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UNIFIED SUPPLEMENTARY LEARNING MATERIALS
(USLeM)

MATHEMATICS

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UNIFIED SUPPLEMENTARY LEARNING MATERIALS
Grade 6 MATHEMATICS

LESSON 1: DETERMINE THE RELATIONSHIP BETWEEN THE VOLUME OF SOLID FIGURES (M6ME-IVa-95)

EXPECTATIONS

Specifically, this module will help you to:

- Determine the relationship between the volumes of:
  - a rectangular prism and a pyramid
  - a cylinder and a cone
  - a cylinder and a sphere

PRE-TEST

Directions: Choose the letter of your answer. Write your answer on the space provided before each number.

_____1. Which solid figure is related to a rectangular prism with regard to its volume?
A. Cone B. Cylinder C. Pyramid D. Sphere

_____2. What is the relationship of the volume of the pyramid to the volume of prism?
The volume of a pyramid is _____ the volume of a prism with equal base area and height.
A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{5}$

_____ 3. Which statement is true about the volumes of a cylinder and a cone with equal diameters and heights?
A. Their volumes are equal.
B. Their surface areas are equal.
C. The volume of a cylinder is twice the volume of a cone.
D. The volume of a cone is three times the volume of a cylinder.

_____ 4. How is the volume of a cylinder compared to the volume of a cone?
A. The cone's volume is exactly one-third $\left(\frac{1}{3}\right)$ the cylinder's volume.
B. The cone's volume is equal to the cylinder's volume.
C. The cylinder's volume is half $\left(\frac{1}{2}\right)$ the cone's volume.
D. The cylinder's volume is two-thirds $\left(\frac{2}{3}\right)$ the cone's volume. .

_____5. Which statement tells the relationship between the volumes of sphere and cylinder with equal radius and height?
A. The sphere fills the whole cylinder.
B. Both solid figures have equal volumes.
C. Sphere's volume is only $\frac{1}{3}$ the cylinder's volume.
D. The volume of the sphere is $\frac{2}{3}$ the volume of the cylinder.

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LOOKING BACK

DIRECTIONS: Match the solid figures in column A with the formula in finding the surface area in column B. Write the letter of the correct answer.

A	B
_____ 1. Rectangular Prism	A. $SA = 4\pi r^2$
_____ 2. Square Pyramid	B. $SA = 2(l \times h) + 2(w \times h) + 2(l \times w)$
_____ 3. Cylinder	C. $SA = \pi r^2 + \pi rs$
_____ 4. Cone	D. $SA = 2(b \times h) + s^2$
_____ 5. Sphere	E. $SA = 2\pi r^2 + 2\pi rh$

BRIEF INTRODUCTION

The amount of space an object occupies is called the **Volume**. The more space an object occupies, the greater is its volume (Cruz et al., 2017).

To measure the volume, we use a unit cube such as cubic centimeters (cm^3) or cubic meters (m^3). A ***unit cube*** is a cube having 1 unit on each  side.

Relationship between the Volumes of Rectangular Prism and Pyramid

To measure volume means to find how many cubic units it will take to fill the solid figure.

If a rectangular prism is 8 units long, 3 units wide and 5 units high, how many cubes could fit in it? (See Figure 1 and 2)

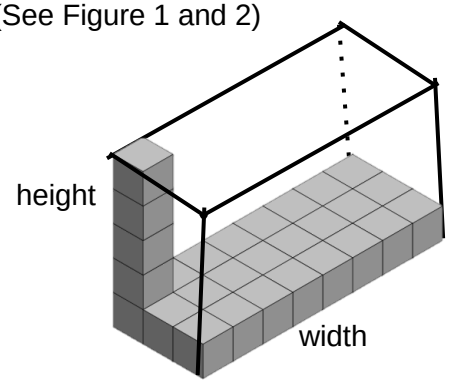


Figure 1

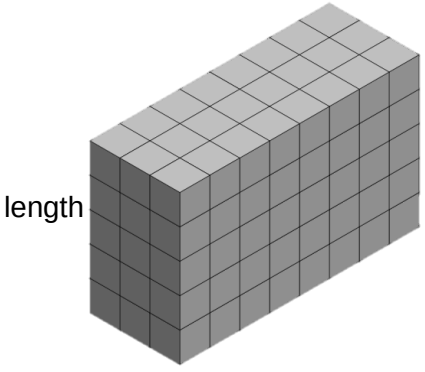


Figure 2

$V_{\text{rectangular prism}} = Bh$

- To get the **area of the base (B)**, multiply the length and the width (l x w)
  = (8 unit x 3 unit) x 5 unit
  = **120 cubic units**

Thus, 120 cubic units can fit in the rectangular prism. So the **volume of the prism ($V_{\text{rectangular prism}} = Bh$)** is the product of the base and the height, where B is the area of the base or the product of the length, and width multiplied by the height (V=l x w x h).

