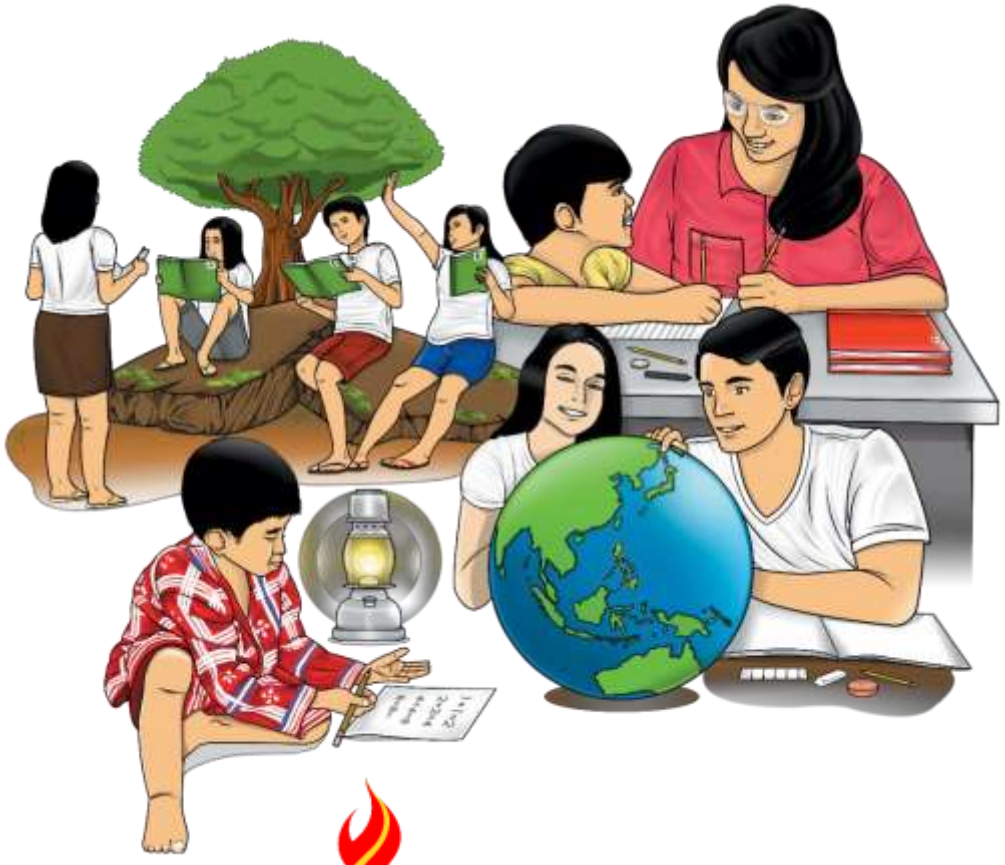
8

DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1– Module 1B
Factoring Perfect Square Trinomials and General Trinomials



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 8
Alternative Delivery Mode
Quarter 1 – Module 1B Factoring Perfect Square Trinomials and General Trinomials
First Edition, 2020

Published by the Department of Education
Secretary: Leonor M. Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers:	Lee C. Apas**,** Clint R. Orcejola
Language Editor:	Merjorie G. Dalagan
Content Evaluator:	Isabelita R. Lindo
Layout Evaluator:	Jake D. Fraga
Reviewers:	Rhea J. Yparraguirre, Nilo B. Montaño, Lilibeth S. Apat, Liwayway J. Lubang, Rhodora C. Luga, Lee C. Apas, Jenny O. Pendica, Vincent Butch S. Embolode, Emmanuel S. Saga
Illustrator:	Fritch A. Paronda
Layout Artist:	Clint R. Orcejola, Fritch A. Paronda
Management Team:	Francis Cesar B. Bringas, Isidro M. Biol, Jr., Maripaz F. Magno, Josephine Chonie M. Obseñares, Josita B. Carmen, Celsa A. Casa, Regina Euann A. Puerto, Bryan L. Arreo, Lieu Gee Keeshia C. Guillen, Leopardo P. Cortes, Jr., Claire Ann P. Gonzaga

Printed in the Philippines by _____

Department of Education – Caraga Region

Office Address: Learning Resource Management Section (LRMS)
J.P. Rosales Avenue, Butuan City, Philippines 8600
Telefax: (085) 342-8207 / (085) 342-5969
E-mail Address: caraga@deped.gov.ph

8

Mathematics

Quarter 1 – Module 1B

"Factoring Perfect Square Trinomials and General Trinomials"

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module is designed and written to help you factor polynomials completely using different techniques. In all lessons, you are given the opportunity to use your prior knowledge and skills in multiplying and dividing polynomials. Activities are also given to process your knowledge and skills acquired, deepen and transfer your understanding of the different lessons. The scope of this module enables you to use it in many different learning situations. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains the following lessons:

Lesson 1: Factoring Perfect Square Trinomials

Lesson 2: Factoring General Trinomials

After going through this module, you are expected to:

1. determine patterns in factoring polynomials; and
2. factor perfect square trinomials and general trinomials completely.

What I Know

Directions: Choose the letter of the correct answer. Write your answers on a separate sheet of paper.

1. What is the result when you square a binomial?
   A. binomial
   B. sum of two squares
   C. difference of two squares
   D. perfect square trinomial

For items 2 – 4. Expand $(2x - 4)^2$and answer what is asked.

