9

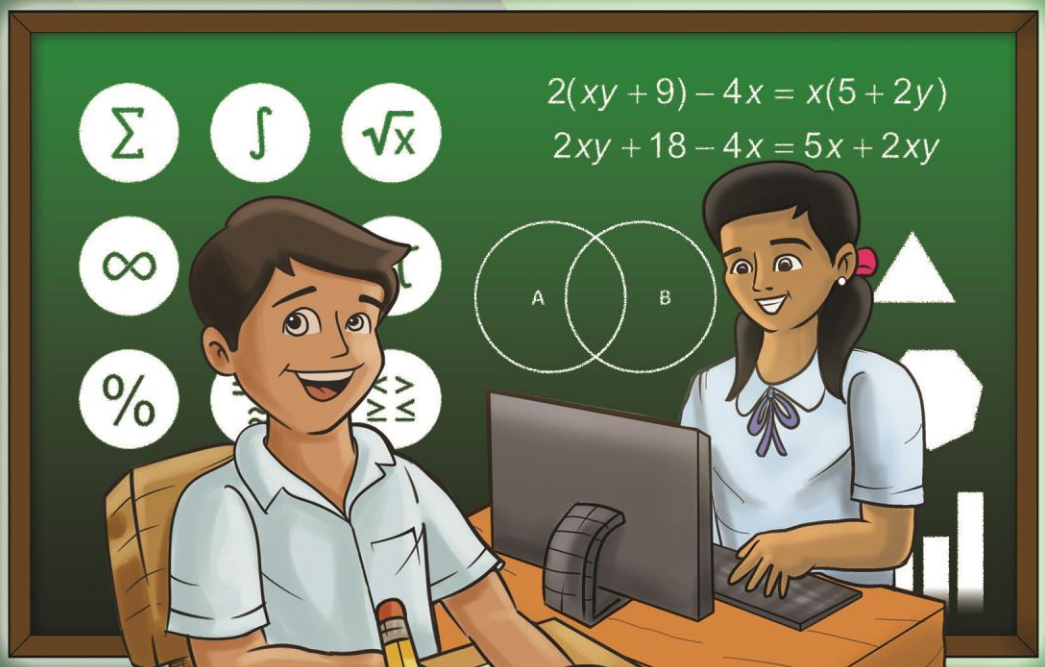
NATIONAL LEARNING CAMP

DepED DEPARTMENT OF EDUCATION

Mathematics

Enhancement Learning Camp

Notes to Teachers



Enhancement Learning Camp
Notes to Teachers

Mathematics Grade 9

Weeks 1 to 3

Contents

Dear Reader,

Every care has been taken to ensure the accuracy of the information provided in this Booklet. Nevertheless, if you identify a mistake, error, or issue, or wish to provide a comment, we would appreciate you informing the **Office of the Director of the Bureau of Learning Delivery** via telephone numbers (02) 8637-4346 and 8637-4347 or by email at bld.od@deped.gov.ph

Thank you for your support.

Notes to Teachers

Part A: Introduction to Mathematics in the 2024 Learning Camp

The Mathematics section of the 2024 Learning Camp for students who have recently completed Grade 9, consists of 15 'main' lessons each focused on a single Key Idea (KI), and 3 'deliberate practice' lessons each week. All 18 lessons are designed for a lesson duration of 45 minutes.

The set of 15 main lessons consists of single lessons addressing each of 15 key ideas of the Grade 9 curriculum content. The Key Ideas are based on selected Most Essential Learning Competencies (MELCs) for the four Quarters of the Grade 9 content.

The three deliberate practice lessons, Lessons 6, 12, and 18, are each delivered at the end of Week 1, 2 and 3 of the 2024 Learning Camp. The lessons are designed to reinforce learning from the Key Idea lessons of the week.

The 15 Key Ideas listed by Quarter are:

Quarter 1:	KI 1	Determine arithmetic means, nth term of an arithmetic sequence, and the sum of the Terms of an arithmetic sequence.
	KI 2	Determine geometric means, nth term of a geometric sequence, and sum of the terms of a finite or infinite geometric sequence.
	KI 3	Solve problems involving sequences.
	KI 4	Solve problems involving polynomials and polynomial equations.
Quarter 2:	KI 5	Solve problems involving polynomial functions.
	KI 6	Solve problems involving circles.
	KI 7	Determine the center and radius of a circle given its equation, and vice versa.
	KI 8	Graph and solve problems involving circles and other geometric figures on the coordinate plane.
Quarter 3:	KI 9	Differentiate permutation from combination of objects taken r at a time.
	KI 10	Solve problems involving permutations and combinations.
	KI 11	Illustrate and find the probability of a union of two events $(A \cup B)$, including for when the events are mutually exclusive.
	KI 12	Solve problems involving probability.
Quarter 4:	KI 13	Calculate and interpret measures of position (quartiles, deciles, and percentiles) of a set of data.
	KI 14	Solve problems involving measures of position.
	KI 15	Use appropriate measures of position and other statistical methods in analyzing and interpreting data.

The Key Ideas are representative of the three content sections of the Grade 9 curriculum as follows:

Patterns and Algebra: KI 1, KI 2, KI 3, KI 4, KI 5

Geometry: KI 6, KI 7, KI 8

Statistics and Probability: KI 9, KI 10, KI 11, KI 12, KI 13, KI 14, KI 15

The 15 main lessons, which each address one (only) of the Key Ideas listed above, are:

Lesson 1: Determining Arithmetic Means, nth term of an Arithmetic Sequence, and Sum of the Terms of an Arithmetic Sequence.

Lesson 2: Determining Geometric Means, nth term of a Geometric Sequence, and Sum of the Terms of a Finite or Infinite Geometric Sequence.

Lesson 3: Solving Problems involving Sequences.

Lesson 4: Solving Problems involving Polynomials and Polynomial Equations.

Lesson 5: Solving Problems involving Polynomial Functions.

Lesson 6: Deliberate Practice: Solving Problems involving Sequences; Solving Problems involving Polynomials and Polynomial Equations; Solving Problems involving Polynomial Functions.

Lesson 7: Solving Problems involving Circles.

Lesson 8: Determining the Center and Radius of a Circle given its Equation, and vice versa.

