which are mirror images of each other

#### **Component 4: Lesson Activity** Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with (or suggest examples such as and give descriptions of any words that may be problematic.

For a quadratic equation of the form  $y = k(x - a)^2 + b$ , the following properties are true:

- The axis of symmetry is x = a
- The vertex is the point (a,b)
- If k > 0, then the parabola is concave up
- If k < 0, then the parabola is concave down

#### **Component 4B**

- Read out the following questions and ask students to answer.
- 1. For the equation  $y = x^2 6x + 11$

Q1. Rewrite the equation in the form  $y = (x - a)^2 + b$  by completing the square

Q2. Use the properties to determine significant points

Q3. Sketch the parabola



#### Component 4C (O)

Read out the following questions and ask students to answer

From the graph below determine the equation of the parabola



- Observe students' answers.
- Ask the students to volunteer their answers, giving positive feedback.

#### Sample answers:

Axis is Symmetry is x = 0, therefore x = a=0Vertex is (0,2), therefore b=2k=1Equation is  $y = x^2 + 2$ 

#### **Component 5: Lesson Conclusion** Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

#### **REMINDER:** Collect student worksheets to review and analyze student's learning.

#### Simplifying Expressions with Rational Components

#### Key Idea

The learner applies knowledge of index laws to simplify algebraic expressions

#### **Component 1: Short Review**

Time: 5 minutes

Ask students to answer the following questions on their worksheet.

Simplify the following expressions

Q1.  $(a^3)^2$ Q2.  $(b^5)(b^7)$ Q3.  $(x^{12}) \div (x^7)$ Q4.  $(2x^{\frac{3}{2}})(x^{\frac{4}{5}})$ 

Q5.  $(x^4)^{\frac{3}{2}}$ 

Q6.  $27^{\frac{2}{3}}$ 

Q7. 2<sup>-3</sup>

 Ask students to volunteer answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. (This may come from one of the students or from the sample answers below).

Sample answers:

Q1.  $a^{6}$ Q2.  $b^{12}$ Q3.  $x^{5}$ Q4.  $2x^{\frac{23}{10}}$ Q5.  $x^{6}$ Q6. 9 Q7. $\frac{1}{8}$ 

# Component 2: Lesson Purpose/Intention

Time: 5 minutes

The lesson is about applying the understanding of index laws to simplify algebraic expressions that include radicals.

#### **Component 3: Lesson Language Practice**

Time: 5 minutes

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence.
- Ask students to volunteer their answers, giving positive feedback and discussing.

*index laws* - the rules for simplifying expressions involving powers to the same base number *surd* - the roots of numbers that cannot be simplified into a whole or rational number

#### **Component 4: Lesson Activity**

Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with and give descriptions of any words that may be problematic.

The lesson is about applying the understanding of index laws to simplify algebraic expressions that include radicals.

#### **Component 4B**

Read out the following questions and ask students to answer in the space on their workbook

Q1. 
$$(8p^4)(4p^8)$$

Q2.  $(7r^6)(8s^5)(9r^4)$ 

Q3.  $(5^5)(a^3)(3)(5^5)(3a^5)$ 

Q4.  $(x^2y^3) \div (xy^2)$ 

Q5.  $(16f^7g^2) \div (4f^3g)$ 

- Q6.  $(21j^8k^3m^3) \div (3k^2m)$
- Q7. (8<sup>9</sup>)<sup>3</sup>
- Q8. (h<sup>4</sup>)<sup>6</sup>
- Q9.  $(c^2 d^3)^5$
- Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.
- Select a good sample answer for all students to write down. This may come from one of the students or from the following sample answer.

#### Sample answers:

Q1.  $32p^{12}$ Q2.  $504r^{10}s^5$ Q3.  $(5^{10})(9a^8)$ Q4. xyQ5.  $4f^4g$ Q6.  $7j^8km^2$ 

```
Q7. 8<sup>27</sup>
Q8. h<sup>24</sup>
Q9. c<sup>10</sup>d<sup>15</sup>
```

#### Component 4C (O)

- Read out the following questions and ask students to answer in the space on their worksheet.
- Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.

These questions are more challenging for your skills of manipulating expressions using index laws.

Q1.  $(27p^6)^{\frac{5}{3}}$ Q2.  $(25b^6)^{\frac{-3}{2}}$ Q3.  $\frac{4x^2}{2x^{\frac{1}{2}}}$ 

 Select a good sample answer for all students to write down. This may come from one of the students or from the following sample answer.