2. What is the last term?
   A. $4x^2$ B. -16 C. 16 D. $-4x^2$

3. What is the middle term?
   A. 8x B. -8x C. 16x D. -16x

4. What is the first term?
   A. $4x^2$ B. $-4x^2$ C. $2x^2$ D. $-2x^2$

5. Which of the following is the expanded form of $(x - 5)^2$?
   A. $x^2 - 25$
   B. B. $x^2 - 5x - 25$
   C. $x^2 - 10x + 25$
   D. $x^2 - 10x + 25$

6. Which of the following is a perfect square trinomial?
   A. $x^2 - 2x + 4$
   B. $4x^2 + 12x + 9$
   C. $x^2 - 4y + 2y^2$
   D. $9x^2 - 6xy - y^2$

7. All of the following are perfect square trinomials EXCEPT one. Which is it?
   A. $4x^2 + 12xy + 9y^2$
   B. $9a^2 - 36a + 36$
   C. $x^2y^2 - 4x^2y^2 + 4x^2y^2$
   D. $a^2b^2 + 4a^3b + 4a^2$

8. What is the complete factored form of $4x^2 + 16xy + 16y^2$?
   A. $(2x + 4y)^2$
   B. $(2x + 4y)(2x - 4y)$
   C. $(2x - 4y)^2$
   D. $(2x - 4y)(2x - 4y)$

9. What are the two numbers whose sum is 1 and whose product is -12?
   A. 3 and -4
   B. B. -3 and 4
   C. -3 and -4
   D. 3 and 4

10. What are the two expressions whose sum is 7x and whose product is $10x^2$?
    A. 2x and 5x
    B. B. -2x and 5x
    C. 2x and -5x
    D. -2x and – 5x

11. If one factor of $x^2 - 5x - 24$ is $x + 3$, what is the other factor?

A. $x - 3$ C. $x + 8$

B. $x - 8$ D. $2x - 8$

12. What is the complete factored form of $x^2 + 7x + 10$?

A. $(x + 2)(x + 5)$ C. $(x + 1)(x + 10)$

B. $(x - 2)(x + 5)$ D. $(x - 1)(x + 10)$

13. If one factor of $3x^2 + 17x + 10$ is $3x + 2$, what is the other factor?

A. $x - 5$ C. $3x - 2$

B. $x + 5$ D. $2x + 5$

14. What is the complete factored form of $2x^2 + 5x - 3$?

A. $(2x - 1)(x - 3)$ C. $(2x - 1)(x + 3)$

B. $(2x + 1)(x + 3)$ D. $(2x + 1)(x - 3)$

15. The rectangle has an area of $(2x^2 + 8x + 8)$ square units. If the length is $(2x + 4)$ $units$, find the width of the rectangle.

A. $(x + 2)units$ C. $(-x - 2)units$

B. $(x - 2)units$ D. $(-x + 2)units$

Lesson 1 Factoring Perfect Square Trinomials

Another factoring technique that you are going to explore is factoring perfect square trinomials. Before you will start learning this topic, recall the pattern to square a binomial as this is very important in understanding this factoring technique. Do the following activity to refresh your learning in squaring a binomial.

What's In

Patterns in squaring binomial

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$

Activity: Remember Me!

Directions: Following the pattern in squaring a binomial, fill-in the missing term. Write your answer on your answer sheet.

1. $(x + 2)^2 = \_\_\_ + 4x + 4$
2. $(2x - 3)^2 = 4x^2 - \_\_\_\_ + 9$
3. $(3x + 4)^2 = 9x^2 + 24x + \_\_\_\_$
4. $(x - y)^2 = \_\_\_ - 2xy + y^2$
5. $(2x + 3y)^2 = 4x^2 + \_\_\_\_ + 9y^2$

Questions:

1. What did you do to find the first term of the product? the second term? the last term?
2. How will you determine the sign of the middle term?
3. What do you call the product of squaring a binomial?

What's New

Recall that squaring a binomial is a perfect square trinomial. Say, $(2x + 1)^2 = 4x^2 + 4x + 1$ thus, $4x^2 + 4x + 1$ is a perfect square trinomial. The following activity will test your ability in determining perfect square trinomial.

Activity: Am I Perfect?

Directions: Determine whether the given expressions are perfect square trinomials. Write P if it is a perfect square trinomial and N if not. Write your answer on your answer sheet.

1. $a^2 - 22a + 121$
2. $b^2 - 8b + 16$
3. $4d^2 + 20d - 25$
4. $100 + 50e + e^2$
5. $36r^2 - 60rt + 25t^2$

Questions:

1. How did you determine whether the given expression is a perfect square trinomial?
2. Did you encounter difficulties in determining it?
3. Do you see pattern in determining perfect square trinomials?
4. What are your observations on the terms of a perfect square trinomial?

What is It

Perfect Square Trinomial is the result of squaring a binomial. A perfect square trinomial has first and last terms which are perfect squares and the middle term is twice the product of the first and last terms.

That is,

$(a + b)^2 = a^2 + 2ab + b^2$ or

$(a - b)^2 = a^2 - 2ab + b^2$

To factor the given trinomial,

1. Examine whether the first term and last term are perfect squares.
2. Look at the middle term. Check whether it is twice the product of the square root of the first term and last term.
3. If conditions 1 and 2 were satisfied then, the expression is a perfect square trinomial.
4. Factor completely the given trinomial following the pattern $a^2 + 2ab + b^2 = (a + b)^2$ or $(a + b)(a + b)$. Similarly, $a^2 - 2ab + b^2 = (a - b)^2$ or $(a - b)(a - b)$.

The steps given above are applicable for perfect square trinomial. If what is to be factored is not a perfect square trinomial then other possible techniques may be applied.

To fully understand the process, consider the following examples.

Example 1: Factor $n^2 + 16n + 64$

Solution:

Step 1: Determine whether the first term and the last term are perfect squares.

First Term: $n^2 = n \cdot n = (n)^2$

Last Term: $64 = 8 \cdot 8 = (8)^2$

Both are perfect squares

Step 2: Determine whether the middle term is twice the product of the square root of the first term and the last term.

$$16n = 2(n)(8)$$

Step 3: Since the conditions are satisfied then $n^2 + 16n + 64$ is a perfect square trinomial.

Step 4: Factor completely $n^2 + 16n + 64$

$$n^2 + 16n + 64 = (n + 8)^2 \text{ or } (n + 8)(n + 8)$$

To factor perfect square trinomial, use the following relationships:

$$(a)^2 \quad 2ab \quad (b)^2 \quad = \quad (a + b)^2$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \qquad \downarrow \qquad \downarrow$$

$$(\text{First term})^2 + 2(\text{First term})(\text{Last term}) + (\text{Last term})^2 = (\text{First term} + \text{Last term})^2$$

$$(a)^2 \quad 2ab \quad (b)^2 \quad = \quad (a - b)^2$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \qquad \downarrow \qquad \downarrow$$

$$(\text{First term})^2 - 2(\text{First term})(\text{Last term}) + (\text{Last term})^2 = (\text{First term} - \text{Last term})^2$$

Example 2: Factor $4x^2 + 4x + 1$.