Lesson 9: Graphing and Solving Problems involving Circles and other Geometric Figures on the Coordinate Plane.

Lesson 10: Differentiating Permutation from Combination of Objects taken r at a time.

Lesson 11: Solving Problems involving Permutations and Combinations.

Lesson 12: Deliberate Practice: Solving Problems involving Circles; Graphing and Solving Problems involving Circles and Other Geometric Figures on the Coordinate Plane; Solving Problems involving Permutations and Combinations.

Lesson 13: Illustrating and finding the Probability of a Union of Two Events $(A \cup B)$, including for when the events are mutually exclusive.

Lesson 14: Solving Problems involving Probability.

Lesson 15: Calculating and Interpreting Measures of Position (quartiles, deciles, and percentiles) of a Set of Data.

Lesson 16: Solving Problems involving Measures of Position.

Lesson 17: Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data.

Lesson 18: Deliberate Practice: Solving Problems involving Probability; Solving Problems involving Measures of Position; Using Appropriate Measures of Position and Other Statistical Methods in Analyzing and Interpreting Data.

Note: The three deliberate practice lessons each address and correspond to multiple Key Ideas. The Deliberate Practice lessons, with the associated Content Section/s and Key Ideas are:

Lesson 6: Content Section: Patterns and Algebra; Key Ideas: KI 3, KI 4, KI 5

Lesson 12: Content Sections: Geometry, Statistics and Probability; Key Ideas: KI 6, KI 8, KI 10

Lesson 18: Content Section: Statistics and Probability; Key Ideas: KI 12, KI 14, KI 15

Each of the 15 main lessons and three deliberate practice lessons is written in a standard format, made up of five sequential lesson components.

The components are:

Lesson Component 1: Lesson Short Review

Lesson Component 2: Lesson Purpose/Intention

Lesson Component 3: Lesson Language Practice

Lesson Component 4: Lesson Activity

Lesson Component 5: Lesson Conclusion – Reflection/Metacognition on Student Goals

The nature of these lesson components for all 18 Mathematics KI lessons is described and discussed in Part B.

Part B: Commentary on Lesson Components in All Lessons

Overview

The NLC lessons emphasize consolidating and, where possible, extending student knowledge in previously covered topics. Lesson sets are designed to strengthen students' current foundational knowledge ready for future learning. The review lessons have been designed to be interactive among teachers and their students, as well as with students and their peers.

At the same time, the expectation is that teachers will enhance their own pedagogical practices and subject knowledge as well as refine their teaching methods further. The thinking behind the Camp lessons is grounded in the 'Science of Learning' framework, creating a dynamic, learning environment employing the findings of cognitive research and evidence-informed approaches.

Lesson Component 1 (Lesson Short Review)

Component 1 offers teachers the chance to:

- settle the class quickly;
- review or preview previously encountered information;
- address previous content in the form of a few targeted questions that are *relevant to the current lesson;*
- note what students already know;
- elicit answers from the class to reinforce the important content needed for the lesson; and
- briefly address issues that may arise.

Overall, Component 1 acts as a partial advance organizer designed to remind students of previous work that has relevance to activities to be undertaken in the current lesson. When done carefully, this replay of previous information directed at what will come in the lesson helps students prepare for future memory recall and decision-making.

Reminding students of *relevant* information at the beginning of a lesson, prior to encountering the main lesson learning focus, can enhance the brain's ability to:

- access information to be used in problems/questions/information to come;
- prepare, and have some oversight, for the direction of future learning; and
- further consolidate ideas in long-term memory.

The teacher should note any issues that may arise in student answers. This may be addressed later in the lesson or in later lessons, if relevant.

Lesson Component 2 (Lesson Intention)

This component allows teachers the opportunity to explain to the class the intention or purpose of the lesson. The explanation should link with student prior knowledge or experience. This may mean connecting the purpose to the responses and levels of understanding in Component 1. The words and phrases used by the teacher should be familiar to, and understandable by, students. Information could include ideas personal to students that could facilitate student engagement in the lesson such as:

- the provision of a relevant context;
- asking a question that sounds interesting to that age group; and/or
- addressing an aspect that has a special interest to the class.

In addition, this component is an appropriate time to address what students might expect/aim to achieve, i.e., the lesson goal(s). Teachers should clarify, in clear language, the learning intention and what success looks like.

(Note**:** Evaluation of the degree of success or partial success of student learning intention should occur as part of Component 5.)

Lesson Component 2 is about activating, in the student's brain, ideas already relevant to the students. The purpose is to help students contextualize their new learning experiences and to help them make sense of any new information.

Design considerations in statements of the lesson intention are about promoting student engagement and enthusiasm. This is best done by stating things in ways that make sense to as many students as possible in the class. In terms of timing, this component is relatively brief. Its presence, as one of five components, lies in *its importance* to the student brain and learning. Finally, it is important *not to* overwhelm students with excessive and unnecessary detail that could disengage them at this early point in the lesson.

Lesson Component 3 (Lesson Language Practice)

Component 3 concerns language use – speaking, hearing, listening, and comprehending. The focus is on words or phrases that are important to this lesson. It may be language that has the potential to cause difficulties for the students through speech, interpretation, or understanding, or simply a reminder that these words are important to, or will be used in, this lesson. Typically, the language identified is restricted to about six words/phrases so that there is enough time for students to use them in practice.

Deliberate practice concerns repeating aspects of learning that the teacher has deliberately identified/selected because it is where students are making an error that needs to be corrected, or because of its important role in learning. In the case of unfamiliar or unknown textual or symbolic language, deliberate practice can help students reduce cognitive load (reduce working memory) by making some aspects more familiar, enabling students to re-allocate resources to a problem solution, comprehending a passage, answering a question, explaining a concept, or describing some event or story, etc.

Overall, Component 3 can help achieve language familiarity by saying the word/phrase, being able to spell it, or using it in a specific context. This may also involve assisting the students to understand or unpack a visual text, diagram or graph, e.g., for a graph, the teacher may need to point out such things as the graph heading, the axes, units, data points, or trend lines.