Sample answers:

Q1.  $3^{5}p^{10}$ Q2.  $5^{-3}b^{-9}$ Q3.  $2x^{\frac{3}{2}}$ 

**Component 5: Lesson Conclusion** Time: 5 minutes

In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

**REMINDER:** Collect student worksheets to review and analyze student's learning.

#### **Radical Expressions**

#### Key Idea

Performs Operations on Radical Expressions

onent 1: Short Review
8 mins
nplify:
$\sqrt{98}$
$\sqrt{80}$
$\sqrt{99}$
$3\sqrt{2} + 5\sqrt{2}$
$\sqrt{3} - \sqrt{48}$
$4\sqrt{3}$ + $\sqrt{2}$ - $\sqrt{3}$
$(\sqrt{3})(\sqrt{6})$
$\sqrt{\frac{75}{3}}$

- Teacher provides answers to questions and highlights particular concepts, as well as issues that may have arisen in student answers (issues not addressed to be noted at end of lesson).
- Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.

Sample answers:

- 1.  $7\sqrt{2}$ 2.  $4\sqrt{5}$ 3.  $3\sqrt{11}$ 4.  $8\sqrt{2}$
- 4.  $8\sqrt{2}$ 5.  $-3\sqrt{3}$
- 6.  $3\sqrt{3} + \sqrt{2}$
- 7.  $\sqrt{18} = 3\sqrt{2}$
- 8. <u>+</u>5

### Component 2: Lesson Purpose/Intention

Time: 2 mins

The purpose of this lesson is to simplify radical expressions.

#### **Component 3: Lesson Language Practice**

Time: 5 mins

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

*index laws* - the rules for simplifying expressions involving powers to the same base number *surd* - the roots of numbers that cannot be simplified into a whole or rational number *radical* - is the square root or cubed root etc.

• Ask students what the difference is between the words.

#### **Component 4: Lesson Activity** Time: 25 mins

#### **Component 4A**

Refer students to the main lesson stimulus and orient them to the text

Simplify the following expressions. Some may require rationalizing the denominator.

#### **Component 4B**

• Read out the following questions and ask students to answer in the space on their worksheet.

Q1. 
$$3\sqrt{7} - 2\sqrt{8} + \sqrt{63}$$
  
Q2.  $(3\sqrt{3} - 2\sqrt{2})^2$   
Q3.  $2\sqrt{5} + \sqrt{12} - \sqrt{27}$   
Q4.  $(2 - 2\sqrt{2})^2$   
Q5.  $\sqrt{15} \times \sqrt{5}$   
Q6.  $5\sqrt{5}(2 - 5\sqrt{6})$   
Q7.  $(5 - 4\sqrt{5})(5 + 3\sqrt{5})$   
Q8.  $(-5\sqrt{3} + 2)(-2\sqrt{3} - 4)$   
Q9.  $\frac{\sqrt{5}}{\sqrt{2}}$   
Q10.  $\frac{4}{\sqrt{3}}$ 

 Observe students' answers. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample Answers:

```
Q1. \ 6\sqrt{7} - 4\sqrt{2}
Q2.35 - 12\sqrt{6}
Q3. \ 2\sqrt{5} - \sqrt{3}
Q4. \ 12 - 8\sqrt{2}
Q5. \ 3\sqrt{5}
Q6. \ 10\sqrt{5} - 25\sqrt{30}
Q7. - 35 - 5\sqrt{5}
Q8. \ 22 + 16\sqrt{3}
Q9. \ \frac{\sqrt{10}}{2}
Q10. \ \frac{4\sqrt{3}}{3}
```

#### Component 4C

• Read out the following questions and ask students to answer in the space on their worksheet.

Simplify these radical expressions. These questions may require you to simplify more than once.

Q1. 
$$\frac{\sqrt{6}}{\sqrt{3}}$$
  
Q2.  $\frac{3\sqrt{3}}{5\sqrt{75}}$   
Q3.  $(\sqrt{2} - 4\sqrt{3})(\sqrt{2} + 4\sqrt{3})$   
Q4.  $\frac{\sqrt{8}}{-2\sqrt{24}}$   
Q5.  $\frac{\sqrt{15xy}}{3\sqrt{10xy^3}}$  (Optional)

 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample Answer:

 $Q1. \sqrt{2}$  $Q2.\frac{3}{25}$ Q3. -46 $Q4. \frac{\sqrt{3}}{-6}$  $Q5. \frac{\sqrt{6}}{6y}$ 

#### **Lesson Component 5: Reflection**

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### Parallelograms

#### Key Idea

Uses the properties of parallelograms to solve problems

#### Component 1: Short Review

#### Time: 8 mins

- Review the properties of parallelograms:
  - 1. Opposite sides are parallel and equal in length
  - 2. Diagonally opposite angles are equal
  - 3. Adjacent (consecutive) angles are supplementary
  - 4. The two diagonals bisect each other
- For the parallelogram below find the values of the unknowns and give reasons.