Solution:

Step 1: Determine whether the first term and the last term are perfect squares.

First Term: $4x^2 = 2x \cdot 2x = 4x^2$

Last Term: $1 = 1 \cdot 1 = 1^2$

Both are perfect squares

Step 2: Determine whether the middle term is twice the product of the square root of the first term and the last term.

$$4x = 2(2x)(1)$$

Step 3: Since the conditions are satisfied then $4x^2 + 4x + 1$ is a perfect square trinomial.

Step 4: Factor completely $4x^2 + 4x + 1$ you have,

$$4x^2 + 4x + 1 = (2x + 1)^2 \text{ or } (2x + 1)(2x + 1)$$

Example 3: Factor $x^2 + 14xy + 49y^2$

Solution:

Step 1: Determine whether the first term and the last term are perfect squares.

First Term: $x^2 = x \cdot x = (x)^2$

Last Term: $49y^2 = 7y \cdot 7y = (7y)^2$

Both are perfect squares

Step 2: Determine whether the middle term is twice the product of the square root of the first term and the last term.

$$14xy = 2(x)(7y)$$

Step 3: Since the conditions are satisfied then $x^2 + 14xy + 49y^2$ is a perfect square trinomial.

Step 4: Factor completely $x^2 + 14xy + 49y^2$ you have,

$$x^2 + 14xy + 49y^2 = (x + 7y)^2 \text{ or } (x + 7y)(x + 7y)$$

There are some cases in which you need to factor out first the **greatest common monomial factor** before factoring the perfect square trinomial. To fully understand this, take the following examples.

Example 4: Factor $3x^2 - 18xy + 27y^2$

Solution:

At first glance, we can't find the perfect square trinomial in it. But if we factor out its greatest common monomial factor, like the following:

Step 1: Factor $3x^2 - 18xy + 27y^2$ by GCF.

$$3x^2 - 18xy + 27y^2 = 3(x^2 - 6xy + 9y^2)$$

Step 2: Determine whether the trinomial is a perfect square. The first term and the last term should be perfect squares.

First Term: $x^2 = x \cdot x = (x)^2$

Last Term: $9y^2 = 3y \cdot 3y = (3y)^2$

Both are perfect squares

Step 3: Determine whether the middle term is twice the product of the square root of the first term and the last term.

$$-6xy = -2(x)(3y)$$

Step 4: Since the conditions are satisfied then $x^2 - 6xy + 9y^2$ is a perfect square trinomial.

Step 5: Factor completely $3x^2 - 18xy + 27y^2$ you have,

$$3x^2 - 18xy + 27y^2 = 3(x - 3y)^2 \text{or } 3(x - 3y)(x - 3y)$$

Example 5: Factor $75t^3 + 30t + 3t$

Solution:

Step 1: Factor $75t^3 + 30t + 3t$ by GCF.

$$75t^3 + 30t + 3t = 3t(25t^2 + 10t + 1)$$

Step 2: Determine whether the trinomial is a perfect square. The first term and the last term should be perfect squares.

First Term: $25t^2 = 5t \cdot 5t = (5t)^2$

Last Term: $1 = 1 \cdot 1 = (1)^2$

Both are perfect squares

Step 3: Determine whether the middle term is twice the product of the square root of the first term and the last term.

$$10t = 2(5t)(1)$$

Step 4: Since the conditions are satisfied then $25t^2 + 10t + 1$ is a perfect square trinomial.

Step 5: Factor completely $75t^3 + 30t + 3t$ you have,

$$75t^3 + 30t + 3t = 3t(5t + 1)(5t + 1)$$

Example 6: Factor $9x^2 + 12xy + 16y^2$.

Step 1: Determine whether the trinomial is a perfect square. The first term and the last term should be perfect squares.

First Term: $9x^2 = 3x \cdot 3x = (3x)^2$

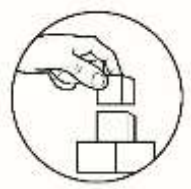
Last Term: $16y^2 = 4y \cdot 4y = (4y)^2$

Both are perfect squares

Step 2: Determine whether the middle term is twice the product of the square root of the first term and the last term.

$12xy \neq 2(3x)(4y)$ **,** as $2(3x)(4y) = 24xy$

This means that the trinomial is not a perfect square. Thus, you don't have to proceed to factoring.



What's More

Activity 1: You're the One

Directions: Supply the missing term of the factor of the given perfect trinomials below. Write your answers on a separate sheet of paper.

1. $x^2 - 6x + 9 = (x - \_\_)^2$
2. $4x^2 - 4x + 1 = (\_\_\_ - 1)^2$
3. $9x^2 + 12x + 4 = (3x + \_\_\_)^2$
4. $4x^2 + 16xy + 16y^2 = (2x + \_\_\_)^2$
5. $16a^2 - 24ab + 9b^2 = (\_\_\_ - 3b)^2$

Activity 2: Break it Perfectly

Directions: Factor the following completely by writing each of the perfect square trinomial as the square of a binomial. Write your answers on a separate sheet of paper.

1. $y^2 + 20y + 100$
2. $k^2 - 8k + 16$
3. $16m^2 + 48m + 36$
4. $49b^2 - 14b + 1$
5. $3x^2y - 24xy + 48y$

What I Have Learned

Complete the paragraph below by filling in the blanks with correct word/s or figure/s which you can choose from the box below. Each word or figure may be used repeatedly. Write your answers on your answer sheet.

first term	$a^2 + 2ab + b^2 = (a - b)^2$	*third term*	*squaring a binomial*
multiplied	*two*	*factoring*	*perfect square*
polynomials	*second term*	*square*	$a^2 - 2ab + b^2 = (a - b)^2$
$a^2 + 2ab + b^2 = (a + b)^2$	*Simplifying*	$a^2 - 2ab - b^2 = (a - b)^2$	*technique*

Factoring is an important process that helps us understand more about mathematical expressions or equations. Through **(1)**, you can rewrite your **(2)** in a simpler form, and when you apply the factoring **(3)** to mathematical expressions or equations, it can yield a lot of useful information. You have learned in this lesson about factoring perfect square trinomial. This trinomial is a result of **(4)**. The first term of the trinomial is the square of the **(5)** of the binomial. The second term is the product of the **(6)** and **(7)** of the binomial which will always be **(8)** by **(9)**. The third term of the trinomial is the **(10)** of the **(11)** of the binomial. If the trinomial follows the pattern in squaring a binomial, then it is a **(12)**. To factor this, you should recognize first, whether the **(13)** and **(14)** are **(15)**. After recognizing whether the given trinomial is a perfect square, then you can proceed to factoring following the pattern **(16)** or **(17)**.