Lesson Component 4 (Lesson Activity)

Addressing the key idea for the lesson is the focus of Component 4. It involves students applying known content to solve non-routine problems or interpreting new texts. This requires students to interpret/understand the meaning of the stem of the problem correctly and then answer a few questions of varying degrees of complexity related to the stem. The stem holds the needed information that will be the basis for the questions. Following the stem is a small number of questions that can be answered by utilizing students' background content knowledge and understanding, together with information in the stem.

From a learning perspective, the lessons are intended to help students consolidate their understanding at different levels of difficulty, e.g., the early questions are at an elementary level allowing the students to get started, then the next level is directed at the majority of students and usually requires a number of steps to reach a conclusion, and, finally, the third question attempts to offer all students the opportunity to be challenged and experience enhancements of their learning through seeing how ideas are connected or applied.

(Note: The level of difficulty of the questions should not stop any student from being given the opportunity to experience, with support, questions at higher levels, including the more challenging questions, and to hear about, and be involved in, discussions about the answers.

Most students should be able to make some progress and be acknowledged for that. The point of question levels is to at least have students experience these more demanding questions and their answers as the start of the

process for their learning journey. It is also designed to offer teachers a more realistic view of potential expectations of students in their class.)

Component 4 has three aspects, 4A, 4B, and 4C. Students are first presented in 4A with the stem. This can be a stimulus or passage/text or diagram or … and are given the time/opportunity to understand the stem.

Then, in 4B and 4C, two separate set of questions related to the same stem are presented. This process involves a set of three questions based on the same stem, which is then repeated, resulting in one set of questions in each of 4B and another set of questions in 4C.

Note: The early components, Components 1, 2 and 3, can be seen as bringing together the pre-requisite information that will place the student in the best possible position to be successful in Component 4. Component 4 begins with 4A.

4A Reading and Understanding the Stem

4A involves understanding the language of the stem. The purposes here are for – _
the teacher:

- to model fluent reading of the stem (first);
- to identify any unfamiliar language the student (possibly addressed in Component 3);
- to read the passage or describe the figure, etc
- to hear and experience fluency in reading the stem.

Other activities here could include students:

- reading to each other;
- reading silently to themselves, and
- exploring the meaning of the vocabulary.

4B Solving the First Set of Questions

4B involves students answering questions associated with the stem. The students will recognize that they have a stem (previously met in **4A**) and that this is followed by a small set of questions. Students find their own way to a response for each question in the set. The students write down responses or attempts at each question. It is important that every student in the class is expected to have a response. To achieve this desired result, it is important for teachers to ensure all students start on time at the same time.

When the students are finished, or sufficient time has been allocated, students provide answers to the questions and the teacher marks the questions. Discussion takes place about:

- the quality of the answers;
- the implications of errors; and
- what this information tells the class about the content.

The time allocated for 4B provides teachers with an opportunity to observe the quality and levels of student response, which they can build on as a base of what the student knows.

Note: It is important that students start the questions promptly. This involves student self-regulation concerning focus and attitude to work, and may need to be consistently encouraged or reinforced by the teacher.

Teachers can seek out different responses or approaches or thinking exhibited. Errors made by students should be *acknowledged and valued* for their contribution to the class discussion and student learning. Those who achieve correct answers on different questions should also be acknowledged. **Note:** The questions are usually arranged in increasing difficulty from basic to more challenging.

4C Solving the Second Set of Questions

4C uses the same Stem as **4B** and repeats the same process as **4B** but offers students a second (different) batch of questions, again in order of increasing difficulty. When all questions are completed, as was the case in **4B**, students provide answers to all questions, i.e., the students write down responses to, or attempts at, each question. When they are finished, the questions are marked (either using teacher or student answers) and discussion takes place about the quality of correct answers and the implications of errors and what this tells the class about the content.

Note: 4C offers a new start for students regardless of how they performed in **4B**. It allows all students to see **4C** as a new starting point and the class focus for all students should now be around the content and answers in **4C**.

For teachers this approach serves two purposes. *First,* it is a practical way to ensure all students have experiences and are able to contribute perspectives with all questions asked. *Second,* the teacher will have the opportunity to practice further problem-solving questions where different sets of questions can be used with a familiar Stem. This approach is efficient as students obtain more problem-solving practice on the same underlying content.

Reducing cognitive load (working memory demands) is important in writing a stem. Stems in the lessons are designed to facilitate students' reading and interpretation. This is achieved by restricting materials to several sentences and a few paragraphs in length, with no more than one diagram for each item. The teacher could have students read the stems together or individually to assist the development of their fluency with the language used.

In Component 4 students are expected to provide answers using:

- factual knowledge
- application of skills and procedures (fluency)
- understanding
- communicating skills
- reasoning and justification.

Clear feedback to students is very important. Teachers should assist students at a level that they can understand in addressing issues, misconceptions or errors that have arisen.

Lesson Component 5 (Lesson Conclusion)

Component 5 offers a student-focused summary of the lesson intention. **Students** reflect on their progress, achievement, or partial achievement of goals (lesson intention) and their performance and understandings. It takes up comments from Component 2 about teacher expectations. Here teachers can confirm student progress. Honesty is needed, as positive as circumstances permit, including the long-term impact of student effort and persistence.

Component 5 has a high metacognitive aspect for students – thinking about their own thinking – which can be further enhanced by teacher modelling.

Part C: Syllabus References, Purpose, and Worked Answers for the Individual Lessons

Information provided in Part C includes matters that are important for students to observe in the 15 main lessons and three deliberate practice lessons. This information, together with the 'Syllabus Codes' (if applicable) from the **K to 12 Mathematics Curriculum Guide (Grade 1 to Grade 10) August 2016**, is listed, as well as worked answers to the Component 1 and Component 4 questions. This is provided for assistance, as needed, in working through the respective lessons.

Lesson 1: Quadratic Equations

Curriculum references: M9AL-Ib-1

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –1st Quarter*

Content Section:

Patterns and Algebra

Learning Competency (**Code**):

1. M9AL-Ib-1

Content Standard:

The learners demonstrate understanding of key concepts of quadratic equations.

Performance Standard:

The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies.