The volume of a pyramid is closely related to the volume of a prism. A prism can be divided into 3 pyramids as shown below.

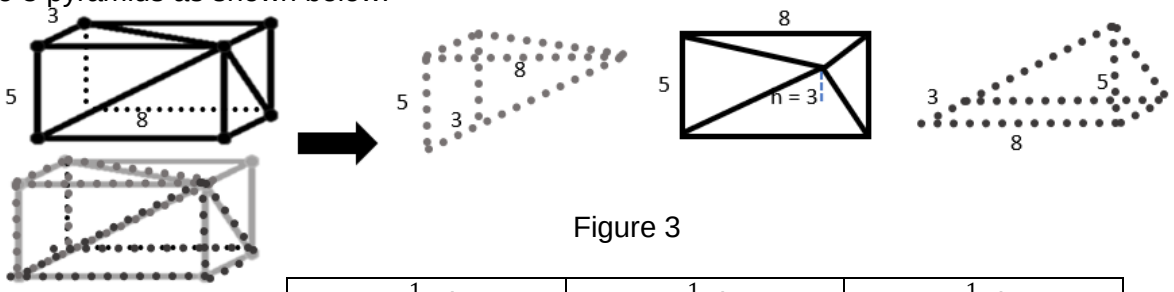


Figure 3

$V_{\text{pyramid}} = \frac{1}{3}$ Bh $=\frac{1}{3}[(5 \times 3) \times 8]a$ $=\frac{1}{3}(120)$ **= 40 cubic units**	$V_{\text{pyramid}} = \frac{1}{3}$Bh $=\frac{1}{3}[(8 \times 5) \times 3]$ $=\frac{1}{3}(120)$ **= 40 cubic units**	$V_{\text{pyramid}} = \frac{1}{3}$Bh $=\frac{1}{3}[(8 \times 3) \times 5]$ $=\frac{1}{3}(120)$ **= 40 cubic units**

Though the three pyramids have different dimensions, we can say that they have the same volume as shown in the computation. Therefore, the **volume of each pyramid is $\frac{1}{3}$ the volume of the prism**. The formula is **$V_{\text{pyramid}} = \frac{1}{3}$ Bh**, where B is the area of the base.

Relationship between the Volumes of Cylinder and Cone

The relationship between the volumes of a pyramid and a prism is similar to the relationship between the volumes of a cylinder and a cone.

The volume of a cylinder is given by the product of the area of its circular base and its height. That is,

$V_{\text{cylinder}} = A_{\text{circle}} \times h$
$= \pi r^2 \times h$
$= \pi r^2 h$

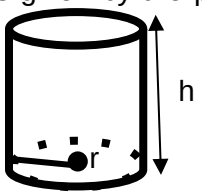


Figure 4

The volume of a cone can be determined based on the volume of a cylinder. We use a cylinder and a cone whose heights and radii are congruent. If you fill the cone with liquid and pour it into the cylinder, how many cones are needed to completely fill the cylinder? (see figure 5).

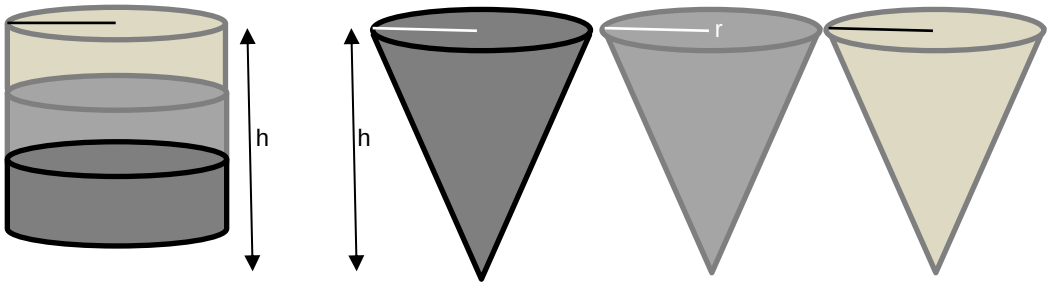


Figure 5

As shown in Figure 5, three cones of liquid will completely fill the cylinder. This means that the **volume of the cylinder is equal to three times the volume of the cone. Thus, the volume of the cone is one-third the volume of the cylinder.** The volume of the cylinder is $V = \pi r^2 h$, and the volume of the cone is $V = \frac{1}{3} \pi r^2 h$.