What I Can Do

Suppose you have a square room with an area of $4x^2 + 16x + 16$.

1. Find the binomial that represents each side of your room.
2. If the measure of the side of your room is 300 inches, what is the value of x?
3. What is the area of the square?
4. Suppose you want to tile the floor of your room; how many 30 x 30 inches tiles will be used?

Lesson 2 Factoring General Trinomials

In this lesson you will learn how to factor general trinomials. There will be two types of trinomials that you are going to deal with. First is the trinomial in the form $ax^2 + bx + c$, where a = 1, and $ax^2 + bx + c$, where a ≠ 1. Before you will start exploring this lesson, let us first reactivate your basic mathematical skills.

What's In

Activity: The Two of Us

Think of two numbers whose sum and product are given in the table below. Write your answers on your answer sheet. Item 1 is done for you.

Item	Sum	Product	The Two Numbers
1	6	8	2 and 4
2	6	-16	
3	-2	-15	
4	4	-12	
5	6	9	
6	-5	-14	

Notice that in item 1, the numbers 2 and 4 in Column 4 when added will give a sum of 6 and when multiplied will give a product of 8.

Questions:

1. How did you find the two numbers in column 4 to satisfy the conditions in columns 2 (sum) and 3(product)?
2. Was it easy to find the two numbers?
3. What does it mean when the product of the numbers is negative? positive?
4. Did you recognize a pattern or technique on how to find the two numbers given its sum and product? What is it?

What's New

Recall that FOIL method is a technique used to remember the steps in multiplying two binomials. The acronym FOIL stands for:

F - first terms
O – Outer terms
I – Inner terms
L – last terms

Activity: Grilling with F O I L

Directions: Given below are expressions in factored form in which both factors are binomials. Follow the process in multiplying the binomials using FOIL method and answer the questions that follow. Write your answers on your answer sheet.

		F	O	I	L
1. (x + 1)(x + 2)	=	x^2	+ 2x	+ x	+ 2
	=	$x^2 + 3x + 2$			

		F	O	I	L
2. (x – 2)(x -3)	=	x^2	– 3x	– 2x	+ 6
	=	$x^2 - 5x + 6$			

		F	O	I	L
3. (x + 3)(x – 4)	=	x^2	– 4x	+ 3x	– 12
	=	$x^2 - x - 12$			

Questions:

1. What did you observe with the numerical coefficients of the x^2 term?
2. What did you notice about the last terms of the trinomial? How are the last terms of each trinomial related to the last terms of the given binomials?
3. What did you notice with the numerical coefficients of the middle terms? How are the coefficients of the middle term related to the last terms of the given binomials?
4. Suppose a trinomial is given, how are you going to find its two binomials factors?

What is It

General trinomials can be classified into two (2) ways:

1. Trinomial in the form $ax^2 + bx + c$, where a = 1; and
2. Trinomial in the form of $ax^2 + bx + c$, where $a \neq 1$.

In this lesson you will first learn factoring general trinomial where a = 1. The following are some examples of trinomials of the form $ax^2 + bx + c$, where a = 1.

$x^2 + 5x + 6$ $x^2 - 7x + 12$ $x^2 + 2x - 15$

Trinomials of this form are the product of two binomials having leading coefficients of 1. Consider the illustration below where the **FOIL** method is being applied in multiplying two binomials having leading coefficients of 1.

F O I L

$(x + 1)(x + 2) = x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

Sum of 2 and 1

Product of 2 and 1

Notice that the coefficient of the middle term is the sum of the last terms of the two binomials and the third term is the product of the last terms of the two binomials. If you are going to factor trinomials of the form $ax^2 + bx + c$, where a = 1, you will reverse the FOIL method.

These are the steps in factoring this trinomial.

1. Find two factors with a product equal to the last term (c) of the trinomial and a sum equal to the middle term (b) of the trinomial.

2. Write the factored form of the trinomial following the pattern:
   $(x \pm first\ factor)\ (x \pm second\ factor)$

The sign in the last term of the binomial factors can be minus signs, depending on the signs of **b** and **c.**

Let us take the following examples.

Example 1: Factor $x^2 + 7x + 10$.

Solution:

Step 1. Find two factors with a product equal to the last term (c) of the trinomial and a sum equal to the middle term (b) of the trinomial.

Here, you are going to find two factors whose product is 10 (last term) and whose sum is 7 (middle term).

Refer to the illustration below.

Product	Sum
$(1)(10) = 10$	$1 + 10 = 11$
$(\mathbf{2})(\mathbf{5}) = \mathbf{10}$	$\mathbf{2 + 5 = 7}$

This is the correct combination. So, 2 and 5 are the factors.

Step 2: Write the factored form of the trinomial following the pattern $(x + first\ factor)(x + second\ factor)$.

$$x^2 + 7x + 10 = (\mathbf{x + 2})(\mathbf{x + 5})$$

Example 2: Factor $x^2 + 2x - 15$.

Solution:

Step 1: Find two factors with a product equal to the last term (c) of the trinomial and a sum equal to the middle term (b) of the trinomial.

Here, you are going to find two factors whose product is -15 (last term) and whose sum is 2 (middle term). Since the product is negative, the two numbers must have different signs. And since the sum is positive, the bigger number (the number with greater absolute value) must be positive.

Refer to the table below.