Most Essential Learning Competency (MELC/s):

'Same as **Learning Competency**

Key Idea

Solving quadratic equations

Purpose

This lesson is about factorizing to solve quadratic equations. This is an important process that helps us understand more about equations which will eventually assist in solving problems that rely on the solution to a quadratic equation.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. *There is another answer x = -4 because* $(-4)^2 = 16$
2. $y = 0$
3. $x = -3 \; or \; 0$
4. $x = 1 \; or \; 2$
5. $x = \pm\sqrt{5}$
6. $x = \pm 3$
7. *There is no solution. The answer is imaginary since the square root of -9 is* $3i$.

Component 4

Part 4B

1. $x = 0, -4$
2. $x = \pm\sqrt{5}$
3. $x = -3, 7$
4. $x = \frac{-5}{3}, 8$
5. $x = -4, 1$
6. $y = 7, -3$
7. $m = 6, -5$
8. $x = -6, 1$
9. $n = -8, 4$
10. $x = -2 \pm \sqrt{5}$

Part 4C

Russell is 9 years old and Sandra is 18 years old.

Lesson 2: The Discriminant

Curriculum references: M9AL-Ic-1

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –1st Quarter*
Content Section:
Patterns and Algebra

Learning Competency (**Code**):
1. M9AL-Ic-1

Content Standard:
The learners demonstrate understanding of key concepts of quadratic equations.

Performance Standard:
The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies.

Most Essential Learning Competency (MELC/s):
Same as **Learning Competency**

Key Idea
Using the discriminant to determine the nature of the roots of a quadratic equation

Purpose

The lesson is about using the discriminant to determine the nature of the roots of quadratic equations. The discriminant allows us to determine if there are no solutions to a quadratic equation, only one solution, two solutions and whether those two solutions are unequal and rational or not.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. *Q1.* $x = -1 \pm \sqrt{6}$

   Q2. $x = 6 \text{ or } -3$

   Q3. $x = 2 \text{ or } 7$

   Q4. $x = \pm\frac{3}{2}$

2. *a)* $7(x^2 + 6x - 16) = 0$

   $7(x + 8)(x - 2) = 0$

$$x = -8 \text{ or } 2$$

b) $x = \frac{-42 \pm \sqrt{42^2 - 4 \times 7 \times (-112)}}{2 \times 7}$ $x = -8 \text{ or } 2$

Factorizing is much easier because the numbers are cumbersome in the formula

3. *Q1* $\Delta = 24$

   Q2. $\Delta = 81$

   Q3. $\Delta = 25$

   Q4. $\Delta = 144$

Component 4

Part 4B

Q1.

1. $\Delta > 0, roots\ are\ real\ and\ unequal$
2. $\Delta = 0, roots\ are\ real\ and\ equal$
3. $\Delta < 0, no\ real\ roots$

Q2.

a) $\Delta = 0$, k=3
b) $\Delta < 0, k > 3$
c) $\Delta > 0, k < 3$

Q3. $\Delta = 0, m = \frac{25}{24}$

Q4.

a) $\Delta = 25, 2\ real\ rational\ roots$
b) $\Delta < 0$*, no real roots*

Part 4C

Q1. $\Delta = (2r)^2 - 4(r)(r) = 0$

The roots are real and equal, since Δ=0.

Lesson 3: Solves Quadratic Inequalities

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade 9– 1st Quarter

Content Section:
Patterns and Algebra M8AL-If-1

Learning Competency (**Code**):

Content Standard:
The learners demonstrate understanding of quadratic equations, inequalities and functions, and rational algebraic equations

Performance Standard:
The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies

Most Essential Learning Competency (MELC/s):
Same as **Learning Competency**

Key Idea
Using knowledge of quadratic inequalities to illustrate solutions graphically

Purpose

The lesson is about using quadratic inequalities to illustrate solutions to quadratic inequalities both algebraically and graphically. This skill will assist in solving problems with quadratic inequalities.

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. $x = \pm 8$

Q2. $x > 8, x < -8$

Q3. $x > -8, x < 8, or -8 < x < 8$

Q4. $x \geq 8, x \leq -8$

Q5. No solution. No real number can be satisfied by the inequality.

Q6. $x > 3, x < -3$

Q7. $-1 < x < 3$

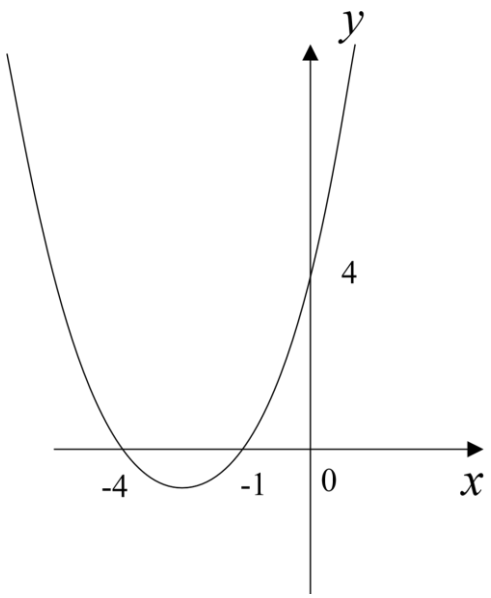
Component 4

Part 4B

Q1. $x^2 + 5x + 4 \geq 0$

Q2. $(x + 1)(x + 4) \geq 0$

Q3.



Q4. For graph to be above the x-axis, $x < -4$ *or* $x > -1$

Q5. The solution for $x^2 + 5x \geq -4$ *is* $-4 > x > -1$

Part 4C

Q1. $x < 0$ *or* $x > 3$

Q2. $2 < x < 8$

Lesson 4: Solving Problems with Quadratic Inequalities

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –1st Quarter*
Content Section:
Patterns and Algebra
Learning Competency (**Code**): M9AL-If-g-1
Content Standard:

The learners demonstrate understanding of quadratic equations, inequalities and functions, and rational algebraic equations

Performance Standard:

The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies

Most Essential Learning Competency (MELC/s): M9AL-If-g-1
Key Idea
Students use their skills with quadratic inequalities to solve problems

Purpose

The lesson is about solving quadratic inequalities in order to find the answer to real problems.

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. $x < -5, x > 3$

Q2. $-1 \leq x \leq 3$

Q3. $2 < x < 6$

Q4. $x < 0, x > 5$

Q5. $x < 6, x > -\frac{3}{2}$ *or* $-\frac{3}{2} < x < 6$

Component 4
Part 4B

Q1. $11 < x < 2$

Q2. 1 second

Part 4C

When inequation is solved, there is also a negative answer that is not possible, so $x > 3.55$

Lesson 5: Graphs of Quadratic Functions

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –1stQuarter*

Content Section:
Patterns and Algebra

Learning Competency (**Code**): M9AL-Ig-h-i-1

Content Standard:
The learners demonstrate understanding of quadratic equations, inequalities and functions, and rational algebraic equations

Performance Standard:
The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies

Most Essential Learning Competency (MELC/s): M9AL-Ig-h-i-1

Key Idea
Uses the key features of a parabola to sketch its graph

Purpose

The lesson is about using the number plane to determine the position or location of a parabola on a number plane. Being able to draw accurate sketches of parabola is a tool that is used to solve Mathematical problems. Confidence in using a Number Plane to sketch graphs will assist in many other aspects of Mathematics

Worked Answers to Component 1 and Component 4 Questions

Component 1

a) $a > 0$*, the parabola is concave up*

b) The axis of symmetry is $x=3$

c) The vertex is (3, -1)

d) The y*-intercept is* $y=8$

e) The x-intercept/s: x = 4, 2

f) Sketch of the parabola including the above points

Component 4

Part 4B

Q1 $f(x) = 3x^2 - 6x + 5$

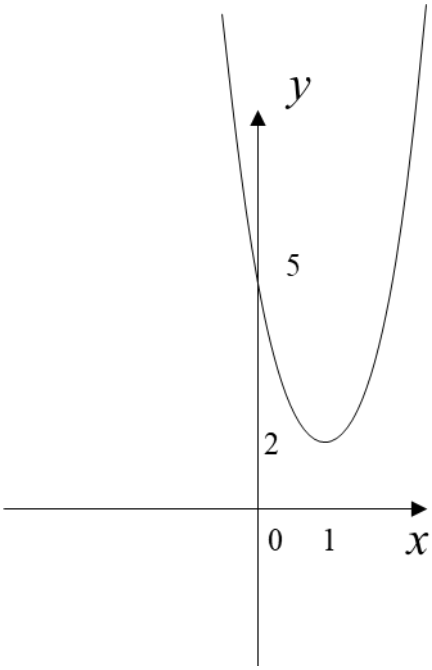
Q2. Concave up

Q3. axis of symmetry is $x = 1$

Q4. vertex is (1,2)

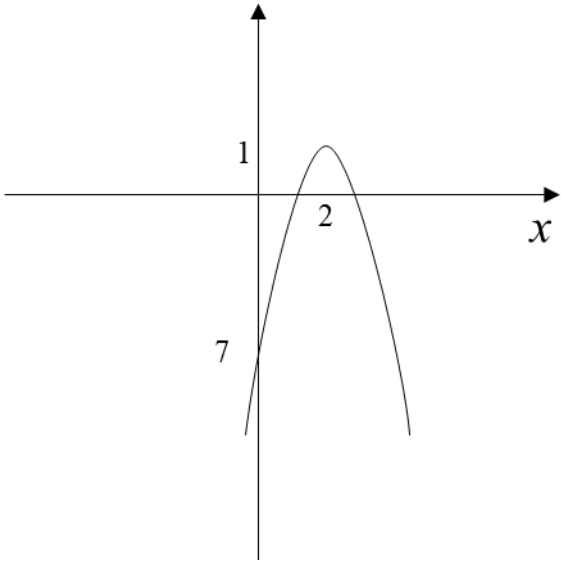
Q5. y-intercept is y=5, there is no x-intercept

Q6.



Part 4C

- Concave down
- Axis of symmetry $x = 2$
- Vertex (2,1)
- *y*-intercept *y*=-7, *x* -intercept *x*=1.29 and *x*=2.71



Lesson 6 Deliberate Practice: Consolidation Week 1

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 – Ist Quarter*

Content Section:
Patterns and Algebra

Content Standard:

The learners demonstrate understanding of key concepts of quadratic equations, inequalities and functions, and rational algebraic equations

Performance Standard:

The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies.

Most Essential Learning Competency (MELC/s): *M9AL-Ia-b-1,* M9AL-Ic-1, M8AL-If-1, M9AL-If-g-1, M9AL-Ig-h-i-1

Key Ideas

review Patterns and Algebra

Purpose

The lesson is about ensuring the skills learnt during the week will be consolidated in this review lesson

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. x= 6,-5
Q2. x= -2,-5

Q3. $x = \frac{-4 \pm \sqrt{46}}{6}$

Q4. $x > \frac{-5}{4}, x < \frac{5}{4}$

Q5. $y \geq 5, y \leq -4$

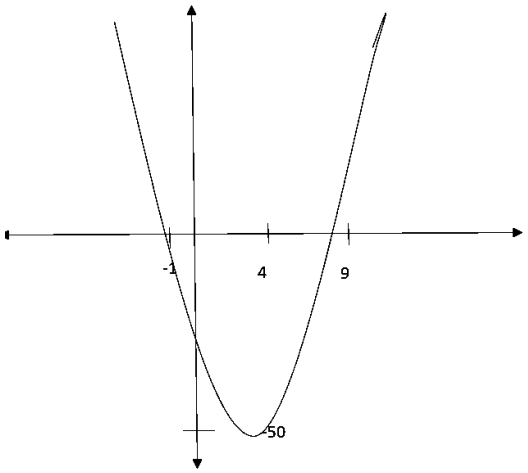
Component 4

Part 4B

1. $2x^2 - 16x - 18 = 0$
2. *The nature of the roots is real since the computed discriminant is greater than 0 which is 400.*
3. $\Delta = 400$
4. $x = 4, y = -50$
5. x-intercepts:

   x = 9, -1

   y-intercept:
   y = -18

6. 