- Teacher provides answers to questions and highlights particular concepts, as well as issues that may have arisen in student answers (issues not addressed to be noted at end of lesson).
- Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.

#### Sample answers:

- a = 12 because opposite sides of a parallelogram are equal
- b = 7 because opposite sides of a parallelogram are equal
- $x^{\circ} = 112^{\circ}$  because diagonally opposite angles are equal
- $y^{\circ} = 68^{\circ}$  because adjacent (consecutive) angles are supplementary

#### **Component 2: Lesson Purpose/Intention** Time: 2 mins

The purpose of this lesson is to use the properties of parallelograms to find unknown length values and unknown angles. This skill will be useful when proving geometric facts and solving problems.

#### **Component 3: Lesson Language Practice**

Time: 5 mins

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

Complementary angles are two angles whose sum is 90 degrees.

Supplementary angles are two angles whose sum is 180 degrees.

Adjacent angles are pair of angles that share a common vertex and a common side that do not overlap. Alternate angles are equal angles formed by two parallel lines crossed by a transversal.

Transversal is a line that intersects two or more lines at different points.

*Vertically opposite angles/vertical angles are* the angles formed on opposite sides of the vertex of intersecting lines.

Ask students the difference between the words 'complementary' and 'supplementary'.

#### Component 4: Lesson Activity

Time: 25 mins

**Component 4A** 

Refer students to the main lesson stimulus and orient them to the text

Find the indicated values on each of the diagrams below. Give reasons for your answers.

#### Component 4B

- Read out the following questions and ask students to answer in the space on their worksheet.
- 1.



 Observe students' answers. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample answers:

- a=10 because the two diagonals bisect each other b=10 because opposite sides of a parallelogram are equal x° = 62° because x° and 62° are alternate angles when a transversal crosses two parallel lines y° = 33° because the two angles are vertically opposite each other and are equal
- 2. a = 20 because opposite sides of a parallelogram are equal
  - b = 14 because opposite sides of a parallelogram are equal
  - $x^{\circ} = 17^{\circ}$  because 4x=68° adjacent angles are supplementary
  - $y^{\circ} = 14^{\circ}$  because  $3y^{\circ}$  and  $42^{\circ}$  are alternate angles

#### Component 4C(O)

• Read out the following questions and ask students to answer in the space on their worksheet.

For the parallelograms below find the indicated values. Q1.



 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample Answer:

- 1.  $x^{\circ} = 31^{\circ}$
- 2. x = 9, y = 10.5
- 3. x = 15, y = 7

#### **Lesson Component 5: Reflection**

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

**NOTE:** Remember to collect student worksheets to review and analyze student's learning.

#### Parallelograms, Trapezoids and Kites

#### Key Idea

Time: 8 mins

**Component 1: Short Review** 

Solve problems involving Parallelograms, Trapezoids and Kites



 Teacher provides answers to questions and highlights particular concepts, as well as issues that may have arisen in student answers (issues not addressed to be noted at end of lesson).

<ul> <li>Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.</li> </ul>
Sample answers:
<ol> <li>Check that what students have written is correct for each shape.</li> <li>a).200m<sup>2</sup></li> <li>b). 30m<sup>2</sup></li> <li>c). 48m<sup>2</sup></li> <li>d). 126m<sup>2</sup></li> </ol>
Component 2: Lesson Purpose/Intention Time: 2 mins
The purpose of this lesson is to have the knowledge and understanding to solve problems that involve parallelograms, trapezoids and kites.
Component 3: Lesson Language Practice
<ul> <li>Time: 5 mins</li> <li>Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.</li> </ul>
Parallelogram - is a simple quadrilateral with two pairs of parallel side
Trapezoid - is a quadrilateral that has at least one pair of parallel sides
<i>Kite</i> - a quadrilateral in which 2 pairs of adjacent sides are equal <i>Perpendicular</i> - two lines are perpendicular if they form a right angle at the point of intersection
<b>Rhombus</b> - a quadrilateral whose 4 sides all have the same length
<ul> <li>Ask students the difference between the words 'kite' and 'trapezoid'.</li> </ul>
Component 4: Lesson Activity
Time: 25 mins
<ul> <li>Refer students to the main lesson stimulus and orient them to the text</li> </ul>
John is tiling the floor of a rectangular room that is 4m wide and 3m long. The tiles are in the shape of a parallelogram.
4cm 5cm