Product	Sum
$(-1)(15) = -15$	$-1 + 15 = 14$
$\mathbf{(-3)(5) = -15}$	$\mathbf{-3 + 5 = 2}$

This is the correct combination. So -3 and 5 are the factors.

Step 2: Write the factored form of the trinomial following the pattern $(x + first\ factor)\ (x + second\ factor)$.

$$x^2 + 2x - 15 = (\mathbf{x - 3})(\mathbf{x + 5})$$

Example 3: Factor $x^2 - 5x - 24$.

Solution:

Step 1. Find two factors with a product equal to the last term (c) of the trinomial and a sum equal to the middle term (b) of the trinomial.

Here, you are going to find two factors whose product is -24 (last term) and whose sum is -5 (middle term). Since the product is negative, the two numbers must have different signs. And since the sum is also negative, the bigger number (number with the greater absolute value) must be negative.

Refer to the table below.

Product	Sum
$(1)(-24) = -24$	$1 + (-24) = -23$
$(2)(-12) = -24$	$2 + (-12) = -10$
$\mathbf{(3)(-8) = -24}$	$\mathbf{3 + (-8) = -5}$

This is the correct combination. So, 3 and -8 are the two factors.

Step 2. Write the factored of the trinomial following the pattern (x+ first factor)(x+ second factor).

$$x^2 - 5x - 24 = (\mathbf{x + 3})(\mathbf{x - 8})$$

Example 4: Factor $x^2 + 6x + 14$.

Solution

Step 1. Find two factors with a product equal to the last term (c) of the trinomial and a sum equal to the middle term (b) of the trinomial.

Here, you are going to find two factors whose product is 14 (last term) and whose sum is 6 (middle term). Since the product is positive and the sum is also positive so both numbers must also be positive.

Refer to the table below.

Product	Sum
$(1)(14) = 14$	$1 + 14 = 15$
$(2)(7) = 14$	$2 + 7 = 9$

Based on the table, all the possible factors of 14 were already listed. However, there are NO two numbers having a product of 14 and the sum of 6. Thus, the given polynomial, $x^2 + 6x + 14$, is a ***PRIME***. However, it can be factored using different method to be discussed in other lesson.

Let us discuss the second classification of general trinomial $ax^2 + bx + c$, where $a \neq 1$.

Consider now factoring trinomials in which the coefficient of the squared term is other than one such as the following:

$$6x^2 - 5x - 6 \qquad\qquad 3x^2 + 17x + 10$$

Trinomials of these forms also have two binomial factors in which you need to consider the ax^2 term (first term of the trinomial), the bx term (second term of the trinomial) and the c term (third term of the trinomial) in getting the two binomial factors.

There are many ways of factoring this type of trinomials. One of those is through trial and error. Here is an example.

Example 1: Factor $6x^2 - 5x - 6$ through trial and error.

Solution:

Give all the factors of $6x^2$ and -6.

Factors of $6x^2$	Factors of -6
$(3x)(2x)$	$(3)(-2)$
$(6x)(x)$	$(-3)(2)$
	$(1)(-6)$
	$(-1)(6)$

Write all the possible factors using the values above and determine the middle term which is $-5x$ by getting the sum of the product of the outer terms and the product of the inner terms in FOIL method.

Possible Factors	Product of the Outer Terms	Product of the Inner terms	Sum of the product of the outer terms and the product of the inner terms
$(3x-2)(2x+3)$	$(3x)(3)=9x$	$(-2)(2x)=-4x$	$9x+(-4x)=5x$
$(3x+3)(2x-2)$	$(3x)(-2)-6x$	$(3)(2x)=6x$	$-6x+6x=0$
$(3x-3)(2x+2)$	$(3x)(2)=6x$	$(-3)(2x)=-6x$	$6x+(-6x)=0$
$\boldsymbol{(3x+2)(2x-3)}$	$\boldsymbol{(3x)(-3)=-9x}$	$\boldsymbol{(2)(2x)=4x}$	$\boldsymbol{-9x+4x=-5x}$
$(3x+1)(2x-6)$	$(3x)(-6)=-18x$	$(1)(2x)=2x$	$-18x+(2x)=-16x$
$(3x-6)(2x+1)$	$(3x)(1)=3x$	$(-6)(2x)=-12x$	$3x+(-12x)=-9x$
$(6x+3)(x-2)$	$(6x)(-2)=-12x$	$(3)(x)=3$	$-12x+3x=-9x$
$(6x-2)(x+3)$	$(6x)(3)=18x$	$(-2)(x)=-2x$	$18x+(-2x)=16x$
$(6x-3)(x+2)$	$(6x)(2)=12x$	$(-3)(x)=-3x$	$12x+(-3x)=9x$
$(6x+2)(x-3)$	$(6x)(-3)=-18x$	$(2)(x)=2x$	$-18x+2x=-16x$
$(6x+1)(x-6)$	$(6x)(-6)=-36x$	$(1)(x)=x$	$9x+(-4x)=5x$
$(6x-6)(x+1)$	$(6x)(1)=6x$	$(-6)(x)=-6x$	$6x+(-6x)=0$

With the factors above, $(3x+2)(2x-3)$ has the sum of the product of the outer terms and the product of the inner terms of $-5x$, thus making it as the factors of the trinomial $6x^2-5x-6$.

Factoring using trial and error is a long process. Knowing another way of factoring trinomials of ax^2+bx+c where a $\neq 1$ is very important and it is up to you which method you are going to use.

Another way of factoring this kind of trinomial is by grouping.

The following are the steps in factoring trinomials of the form ax^2+bx+c where $a \neq 1$.

1. Multiply the first term and the last term of the trinomial.

   $(ax^2)(c) = (ac)x^2$ (ac) = constant

2. Get the possible factors of the product of the first term and the last term of the trinomial in such a way that their sum is equal to the second term of the trinomial.

   $(ac)x^2$

   nx | mx

   Where *nx* and *mx* are the factors of acx^2. And,

   nx + mx = bx (second term of the trinomial)

3. Replace the middle term (*bx)* by the two factors.

   $ax^2+bx+c = ax^2+nx+mx+c$

4. Group $ax^2+nx+mx+c$ as follows,

   $(ax^2+nx)+(mx+c)$

5. Factor out the greatest common monomial factor of each group such that you can obtain the same binomial factor.

6. Combine the greatest common monomial factor of each group and multiply it to same binomial factor obtained in step 5. The result serves as the factors of the trinomial.