Relationship between the Volumes of Cylinder and Sphere

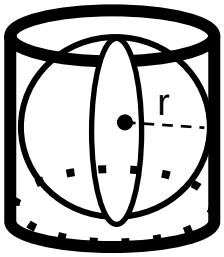
How is the volume of a sphere related to the volume of a cylinder?

Let us investigate.

A sphere is placed inside a cylinder of the same radius.

The cylinder is then filled with water. What happens to the level of the water when the sphere is taken out?

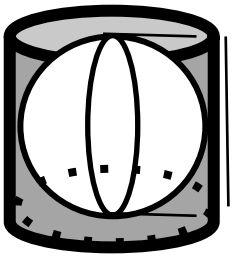
Step 1:



A sphere is placed inside a cylinder with height twice the radius or equal to the diameter of the sphere.

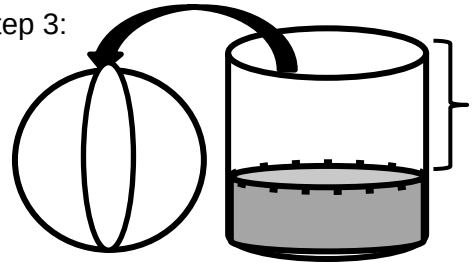
Notice how the sphere occupies a fraction of the cylinder.

Step 2:



The cylinder is filled with water.

Step 3:



of the volume of the cylinder

Removing the sphere from the cylinder leaves the level of the water up to of the volume of the cylinder. This shows us that the **sphere takes up of the volume of the cylinder.**

Volume $_{sphere} = \frac{2}{3}$ volume of cylinder

$= \frac{2}{3} Bh$

$= \frac{2}{3} \pi r^2 h$ (replacing B with πr^2)

$= \frac{2}{3} \pi r^2 \times 2r$ (replacing h with 2r)

$= \frac{4}{3} \pi r^3$ (simplifying)

Volume $_{sphere} = \frac{4}{3} \pi r^3$

ACTIVITIES

Activity 1: Are We Related?

DIRECTIONS: Fill in the blanks to make the relationship true.

1. $V_{rectangular\ prism} = Bh$
   $V_{pyramid} = \frac{1}{3} Bh$
   $V_{pyramid} =$ _____ $V_{rectangular\ prism}$

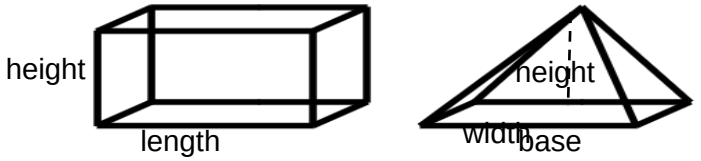
2. $V_{cylinder} = \pi r^2 h$
   $V_{cone} = \frac{1}{3} \pi r^2 h$
   $V_{cone} =$_____ $V_{cylinder}$

3. $V_{cylinder} = \pi r^2 h$
   $V_{sphere} = \frac{4}{3} \pi r^3$
   $V_{sphere} =$ ____ $V_{cylinder}$

Activity 2: The Wonders of Volume

DIRECTIONS: Using the illustrations below, determine the relationship between the volumes of the two given solid figures. Put a check (/) on the blank if the sentence states the correct relationship.

1.



_____ The volume of the prism depends on the volume of the pyramid.

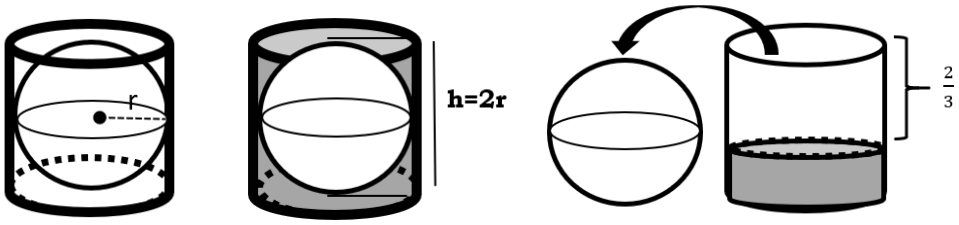
_____ Pyramid's volume is $\frac{1}{3}$ the volume of the prism.