#### Component 4B

- Read out the following questions and ask students to answer in the space on their worksheet.
- 1. a). Calculate the area of the floor to be tiled.
  - b). Calculate the area of a tile.

c). How many whole tiles will be required to cover the floor with tiles? Ignore the number of tiles that may have to be cut to fit next to the walls.

 Observe students' answers. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample answers:

1. a).  $12m^2$  or  $120\ 000cm^2$ 

- b). 20*cm*<sup>2</sup>
- c). 6000

#### Component 4C (O)

• Read out the following questions and ask students to answer in the space on their worksheet.

Jane wants to make a lampshade from a piece of material that is 1m by 1m. The lampshade has a radius of 20cm at the top and the bottom has a radius of 35cm.



Will Jane have enough material to make the lampshade?

 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the sample answer given below.

#### Sample Answer:

The shape of the lampshade is a trapezium when it is opened out. The calculated area of the shde is  $4317.5cm^2 = 0.43175m^2$ Jane will have enough material.

#### Lesson Component 5: Reflection

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### Mathematics Grade 9 Lesson Plan 12 Deliberate Practice

Patterns and Algebra, Quadratic equations, Indices, Radical Expressions, Parallelograms, Trapezoids and Kites

#### Key Idea

Review the learning in the lessons in Week 2

#### **Component 1: Short Review**

Time: 10 minutes

- Ask students to answer the following questions.
  - 1. Rewrite  $y = x^2 6x + 8$  in the form  $y = (x a)^2 + b$
  - 2. What is the difference between concave up and concave down?
  - 3. Simplify:

i) $(a^{5})^{6}$ ii)  $(x^{8})(x^{9})$ iii)  $m^{5} \div m^{4}$ iv)  $\sqrt{\frac{125}{5}}$ v)  $\sqrt{72}$ vi)  $5\sqrt{2}(3 - 2\sqrt{5})$ vii) $\frac{2}{\sqrt{3}}$ 

4. Find the value of x in degrees



Ask students to volunteer answers, giving positive feedback.

#### Sample Answers

```
1.y = (x - 3)<sup>2</sup> - 1
2. Concave up means a parabola faces upwards and has a minimum value and concave down means the parabola faces downwards and has a maximum value
```

```
3.

i) a^{30}

ii) x^{17}

iii) m

iv) 5

v) 6\sqrt{2}

vi) 15\sqrt{2} - 10\sqrt{10}

vii) \frac{2\sqrt{3}}{3}

4. x = 16^{\circ}
```

#### Component 2: Lesson Purpose/Intention

Time: 3 minutes

The lesson is about ensuring the skills learnt during the week will be consolidated in this review lesson

#### **Component 3: Lesson Language Practice**

Time: 5 minutes

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence.
   Ask students to volunteer their answers, giving positive feedback and discussing.

intercept, vertex, concave, axis of symmetry, index laws, surd, radical, complementary, supplementary, corresponding, adjacent, alternate, vertically opposite, parallelogram, trapezoid, kite, perpendicular

#### **Component 4: Lesson Activity** Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus
- Read out the written text.
- Ask the students if there are any words that they are not familiar with and give descriptions of any words that may be problematic.



#### Component 4B

Find the values of the unknowns in the diagram in **4A**. All angles are in degrees and all measurements are in cm

• Observe students' answers. Ask the students to volunteer their answers, giving positive feedback.