Example 1: Factor $6x^2 - 5x - 6$

Solution:

Step 1: Multiply the first term and the last term.

$$(6x^2)(-6) = -36x^2$$

Step 2: Get the possible factors of the product of the first term and the last term of the trinomial in such a way that the sum will be equal to the second term of the trinomial.

Here, the product is $-36x^2$ and the sum is $-5x$ which is the middle term. Since the product is negative, the two numbers must have different signs. And since the sum is also negative, the bigger number (number with the greater absolute value) must be negative.

Refer to the table below.

Product	Sum
$(x)(-36x) = -36x^2$	$x + (-36x) = -35x$
$(2x)(-18x) = -36x^2$	$2x + (-18x) = -16x$
$(3x)(-12x) = -36x^2$	$3x + (-12x) = -9x$
$\boldsymbol{(4x)(-9x) = -36x^2}$	$\boldsymbol{4x + (-9x) = -5x}$
$(6x)(-6x) = -36x^2$	$6x + (-6x) = 0$

This is the correct combination, so 4x and -9x are the two factors.

Step 3: Replace the middle term in such a way that $ax^2 + bx + c = ax^2 + nx + mx + c$, where nx and mx are the factors. Here, the factors are 4x and -9x

$$6x^2 - 5x - 6 = 6x^2 + 4x - 9x - 6$$

Step 4: Group $ax^2 + nx + mx + c$ in this pattern $(ax^2 + nx) + (mx + c)$.

$$(6x^2 + 4x) - (9x + 6)$$

Note: Notice that the operation used in the second group was changed. This will happen if the operation between the two groups is minus (-). Always do this if you encounter this case. In this case -9x - 6 becomes 9x + 6.

Step 5: Factor out the greatest common monomial factor of each group such that you can obtain the same binomial factor**.**

$$(6x^2 + 4x) = 2x(3x + 2)$$

$$-(9x + 6) = -3(3x + 2)$$

$2x$ and -3 are the GCF and $3x + 2$ and $3x + 2$ are the two same binomial factors.

Step 6: Combine the greatest common monomial factor of each group and multiply it to same binomial factor obtained in step 5. The result serves as the factors of the trinomial.

$$(2x - 3)(3x + 2)$$

So, $6x^2 - 5x - 6 = \boldsymbol{(2x - 3)(3x + 2)}$

Example 2: Factor $3x^2 + 17x + 10$.

Solution:

Step 1: Multiply the first term and the last term.

$$(3x^2)(10) = 30x^2$$

Step 2: Get the possible factors of the product of the first term and the last term of the trinomial in such a way that the sum will be equal to the second term of the trinomial.

Here, the product is $30x^2$ and the sum is 17x which is the middle term.

Since the product and the sum are positive, the two factors should also be both positive.

Refer to the table below.

Product	Sum
(x)(30x) = $30x^2$	x + 30x = 31x
(2x)(15x) = $30x^2$	**2x + 15x = 17x**
(3x)(10x) = $30x^2$	3x + 10x = 13x
(5x)(6x) = $30x^2$	5x + 6x = 11x

This is the correct combination, so 2x and 15x are the two factors.

Step 3: Replace the middle term in such a way that $ax^2 + bx + c = ax^2 + nx + mx + c$, where nx and mx are the factors. Here, the factors are 2x and 15x

$$3x^2 + 17x + 10 = 3x^2 + \mathbf{2x} + \mathbf{15x} + 10$$

Step 4: Group $ax^2 + nx + mx + c$ in this pattern $(ax^2 + nx) + (mx + c)$.

$$(3x^2 + 2x) + (15x + 10)$$

Step 5: Factor out the greatest common monomial factor of each group such that you can obtain the same binomial factor**.**

$$(3x^2 + 2x) = x(3x + 2)$$

$$(15x + 10) = 5(3x + 2)$$

x and *5* are the GCF and *(3x + 2)* and *(3x + 2)* are the two same binomial factors.

Step 6: Combine the greatest common monomial factor of each group and multiply it to same binomial factor obtained in step 5. The result serves as the factors of the trinomial.

$$(x + 5)(3x + 2)$$

So, $3x^2 + 17x + 10 = \mathbf{(x + 5)(3x + 2)}$

Example 3: Factor $2x^2 + 5x - 3$.

Solution:

Step 1: Multiply the first term and the last term.

$$\mathbf{(2x^2)(-3) = -6x^2}$$

Step 2: Get the possible factors of the product of the first term and the last term of the trinomial in such a way that their sum is equal to the second term of the trinomial.

Here, the product is $-6x^2$ and the sum is 5x which is the middle term.

Since the product is negative, the two factors must have different signs. And since the sum is positive, the factor with bigger coefficient (the number with greater absolute value) must be positive.

Refer to the table below.

Product	Sum
(-x)(6x) = - 6x²	**(-x) + 6x = 5x**
(-2x)(3x) = - 6x²	(-2x) + 3x = x

This is the correct combination, so -x and 6x are the two factors.

Step 3: Replace the middle term in such a way that $ax^2 + bx + c = ax^2 + nx + mx + c$, where nx and mx are the factors. Here, the factors are $-x$ and $6x$.

$$\mathbf{2x^2 + 5x - 3 = 2x^2 + 6x + (-x) - 3}$$

Step 4: Group $ax^2 + nx + mx + c$ in this pattern $(ax^2 + nx) + (mx + c)$.

$$(\mathbf{2x^2 + 6x}) + [(-x) - 3]$$

Step 5: Factor out the greatest common monomial factor of each group such that you can obtain the same binomial factor.

$$2x^2 + 6x = 2x(x + 3)$$
$$(-x) - 3 = -1(x + 3)$$

2x and *-1* are the GCF and *(x + 3)* and *(x + 3)* are the two the same binomial factors.