Part 4C

$x > 2$

Lesson 7: Determines the Equation of a Quadratic Function

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 – 1st Quarter*
Content Section: Patterns and Algebra M9AL-Ij-1
Content Standard
The learners demonstrate understanding of quadratic equations, inequalities and functions, and rational algebraic equations

Performance Standard:
The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies

Most Essential Learning Competency (MELC/s): M9AL-Ij-1

Key Idea

Uses the graph of a quadratic function to determine its equation

Purpose

The lesson is about using a graph and knowledge of completing the square to find the equation of a parabola.

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1.

a). $(x-2)^2-3$

b). $(x-3)^2+7$

c). $-3[(x-1)^2+5]$

Q2.

a). $x=\sqrt{5}-2,\ x=-2-\sqrt{5}$

b). $x=7+\sqrt{\frac{8}{3}}\ \ x=7-\sqrt{\frac{8}{3}}$

Q3.

a). $x=\pm\sqrt{7}-3$

b). $x=\pm\sqrt{10}+5$

Component 4

Part 4B

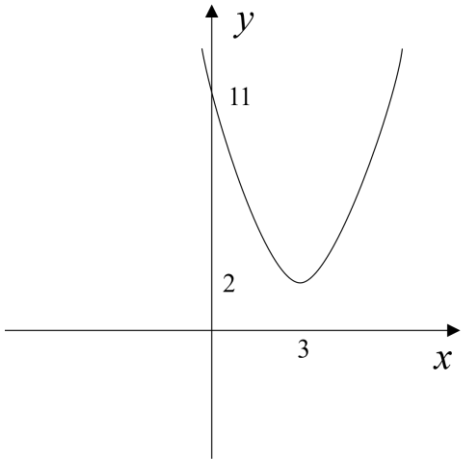
Q1. $y=\ (x-3)^2+2$

Q2. Axis of symmetry is $x=3$

Vertex is (3,2)

Concave up

y-intercept is y=11

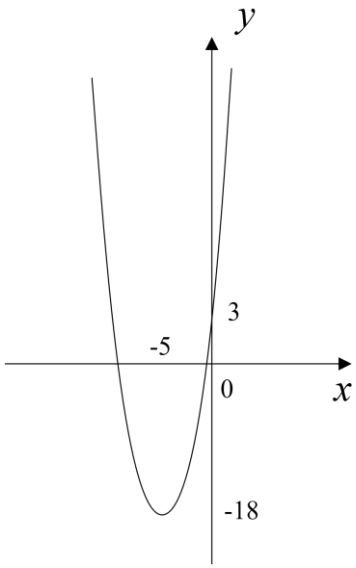


Q3.

y-intercept is y=7

vertex is (-5,-18)

Axis of symmetry is x = -5



Part 4C

Axis is Symmetry is $x = 0$, therefore $x = a$=0

Vertex is (0,2), therefore b=2

k=1

Equation is $y = x^2 + 2$

Lesson 8: Simplifying Expressions with Rational Components

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –2nd Quarter*
Content Section:
Patterns and Algebra
Learning Competency (**Code**): M9AL-II-e-1
Content Standard:
The learners demonstrate understanding of key concepts of variation and radicals.

Performance Standard:
The learner is able to formulate and solve accurately problems involving radicals

Most Essential Learning Competency (MELC/s): M9AL-II-e-1

Key Idea

The learner applies knowledge of index laws to simplify algebraic expressions

Purpose

The lesson is about applying the understanding of index laws to simplify algebraic expressions that include radicals.

Worked Answers to Component 1 and Component 4 Questions

Component 1

Q1. a^6
Q2. b^{12}
Q3. x^5
Q4. $2x^{\frac{23}{10}}$
Q5. x^6
Q6. 9
Q7.$\frac{1}{8}$

Component 4

Part 4B

Q1. $32p^{12}$
Q2. $504r^{10}s^5$
Q3. $(5^{10})(9a^8)$
Q4. xy
Q5. $4f^4g$
Q6. $7j^8km^2$
Q7. 8^{27}
Q8. h^{24}
Q9. $c^{10}d^{15}$

Part 4C

Q1. 3^5p^{10}
Q2. $5^{-3}b^{-9}$
Q3.$2x^{\frac{3}{2}}$

Lesson 9: Radical Expressions

Key Idea: Performs Operations on Radical Expressions

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –3rd Quarter*

Content Section: Patterns and Algebra

Learning Competency (**Code**): M9AL-IIh-1

Content Standard:
The learners demonstrate understanding of key concepts of variation and radicals.

Performance Standard:
The learner is able to formulate and solve accurately problems involving radicals

Most Essential Learning Competency (MELC/s): as above

Key Idea
Performs Operations on Radical Expressions

Purpose
The purpose of this lesson is to simply radical expressions

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $7\sqrt{2}$
2. $4\sqrt{5}$
3. $3\sqrt{11}$
4. $8\sqrt{2}$
5. $-3\sqrt{3}$
6. $3\sqrt{3}+\sqrt{2}$
7. $\sqrt{18}=3\sqrt{2}$
8. ± 5

Component 4

Part 4B

Q1. $6\sqrt{7} - 4\sqrt{2}$

Q2. $35 - 12\sqrt{6}$

Q3. $2\sqrt{5} - \sqrt{3}$

Q4. $12 - 8\sqrt{2}$

Q5. $3\sqrt{5}$

Q6. $10\sqrt{5} - 25\sqrt{30}$

Q7. $-35 - 5\sqrt{5}$

Q8. $22 + 16\sqrt{3}$

Q9. $\frac{\sqrt{10}}{2}$

Q10. $\frac{4\sqrt{3}}{3}$

Part 4C

Q1. $\sqrt{2}$

Q2. $\frac{3}{25}$

Q3. -46

Q4. $\frac{\sqrt{3}}{-6}$

Q5. $\frac{\sqrt{6}}{6y}$

Lesson 10: Parallelograms

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –3rd Quarter*
Content Section: Geometry
Learning Competency (**Code**): M9GE-IIib-1
Content Standard:
The learners demonstrate understanding of key concepts of parallelograms and triangle similarity

Performance Standard
The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation

Most Essential Learning Competency (MELC/s): as above

Key Idea

Uses the properties of parallelograms to solve problems

Purpose

The purpose of this lesson is to use the properties of parallelograms to find unknown length values and unknown angles. This skill will be useful when proving geometric facts and solving problems.

Worked Answers to Component 1 and Component 4 Questions

Component 1

$a = 12$ *because opposite sides of a parallelogram are equal*

$b = 7$ *because opposite sides of a parallelogram are equal*

$x° = 112°$ *because diagonally opposite angles are equal*

$y° = 68°$ *because adjacent (consecutive) angles are supplementary*

Component 4

Part 4B

1. a=10 because the two diagonals bisect each other

   b=10 because opposite sides of a parallelogram are equal

   $x° = 62°$ because $x°$ and $62°$ are alternate angles when a transversal crosses two parallel lines

   $y° = 33°$ because the two angles are vertically opposite each other and are equal

2. a = 20 because opposite sides of a parallelogram are equal

   b = 14 because opposite sides of a parallelogram are equal
   $x° = 17°$ because 4x=68° adjacent angles are supplementary
   $y° = 14°$ because $3y° \; and \; 42°$ are alternate angles

Part 4C

1. $x° = 31°$
2. $x = 9, y = 10.5$
3. $x = 15, y = 7$

Lesson 11: Parallelograms, Trapezoids and Kites

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –3rd Quarter*
Content Section: Geometry
Learning Competency (**Code**): M9GE-IIIe-1
Content Standard:
The learners demonstrate understanding of key concepts of parallelograms and triangle similarity

Performance Standard
The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation

Most Essential Learning Competency (MELC/s): as above

Key Idea

To solves problems involving Parallelograms, Trapezoids and Kites

Purpose

The purpose of this lesson is to have the knowledge and understanding to solve problems that involve parallelograms, trapezoids and kites.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. Check that what students have written is correct for each shape.
2. a).$200m^2$
   b). $30m^2$
   c). $48m^2$
   d). $126m^2$

Component 4

Part 4B

1. a). $12m^2$ or 120 000cm^2

   b). $20cm^2$

   c). 6000

Part 4C

The shape of the lampshade is a trapezium when it is opened out. The calculated area of the shade is $4317.5cm^2 = 0.43175m^2$

Jane will have enough material

Lesson 12 Deliberate Practice: Patterns and Algebra, Quadratic equations, Indices, Radical Expressions, Parallelograms, Trapezoids and Kites

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –1st, 2nd and 3rd Quarter*

Content Section:
Patterns and Algebra

Content Standard:
The learners demonstrate understanding of quadratic equations, inequalities and functions, and rational algebraic equations
The learners demonstrate understanding of key concepts of variation and radicals.
The learners demonstrate understanding of key concepts of parallelograms and triangle similarity

Performance Standard:
The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies

The learner is able to formulate and solve accurately problems involving radicals

The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation

Most Essential Learning Competency (MELC/s): M9AL-Ij-1, M9AL-Il-e-1, M9AL-IIh-1, 1M9GE-IIle1 M9GE-IIib-1

Key Ideas
Review the learning in the lessons in Week 2

Purpose
The lesson is about ensuring the skills learnt during the week will be consolidated in this review lesson

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $y = (x-3)^2 - 1$

2. Concave up means a parabola faces upwards and has a minimum value and concave down means the parabola faces downwards and has a maximum value

3.

i) a^{30}

ii) x^{17}

iii) m

iv) 5

v) $6\sqrt{2}$

vi) $15\sqrt{2} - 10\sqrt{10}$

vii) $\frac{2\sqrt{3}}{3}$

4. $x = 16°$

Component 4
Part 4B

a = 20°, $b = 12°$*,* $x = 7cm, y = 6cm$

Part 4C

1. $24\sqrt{6} - 34$
2. 61
3. $48 - 24\sqrt{3}$
4. $\frac{5\sqrt{15}}{18}$
5. $\frac{\sqrt{3}-3\sqrt{2}}{-5}$

Lesson 13: Pythagorean Theorem

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –3rd Quarter*

Content Section: Geometry

Learning Competency (**Code**): M9GE-IIIe-1

Content Standard:
The learners demonstrate understanding of key concepts of parallelograms and triangle similarity

Performance Standard
The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation

Most Essential Learning Competency (MELC/s): as above

Key Idea

Investigates problems using the Pythagorean Theorem.

Purpose

The purpose of this lesson is to understand the application of the Pythagorean Theorem in real life problems.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. *The square on the hypotenuse is equal to the sum of the squares on the other two sides.*

2. *There will be a variety of answers. Check that they are correct*
3. *a)* $x = 13cm$
   b) $x = 6cm$

Component 4
Part 4B

i) *Check the diagrams to ensure they are correct*
ii) *Check the diagrams to ensure they are correct*
iii) The statue is 13.5 m tall

Part 4C

The tree's height is 16 m so it is long enough to go across the river.

Lesson 14: Trigonometric Ratios

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –4th Quarter*

Content Section: Geometry

Learning Competency (**Code**): M9GE-IVa-1

Content Standard:

The learner demonstrates understanding of the basic concepts of trigonometry

Performance Standard

The learner is able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy

Most Essential Learning Competency (MELC/s): as above

Key Idea

Uses trigonometric ratios

Purpose

This lesson is about using the trigonometric functions to solve problems, expanding on previous lessons on mastering the trigonometric ratios. This is an important process that helps us understand more about the use of trigonometry. Trigonometry and its functions have a large number of uses in our daily life. For instance, it is used in building and construction, in GPS to measure the distance between places, in astronomy to measure the distance of nearby stars and also in the satellite navigation system.

Students will find this skill invaluable as they student the later topics in the review of the Mathematics in Year 9. As students become more confident in their use of the trigonometric ratios, their ability to engage with more challenging problems will become less daunting.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. *a.0.515*

   b. 0.731

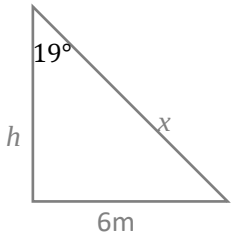
   c. 3.271

2. *a.* $20°$

   b. $44°$

   3. In a right-angled triangle, if one of the angles is $45°$*, then the other angle must also be* $45°$*,as the three angles have a sum of* $180°$*. Therefore, the triangle is isosceles and 2 the 2 sides other than the hypotenuse are equal. So, the 'opposite' side and the 'adjacent' side are equal and so* $sin45° = cos45°$*.*

Component 4
Part 4B



a. $sin19° = \frac{6}{x}$

$$x = \frac{6}{sin19°}$$

$$x = 18.43$$

Therefore, the rope is 18.43m long

b. $tan\ tan\ 19° = \frac{6}{h}$

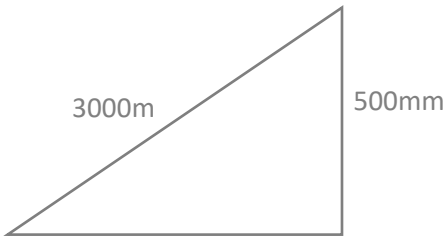
$$h = \frac{6}{tan19°}$$

$$h = 17.43m$$

Therefore, the window is 17.43m high.

Part 4C

Let Ø be the angle between the flight path and the horizontal.



$$sin\ Ø = \frac{500}{3000}$$

$$= 0.16666666$$

$$Ø = 9.6°$$

Therefore, the angle between the ground and the flight path is $9.6°$.
The teacher should discuss whether this is an appropriate answer.

Lesson 15: Similar Triangles

Curriculum references

Solves problems involving similar triangles

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*

Grade – Quarter: *Grade 9 –3rd Quarter*

Content Section: Geometry

Learning Competency (**Code**): M9GE-IIIg-h-1

Content Standard:
The learner demonstrates understanding of key concepts of parallelograms and triangle similarity

Performance Standard:

The learner is able to investigate, analyze, and solve problems and triangle similarity through appropriate and accurate representation

Most Essential Learning Competency (MELC/s): as above

Key Idea

Solves problems involving similar triangles

Purpose

The purpose of this lesson is to apply your knowledge of similar triangles to solve problems

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $a)$ 12.5
   b) x=5

2. *SAS*

Component 4

Part 4B

The height of the palm tree is 11.2 meters.

Part 4C

The river is 39.2 paces wide, which is 31.36m wide.

Lesson 16: Angles of Elevation and Depression

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –3rd Quarter*
Content Section: Geometry
Learning Competency (**Code**): M9GE-Ivd-1

Content Standard:
The learner demonstrates understanding of key concepts of trigonometry

Performance Standard:
The learner is able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy

Most Essential Learning Competency (MELC/s): as above

Key Idea

Apply knowledge of trigonometry and angles of elevation and depression to solve problems.

Purpose

The lesson is about learning how to trigonometry to solve real life problems involving angels of elevation and depression.

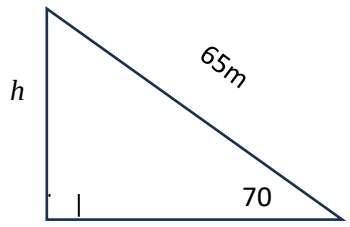
Worked Answers to Component 1 and Component 4 Questions

Component 1

1. *Check markings on the diagram.*
   Line of sight is the hypotenuse of the right-angled triangle.
   The angle of depressions and elevation are alternate angles formed when a transversal crosses a pair of parallel lines. The horizontal line where the angle of depression is formed is parallel to the base of the triangle.

2a). Check the pictures to ensure they represent the information correctly.

2b).



2c). $Height\ is\ 61\ m$

Part 4B

1. *Check the pictures to show the angle of depression in the correct position.*
2. *Check that the diagram is correct. The student could use* $28°$*or* $62°$ *as long as they are in the correct position*
3. *2556 m*

Part 4C

The diagrams drawn here must include the correct information in the correct position. The students may use complementary angles with the correct trigonometric ratio.

The ships are approximately 1944m apart

Lesson 17: Solves problems involving Trigonometric Ratios

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 –3rd Quarter*
Content Section: Geometry
Learning Competency (**Code**): M9GE-IVe-1

Content Standard:
The learner demonstrates understanding of key concepts of trigonometry

Performance Standard:
The learner is able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy

Most Essential Learning Competency (MELC/s): as above

Key Idea

Demonstrates knowledge and understanding of trigonometry to apply skills to solve real world problems

Purpose

In this lesson, students will apply their knowledge of trigonometry to solve real world problems.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. $sin\emptyset = \frac{a}{c}$
   $cos\emptyset = \frac{b}{c}$
   $tan\emptyset = \frac{a}{b}$

2. *h=2.82m*

Component 4

Part 4B

Q1.

a) *Diagrams could be varied. Check to ensure the information is in the correct position.*
b) Most student will use the tan ratio.
c) 1.75 m

Q2.

a) *Diagrams could be varied. Check to ensure the information is in the correct position.*
b) Sin ratio
c) 34.87m

Part 4C

81°

Lesson 18 Deliberate Practice: Using Pythagoras, trigonometry and triangles to solve problems

Curriculum references

Curriculum: *K to 12 Mathematics Curriculum Guide (Grade 3 to Grade 10) August 2016*
Grade – Quarter: *Grade 9 3rd and 4th Quarter*
Content Section:
Geometry
Content Standard:

The learners demonstrate understanding of key concepts of parallelograms and triangle similarity

The learner demonstrates understanding of the basic concepts of trigonometry

Performance Standard:

The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation.

The learner is able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy

Most Essential Learning Competency (MELC/s): M9GE-IIIe-1, M9GE-IVa-1, M9GE-IIIg-h-1, M9GE-Ivd-1, M9GE-IVe-1

Key Ideas

Review the learning from the third week

Purpose

Review the learning for the last week of the camp.

Worked Answers to Component 1 and Component 4 Questions

Component 1

1. The square on the hypotenuse is equal to the sum of the squares on the other two sides

2. i) 4cm *ii)* $\sqrt{74} \approx 8.6$ *cm*

3. x = 3

4. i) 24° *ii) 35.34*

Component 4

Part 4B
Diagram

59.3 m

Part 4C
356.8m

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Delivery (DepEd-BLD)
Contact Numbers: 8637-4366; 8637-4347; 8633-9347

Department of Education - Bureau of Learning Resources (DepEd-BLR)
Contact Numbers: 8634-1072; 8631-6922

Email Address: blr.od@deped.gov.ph

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