2.

_____ The volume of the cylinder equals three times the volume of the cone.

_____ The volume of the cone is only $\frac{1}{2}$ the volume of the cylinder.

3.



_____ Sphere occupies $\frac{2}{3}$ the volume of the cylinder.

_____ Placing the sphere in the cylinder makes their volumes the same.

Activity 3: Complete Me

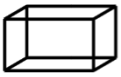
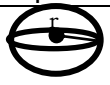
A. Choose the correct word/phrase inside the box to complete and make each statement true.

three	cone	volume	$\frac{1}{3}$
area of the base	$\frac{2}{3}$	$\frac{4}{3}\pi r^3$	pyramid

1. The volume of a _____ is related to the volume of a prism.
2. Though the three pyramids are not congruent to each other, we can say that they have the same _____.
3. The volume of the prism is _____ times the volume of the pyramid.
4. The formula for the volume of pyramid is V = $\frac{1}{3}$ Bh, where B is the _____.
5. The volume of the cone is _____ the volume of the cylinder.
6. The volume of the sphere is _____ the volume of the cylinder.
7. The formula to find the volume of the sphere is V = _____.

REMEMBER

- Volume is the measure of space a solid figure occupies.
- Volume is measured in cubic units (cm^3, m^3).
- The volume of each pyramid is $\frac{1}{3}$ the volume of the prism.
- The volume of a cone can be determined from the volume of a cylinder. The volume of the cone is one-third the volume of the cylinder.
- The volume of the sphere is two-thirds the volume of a cylinder.
- Here are the formula to find the volume of different solids:

Prism	Pyramid	Cylinder	Cone	Sphere
				
$V_{prism} = Bh$	$V_{pyramid} = \frac{1}{3}Bh$	$V_{cylinder} = \pi r^2 h$	$V_{cone} = \frac{1}{3}\pi r^2 h$	$V_{cone} = \frac{4}{3}\pi r^3$

CHECKING YOUR UNDERSTANDING

DIRECTIONS: Write TRUE if the statement is correct. If it is false, change the underlined word/s to make it true.

_____1. The volume of the prism is <u>three times</u> the volume of the pyramid.
_____2. The volume of a pyramid is <u>one-third</u> the volume of the prism.
_____3. The volume of a cone <u>cannot be</u> determined from the volume of a cylinder.
_____4. The total volume of the cylinder equals <u>four times</u> the volume of the cone.
_____5. The volume of the sphere is <u>twice</u> the volume of the cylinder.

POST-TEST

DIRECTIONS: Write the letter of your answer on the space provided before each number.

_____1. How are the volumes of prism and pyramid related?

A. Prism and pyramid have the same volume.
B. The volume of the prism is three times the volume of the pyramid.
C. The volume of the pyramid is three times the volume of the prism.
D. There is no difference between the volumes of both prism and pyramid.

_____2. Which sentence tells the relationship between the volumes of cone and cylinder?

I. They have the same bases.
II. Their heights and radii are congruent.
III. The volume of the cylinder equals three times the volume of the cone.

A. I only
B. II only
C. III only
D. I, II and III

_____ 3. How are the volumes of cylinder and cone related?

A. Their volumes are equal.
B. Their volumes are not the same.
C. The volume of the cylinder is twice as the volume of the cone.
D. The volume of the cylinder equals three times the volume of the cone.

_____ 4. How is the volume of a sphere compared to the volume of a cylinder?

The volume of the sphere is _____ the volume of a cylinder.

A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. $\frac{3}{4}$
D. $\frac{4}{3}$

_____5. Which of the following statements is NOT true?

A. The volume of the sphere is two-thirds the volume of the cylinder.
B. The volume of the pyramid is one-third the volume of the prism.
C. The volume of a cone is one-half the volume of a cylinder.
D. The volume of a cylinder is three times the volume of a cone.

KEY TO CORRECTION

PRE-

ACTIVITY 1 – We are Match

	LOOKING BACK	
A	4. C	
D	3. E	3.
B	2. D	2.
C	1. B	1.

TEST

ACTIVITY 2: THE WONDERS OF VOLUME

1. The volume of the cylinder equals three times the volume of the base
2. Pyramid's volume is of the volume of the prism.

ACTIVITY 3: COMPLETE ME

1. pyramid
2. volume
3. three
4. area of
5. ____

CHECKING YOUR UNDERSTANDING

1. True
2. True
3. can be
4. three
5. times/thrice

POST TEST

1. B 2. D 3. D 4. B 5. C

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