Step 6: Combine the greatest common monomial factor of each group and multiply it to same binomial factor obtained in step 5. The result serves as the factors of the trinomial.

$$(2x - 1)(x + 3)$$

So, $2x^2 + 5x - 3 = \mathbf{(2x - 1)(x + 3)}$

Example 4: Factor $3x^2 - 17x + 10$.

Solution:

Step 1: Multiply the first term and the last term.

$$(3x^2)(10) = 30x^2$$

Step 2: Get the possible factors of the product of the first term and the last term of the trinomial in such a way that their sum is equal to the second term of the trinomial.

Here, the product is $30x^2$ and the sum is 17x which is the middle term.

Since the product is positive and the sum is negative, the two factors must have both negative signs.

Refer to the table below.

Product	Sum
(-x)(-30x) = $30x^2$	(-x) + (-30x) = -31x
(-2x)(-15x) = $30x^2$	**(-2x) + (-15x) = -17x**
(-3x)(-10x) =$30x^2$	(-3x) + (-10x) = -13x
(-5x)(-6x) = $30x^2$	(-5x) + (-6x) = -11x

This is the correct combination, so -2x and -15x are the two factors.

Step 3: Replace the middle term in such a way that $ax^2 + bx + c = ax^2 + nx + mx + c$, where nx and mx are the factors. Here, the factors are $-2x$ and $-15x$.

$$3x^2 - 17x + 10 = 3x^2 + (-2x) + (-15x) + 10 \quad \text{or}$$
$$= 3x^2 - 2x - 15x + 10$$

Step 4: Group $ax^2 + nx + mx + c$ in this pattern $(ax^2 + nx) + (mx + c)$.

$$3x^2 - 2x - 15x + 10 = \quad (3x^2 - 2x) - (15x - 10)$$

Note: Notice that the operation used in the second group was changed. This will happen if the operation between the two groups is minus (–). Always do this if you encounter this case. In this case 15x + 10 becomes 15x – 10.

Step 5: Factor out the greatest common monomial factor of each group such that you can obtain the same binomial factor.

$$3x^2 - 2x = x(3x - 2)$$

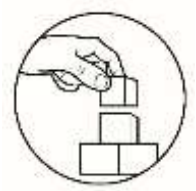
$$-(15x - 10) = -5(3x - 2)$$

x and *-5* are the GCF and *(3x - 2)* and *(3x - 2)* are the two the same binomial factors.

Step 6: Combine the greatest common monomial factor of each group and multiply it to same binomial factor obtained in step 5. The result serves as the factors of the trinomial.

$$(x - 5)(3x - 2)$$

So, $3x^2 - 17x + 10 = (x - 5)(3x - 2)$



What's More

Activity 1: Missing You!

Directions: Fill in the blank of the given equation. Write your answers on your answer sheet.

1. $a^2 + 12a + 11 = (a + \_\_\_)(a + 11)$
2. $b^2 + 6b + 8 = (b + 2)(b + \_\_\_\_)$
3. $c^2 - 7c + 6 = (c - \_\_\_)(c - 1)$
4. $2x^2 - 3x - 9 = (\_\_\_x + 3)(x - 3)$
5. $3x^2 + 5x - 2 = (\_\_\_x - 1)(x + 2)$

Activity 2: Break It to Me Gently

Directions: Factor each trinomial completely, if possible. If the polynomial is not factorable, write PRIME. Write your answers on your answer sheet.

1. $c^2 - 6c - 40$
2. $e^2 + 10e + 16$
3. $h^2 - 5h - 24$
4. $3x^2 - 5x - 12$
5. $4x^2 + 4x - 15$

What I Have Learned

This lesson discusses factoring general type of trinomials, which are divided into two forms. These are trinomials in the form of $ax^2 + bx + c$ where a=1, and $ax^2 + bx + c$ where $a \neq 1$. Write what you have learned about each one, including the step-by-step process on how factoring is done. Write your answers on a separate sheet.

For $ax^2 + bx + c$ where $a = 1$	$ax^2 + bx + c$ where $a \neq 1$.
Concepts Learned:	Concepts Learned:
Give a trinomial of the form $ax^2 + bx + c$ where a = 1 and factor it completely.	Give a trinomial of the form $ax^2 + bx + c$ where $a \neq 1$ and factor it completely.

What I Can Do

Suppose you have a rectangular garden that has an area represented by $15x^2 - 4x - 4$ square meters and its length is represented by $5x + 2$ meters.

1. Find the binomial that represents the width of your garden.
2. Suppose the measure of the length is 12m, what is the measure of the width of the rectangle?
3. What is the area of the rectangular garden?
4. If you are going to plant your garden with okra which is 25cm apart, how many okras can you plant?

Assessment

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which of the following is a perfect square number?
   A. 8 B. 24 C. 125 D. 121

2. All of the following expressions are perfect squares, **EXCEPT** one. What is it?
   A. $12a^2b^2$ B. x^2y^6 C. $16a^2b^6$ D. $25x^6y^8$

3. Which of the following is equal to $(2x - 1)^2$?
   A. $4x^2 + 4x + 1$ C. $4x^2 - 4x + 1$
   B. $2x^2 + 2x + 1$ D. $4x^2 - 4x - 1$

4. Which of the following is a perfect square trinomial?
   A. $x^2 + 6x + 9$ C. $4x^2 + 20xy + 9y^2$
   B. $4x^2 - 20x - 25$ D. $9a^2 - 6a^2bc + a^2b^2c^2$

5. All of the following are perfect square trinomials, EXCEPT one. Which is it?
   A. $x^2 - 4xy + 4y^2$ C. $4x^2 - 12xy + 9y^2$
   B. $9a^2 + 24a - 16$ D. $x^2y^2 - 6x^3y^2 + 9x^4y^2$

6. If one factor of a perfect square trinomial is $2xy - 3x$, what is the other factor?
   A. $2y - 3x$ B. $2y + 3x$ C. $2xy - 3x$ D. $2xy + 3x$

7. Which of the following is equal to $x^2 - 6xy + 9y^2$?
   A. $(x - 3y)^2$ B. $(x + 3y)^2$ C. $(2x - 3y)^2$ D. $(2x + 3y)^2$