#### Sample answers:

 $a = 20^{\circ}, b = 12^{\circ}, x = 7 cm, y = 6 cm$ 

#### Component 4C

# Simplify: 1. $(5\sqrt{2} - 3\sqrt{3})(6\sqrt{3} + 2\sqrt{2})$ 2. $(5\sqrt{5} + 8)(5\sqrt{5} - 8)$ 3. $(2\sqrt{3} - 6)^2$ 4. $\frac{5\sqrt{5}}{6\sqrt{3}}$ 5. $\frac{\sqrt{3}}{1+\sqrt{6}}$ Sample Answer:

1.  $24\sqrt{6} - 34$ 2. 61 3.  $48 - 24\sqrt{3}$ 4.  $\frac{5\sqrt{15}}{18}$ 5.  $\frac{\sqrt{3} - 3\sqrt{2}}{-5}$ 

#### **Component 5: Lesson Conclusion** Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### Pythagorean Theorem

#### Key Idea

Investigates problems using the Pythagorean Theorem

#### **Component 1: Short Review**

Time: 8 mins

- 1. State Pythagorean Theorem
- 2. Draw a diagram, including measurements, that illustrates the Pythagorean Theorem.
- 3. Find the length of the unknown side:



- Teacher provides answers to questions and highlights particular concepts, as well as issues that may have arisen in student answers (issues not addressed to be noted at end of lesson).
- Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.

#### Sample answers:

- 1. The square on the hypotenuse is equal to the sum of the squares on the other two sides.
- 2. There will be a variety of answers. Check that they are correct
- 3. a) x = 13cmb)x = 6cm

#### **Component 2**: Lesson Purpose/Intention Time: 2 mins

The purpose of this lesson is to understand the application of the Pythagorean Theorem in real life problems.

#### **Component 3: Lesson Language Practice**

Time: 5 mins

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

**Pythagorean Theorem** - The square on the hypotenuse is equal to the sum of the squares on the other two sides

Hypotenuse - the side opposite the right angle in a right- angled triangle

Ask students the difference between the words '*hypotenuse*' and '*adjacent*' in terms of right angled triangles.

#### **Component 4: Lesson Activity**

Time: 25 mins

#### **Component 4A**

Refer students to the main lesson stimulus and orient them to the text

Millie is standing 16m away from a statue in the park. Millie is 1.5m tall. The distance from the top of the statue to the top of Millie's head is 20m. How tall is the statue?

#### **Component 4B**

- Read out the following questions and ask students to answer in the space on their worksheet.
  - i) Please add all of the information to the diagram.
  - ii) In the picture mark the unknown value with an *x*
  - iii) Use the Pythagorean Theorem to calculate the required distance.



 Observe students' answers. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample answers:

- i) Check the diagrams to ensure they are correct
- ii) Check the diagrams to ensure they are correct
- iii) The statue is 13.5 m tall

#### Component 4C (O)

• Read out the following questions and ask students to answer in the space on their worksheet.

Mark and his family lived by the side of a river. There was a tall tree right next to the river. Mark would cross the 12m bridge to the other side of the river to fly his kite. One day his kite got stuck in the tree. He knew the string attached to his kite was 20m long. A storm raged and the bridge got washed away. If Mark chopped down the tree from the ground, would it be a way to cross the river?

 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the sample answer given below.

Sample Answer:

The tree's height is 16 m so it is long enough to go across the river.

#### **Lesson Component 5: Reflection**

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### **Trigonometric ratios**

#### Key Idea

Uses trigonometric ratios

#### **Component 1: Short Review**

Time: 10 minutes

- Ask students to answer the following questions on their worksheet.
  - 1. Use an electronic device to find (to 3 decimal places) the value of:
    - a. *sin* 31°
    - b. *cos* 43°
    - c. tan 73°
  - 2. Use an electronic device to find the angle (to the nearest degree) when:
    - a.  $sin \emptyset = 0.34$
    - b.  $\cos \phi = 0.72$
  - 3. Explain why sin  $45^\circ = \cos 45^\circ$ .
  - Ask students to volunteer answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. (This may come from one of the students or from the sample answers below).

#### Sample answers:

- 1. a.O.515
  - b. 0.731
  - с. 3.271
- 2. a. 20°
  - *b.* 44°

3. In a right-angled triangle, if one of the angles is  $45^{\circ}$ , then the other angle must also be  $45^{\circ}$ , as the three angles have a sum of  $180^{\circ}$ . Therefore, the triangle is isosceles and 2 the 2 sides other than the hypotenuse are equal. So, the 'opposite' side and the 'adjacent' side are equal and so  $sin45^{\circ} = cos45^{\circ}$ .

 If students get the earlier Short Review questions incorrect, then time should be devoted to looking at their knowledge of the skills of using an electronic device to calculate using trigonometric ratios, or they may need to revise the trigonometric ratios. Perhaps, give them more similar questions after some basic skills have been reviewed before moving on to the more difficult questions in Component

#### **Component 2: Lesson Purpose/Intention**

#### Time: 3 minutes

This lesson is about using the trigonometric functions to solve problems, expanding on previous lessons on mastering the trigonometric ratios. This is an important process that helps us understand more about the use of trigonometry. Trigonometry and its functions have a large number of uses in our daily life. For instance, it is used in building and construction, in GPS to measure the distance between places, in astronomy to measure the distance of nearby stars and also in the satellite navigation system.

Students will find this skill invaluable as they student the later topics in the review of the Mathematics in Year 9. As students become more confident in their use of the trigonometric ratios, their ability to engage with more challenging problems will become less daunting.

#### **Component 3: Lesson Language Practice**

Time: 5 minutes

The following words would be those used most often in this section. It is necessary for students to
really understand the meaning of all of these words as they are the directions teachers give to inform
students of what is required of them to do.

Cosecant - the reciprocal of the sine function.

*Cosine* - a trigonometric ratio equal to the adjacent side divided by the hypotenuse.

Cotangent - the reciprocal of the tangent function

Secant - the reciprocal of the cosine function.

Sine - a trigonometric ratio equal to the opposite side divided by the hypotenuse

*Tangent - a trigonometric ratio equal to the opposite side divided by the adjacent side.* 

trigonometric ratios - the ratios of the length of two side of a right triangle.

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence.

#### Component 4: Lesson Activity

Time: 25 minutes

#### **Component 4A**

- Refer students to the main lesson stimulus.
- Read out the written text.
- Ask the students if there are any words that they are not familiar with (or suggest and give descriptions of any words that may be problematic.

A rope is stretched from a window in the side of a building to a point on the ground, 6m from the base of the building. The angle between the rope and the side of the building is  $19^{\circ}$ .

- a. How long is the rope?
- b. How high is the window?
- Students should say out loud what the question requires them to do. They could also explain to each other how they will start the problem.

#### Component 4B

Q1. Draw a diagram to show the information in the problem.

Q2. Determine which trigonometric ratios you will need to use to solve the problem.

#### Sample answers:



b.  $\tan 19^\circ = \frac{6}{h}$   $h = \frac{6}{tan 19^\circ}$  h = 17.43mTherefore, the window is 17.43m high

#### Component 4C (O)

Read out the following question and ask students to answer.

As an airplane travels 3000m along a straight flight path, it rises 500m. Calculate the angle between the flight path of the airplane and the horizontal. Give your answer to an appropriate level of accuracy.

- Observe students' answers.
- Ask the students to volunteer their answers, giving positive feedback.
- Select a good sample answer for all students to write down. This may come from one of the students or from the following sample answer.

#### Sample answers:

Let  $\ensuremath{\varnothing}$  be the angle between the flight path and the horizontal.



*Therefore, the angle between the ground and the flight path is* 9.6°. *The teacher should discuss whether this is an appropriate answer.* 

#### **Component 5: Lesson Conclusion**

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### **Similar Triangles**

#### Key Idea

Solves problems involving similar triangles



2. What Similarity test would you use to show that  $\triangle ABC \sim \triangle CDE$  if BC: CE = AC: CD? Justify your answer.



- Teacher provides answers to questions and highlights particular concepts, as well as issues that may have arisen in student answers (issues not addressed to be noted at end of lesson).
- Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.

#### Sample answers:

1. a) 12.5 b) x=5

2. SAS

# Component 2: Lesson Purpose/Intention

Time: 2 mins

The purpose of this lesson is to apply your knowledge of similar triangles to solve problems

#### **Component 3: Lesson Language Practice**

Time: 5 mins

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

**Congruent triangles** - have the same corresponding angle measures and side lengths **Similar triangles** - have the same shape but not the same size **Proportional** - the relationship between two unknowns where the ratios are equivalent

Ask students the difference between the words 'congruent' and 'similar'.

#### **Component 4: Lesson Activity**

Time: 25 mins

#### Component 4A

• Refer students to the main lesson stimulus and **orient them to the text** 

An electricity pole is 10m high and casts a shadow of 8m. At the same time, a palm tree casts a shadow of 14m. How high is the palm tree?

#### Component 4B

- Read out the following questions and ask students to answer in the space on their worksheet.
  - 1. Draw a sketch of the electricity pole and palm tree.
  - 2. Convert the picture into a diagram with the relevant measurements.
  - 3. Determine the height of the palm tree.
- Observe students' answers. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample answers:

Check the diagrams to see if they are correct. The height of the palm tree is 11.2 meters.

#### Component 4C (O)

Read out the following questions and ask students to answer in the space on their worksheet.

Bindi is walking through the jungle and comes to a river. She wants to know how wide the river is. Bindi locates a tree at point A on the other side of the river. She marks the spot and then walks 28 paces to point C where she hammers in a marker. She then walks another 10 paces in the same direction to the point D and turns until she is at right angles to the river and walks 14 paces until point E. What is the width of the river in paces? Bindi measures her pace to be 0.8m. What is the measurement of the width of the river?

 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answer.

#### Sample Answer:

River is 39.2 paces wide, which is 31.36m wide.

#### **Lesson Component 5: Reflection**

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### Angles of Elevation and Depression

#### Key Idea

Apply knowledge of trigonometry and angles of elevation and depression to solve problems.

**Component 1: Short Review** Time: 8 mins

- Teacher asks students questions (designed to remind them of relevant content completed).
  - 1. For the diagram below, mark the angle of elevation with an *a*, the angle of depression with an *x*, and the line of sight. Why is the angle of depression and the angle of elevation equal?



- 2. Gerry is flying a kite in the park. The string from Gerry's hand to the kite is 65m. The angle of elevation from Gerry's hand to the kite is 70°.
  a). Draw a picture to illustrate this information
  - b). Draw a diagram to represent the picture with the relevant measurements included.
  - c). What is the vertical height of the kite above Gerry's hand?
- Teacher provides answers to questions and highlights particular concepts, as well as issues that may have arisen in student answers (issues not addressed to be noted at end of lesson).
- Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.

#### Sample answers:

 Check markings on the diagram. Line of sight is the hypotenuse of the right-angled triangle. The angle of depressions and elevation are alternate angles formed when a transversal cross a pair of parallel lines. The horizontal line where the angle of depression is formed is parallel to the base of the triangle.

*2a). Check the pictures to ensure they represent the information correctly.* 



#### Component 4C (O)

• Read out the following questions and ask students to answer in the space on their worksheet.

Standing on top of a harbour light house 380m above sea level, Locky sees an approaching ship into the harbour and measures an angle of depression of 9°. How far from the tower is the ship? Locky then sights a second ship beyond the first. The angle of depression of the second ship is 5°. How far apart are the ships?

 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answer.

#### Sample answers:

The diagrams drawn here must include the correct information in the correct position. The students may use complementary angles with the correct trigonometric ratio.

The ships are approximately 1944m apart.

#### **Lesson Component 5: Reflection**

#### Time: 5 minutes

In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

NOTE: Remember to collect student worksheets to review and analyze student's learning.

#### Solves problems involving Trigonometric ratios

#### Key Idea

Demonstrates knowledge and understanding of trigonometry to apply skills to solve real world problems



1. For this triangle, write down the expression for sin,  $\cos$  and  $\tan$  of  $\emptyset$ .



- 2. AB is a 3m ladder making an angle of  $70^{\circ}$  with the ground.
- i) Draw a diagram to represent the information
- ii) Find *h*, the height of B above the ground to 2 decimal places.
- Ask students to volunteer their answers, giving positive feedback. Read out a sample answer for all students to listen to and write down. This may come from one of the students or from the sample answers below.

Sample answers:

- 1.  $\sin \phi = \frac{a}{c}$  $\cos \phi = \frac{b}{c}$  $\tan \phi = \frac{a}{b}$
- 2. h=2.82m

# Component 2: Lesson Purpose/Intention

Time: 2 mins

In this lesson, students will apply their knowledge of trigonometry to solve real-world problems.

#### **Component 3: Lesson Language Practice**

Time: 5 mins

 Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.

**Cosecant** - the reciprocal of the sine function. **Cosine** - a trigonometric ratio equal to the adjacent side divided by the hypotenuse. **Cotangent** - the reciprocal of the tangent function

Secant - the reciprocal of the cosine function.

*Sine* - a trigonometric ratio equal to the opposite side divided by the hypotenuse *Tangent* - a trigonometric ratio equal to the opposite side divided by the adjacent side. *trigonometric ratios* - the ratios of the length of two side of a right triangle.

- Ask students the difference between the words 'tangent' and 'cosine'.
   Ask students to volunteer their answers, giving positive feedback and discussing.
- Ask the students to practice saying the words in a sentence.

#### **Component 4: Lesson Activity** Time: 25 mins

Component 4A

- Refer students to the main lesson stimulus and orient them to the text
- Ask the students if there are any words that they are not familiar with

Q1. A block of land slopes at 5°. Find the depth of excavation needed to provide a level base 20m long (to 2 decimal places).

*Q2.* The steepest slope a tractor can climb is 35°. Find the shortest length of road required for a rise of 20m.

#### Component 4B

• Read out the following questions and ask students to answer in the space on their worksheet.

Q1. a). Draw a diagram to represent the information in the question

- b) Determine which trigonometric ratio to use.
- c) Calculate the depth.

Q2. a) Draw a diagram to represent the information in the question

b) Determine which trigonometric ratio to use.

c) Calculate the length of road.

 Observe students' answers. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the following sample answers.

#### Sample answers:

Q1.

- a) Diagrams could be varied. Check to ensure the information is in the correct position.
- b) Most students will use the tan ratio.
- c) 1.75 m

Q2.

- a) Diagrams could be varied. Check to ensure the information is in the correct position.
- b) Sine ratio
- c) 34.87m

#### Component 4C (O)

• Read out the following questions and ask students to answer in the space on their worksheet.

Two wires supporting a 20m flag pole are each anchored with wire 3m from the base of the flag pole. Fine the angles (to the nearest degree) between the wire and the ground.

 Observe students' responses, giving encouragement. Ask the students to volunteer their answers, giving positive feedback. Select good sample answers for all students to write down. These may come from the students or from the sample answer.

Sample answers:

81°

#### **Component 5: Lesson Conclusion**

Time: 5 mins

In this component, the teacher initiates and leads a student-focused wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

**NOTE:** Remember to collect student worksheets to review and analyze student's learning.

#### Mathematics Grade 9 Lesson Plan 18 Deliberate Practice

#### Using Pythagoras, trigonometry and triangles to solve problems

#### Key Idea

Review the learning from the third week



#### Suggested Answers:

1. The square on the hypotenuse is equal to the sum of the squares on the other two sides2. i) 4cmii)  $\sqrt{74} \approx 8.6$  cm3. x = 3ii) 24°ii) 35.34

#### **Component 2**: Lesson Purpose/Intention Time: 3 minutes

Review the learning for the last week of the camp.

#### *Component 3: Lesson Language Practice Time: 5 minutes*

- Read out difficult or unfamiliar words or phrases and ask the students to read them to themselves and then out loud as a class.
- Ask the students to practice saying the words in a sentence.
   Ask students to volunteer their answers, giving positive feedback and discussing.

*Pythagorean Theorem, hypotenuse, sine, cosine, tangent trigonometry, congruence, similar triangles, proportional, angles of elevation, angles of depression, line of sight,* 

• Choose 5 words from the above list and use them in a sentence

#### **Component 4: Lesson Activity**

Time: 25 minutes

#### **Component 4A**

From the tops of a 40m high vertical cliff, the angle of depression of an object that is level with the base of the cliff is 34°. How far is the object from the base of the cliff?

#### Component 4B

- 1. Draw a diagram to represent the information.
- 2. Complete the calculations
- 3. Answer the questions

#### Answers:

Diagram 59.3 m

#### Component 4C

A surveyor wants to know the height of a skyscraper. He places his inclinometer on a tripod 1m from the ground. At a distance of 50m from the skyscraper, he records an angle of elevation of 82°.

Sample Answer:

356.8m

**Component 5: Lesson Conclusion** Time: 5 minutes In this component, the teacher initiates and leads a student-focussed wrap-up to the main objectives of the lesson. The focus is on the whole lesson experience, including how well what students feel that they take from the lesson relates to what their expectations were for the lesson, as outlined in Component 2 ('Purpose/focus' of the lesson).

The teacher should facilitate student reflection and discussion, that addresses such questions as:

- What do you think were the key mathematical concepts addressed in this lesson?
- Would you rate your level of understanding of the material covered in this lesson as high, moderate, or low?
- Has the lesson helped you to gain further insight into aspects of the material covered that represent strengths or weaknesses?
- What would you describe as the main barriers, if any, to your ongoing progress and achievement in relation to the topic area addressed in this lesson?
- What do you think would best assist your ongoing progress and achievement in relation to the topic area?

**REMINDER:** Collect student worksheets to review and analyze student's learning.

#### For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Delivery (DepEd-BLD) Contact Numbers: 8637-4366; 8637-4347; 8633-9347

Department of Education - Bureau of Learning Resources (DepEd-BLR) Contact Numbers: 8634-1072; 8631-6922

Email Address: blr.od@deped.gov.ph