8. What is the complete factored form of $4a^2 - 4a^2b + a^2b^2$?
   A. $(2a + ab)^2$ C. $(2a - ab)^2$
   B. $(a - 2ab)^2$ D. $(ab - 2a)^2$

9. What is the complete factored form of $8x^2 - 24xy + 18y^2$
   A. $2(2x - 3y)(2x - 3y)$ C. $2(3x + 2y)^2$
   B. $2(2x - 3y)(2x + 3y)$ D. $2(3x - 2y)^2$

10. Which of the following trinomials is factorable?
   A. $x^2 - 6x + 7$ C. $x^2 + 3x + 2$
   B. $2x^2 + 5x + 10$ D. $3x^2 - 6x + 12$

11. All of the following trinomials are factorable, EXCEPT one. What is it?
   A. $x^2 - 2x - 3$ C. $2x^2 + 3x - 2$
   B. $x^2 + 5x + 6$ D. $2x^2 + 2x + 4$

12. What is the complete factored form of $x^2 - 4x - 96$?
   A. $(x + 8)(x - 12)$ C. $(x - 8)(x - 8)$
   B. $(x - 8)(x + 12)$ D. $(x + 8)(x + 12)$

13. What is the complete factored form of $2x^2 - 6x - 8$?
   A. $(x + 4)(2x - 2)$ C. $(4x + 2)(x - 2)$
   B. (x-4)(2x+2) D. $(4x - 2)(x + 2)$

14. Which of the following trinomials has factors $(3a - 2b)$and$(2a - b)$?
   A. $6a^2 - 7ab + 2b^2$ C. $6a^2 - 7ab - 2b^2$
   B. $6a^2 + 7ab - 2b^2$ D. $6a^2 + 7ab + 2b^2$

15. A rectangular garden has an area by $6x^2 + x - 2$ square meter. If the length is represented by $3x + 2$, find a binomial that represents the width.
   A. $2x + 1$ C. $2x - 1$
   B. $x - 2$ D. $x - 2$

Additional Activities

Activity: Do More!

Solve each of the following. Write you answer on your answer sheet.

1. What does it mean when we say to completely factor a polynomial?
2. Discuss how you will factor $11x^2 + 19x - 6$.
3. Explain why $16 - 8x + x^2$can be factored as either $(4 - x)^2 or\ (x - 4)^2$.
4. Student A gives $(4x - 1)(x - 3)$ as the answer to a factoring problem. Student B gets $(3 - x)(1 - 4x)$. Who has the correct answer?
5. Is $-6x^2 - x + 2$ factorable? If yes, how will you do it?

Answer Key

What I Know
1. D
2. C
3. D
4. A
5. C
6. B
7. D
8. A
9. B
10. A
11. B
12. A
13. B
14. C
15. A

Lesson 1: Factoring Perfect Square Trinomial

What's In
1. x^2
2. 12x
3. 16
4. x^2
5. 12xy

What's New
1. P
2. P
3. N
4. N
5. P

What's More

Activity 1
1. 3
2. $2x$
3. 2
4. $4y$
5. $4a$

Activity 2
1. $(y+10)^2$
2. $(k-4)^2$
3. $(4m+6)^2$
4. $(7b-1)^2$
5. $3y(x-4)^2$

What I have Learned
1. Factoring
2. Polynomial
3. Technique
4. Squaring a binomial
5. First term
6. First term
7. Second term
8. Multiplied
9. Two
10. Square
11. Second term
12. Perfect square
13. First term
14. Second term
15. Perfect square
16. $a^2 + 2ab + b^2 = (a + b)^2$
17. $a^2 - 2ab + b^2 = (a - b)^2$

Note: Answers to Items 16 & 17 may be interchanged

What I Can Do
1. 2x + 4
2. 148
3. 90 000 in^2
4. 100 pieces

Lesson 2: Factoring General Trinomial

What's In
1. 2 and 4
2. -2 and 8
3. 3 and -5
4. -2 and 6
5. 3 and 3
6. 2 and -7

What's In

Answers may vary

What's More

Activity 1
1. 1
2. 4
3. 6
4. 2
5. 3

What's More

Activity 2
1. (c + 4)(c – 10)
2. (e+2)(e+8)
3. (h+3)(h-8)
4. (x-3)(3x+4)
5. (2x + 5)(2x – 3)

What I Have Learned

Answers may vary

What Can I Do
1. 3x – 2
2. 4m
3. $48m^2$
4. 768 okras

Assessment
1. D
2. A
3. C
4. A
5. B
6. C
7. A
8. C
9. A
10. C
11. D
12. A
13. B
14. A
15. C

Additional Activities
1. A polynomial is completely factored if its factors are considered to be prime.
2. Discussion may vary but leads to (11x-3)(x+2)
3. They will just give the same product when expanded.
4. Both have the correct answers.
5. Yes, (-3x+2)(x+1) or (2-3x)(1+x).

References

Book References

Alferes, M.S., Duro, M.A. (2004). *MSA Intermediate Algebra*. Quezon City. Quezon City MSA Academic Advancement Institute

Glorial, C.J., et.al. (2010). *21st Century Mathematics*. Quezon City. Phoenix Publishing House Inc.

Oronce O.A and Mendoza M.O. (2014). *e – Math Algebra*. Metro Manila. Rex Books Store

Rees K.R., Sparks. F.W., and Rees C.S. (1990). *College Algebra Tenth Edition*. Pennsylvania Plaza.McGraw-Hill Publishing Company

Electronic References

Interactive Mathematics (2018). *Special Products*. Retrived from http://www.intmath.com/factoring-fractions/1-special-products.php

Math Planet (nd). *Special products of polynomials*. http://www.mathplanet.com/education/algebra-1/factoring-and-polynomials/special-products-of-polynomial

Paul's Online Notes (2003). *Factoring Polynomials.* Retrieved from http://tutorial.math.lamar.edu/Classes/Alg/Factoring.aspx

Purple Math (nd). *Factoring Polynomials.* Retrieved from http://www.purplemath.com/modules/rtnladd2.htm

For inquiries or feedback, please write or call:

Department of Education – Bureau of Learning Resource
Ground Floor, Bonifacio Building, DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax. Nos.: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph