General Mathematics
Quarter 2 – Module 17:
Logical Equivalence and Conditional Propositions

ALTERNATIVE DELIVERY MODE

ADM

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Alternative Delivery Mode
Quarter 2 – Module 17: Logical Equivalence and Conditional Propositions
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DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master logical equivalence and conditional propositions. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:
1. illustrate the different forms of conditional propositions;
2. verify the logical equivalences;
3. state the converse, contrapositive, and inverse of a conditional proposition; and
4. cite a real-life situation using logical equivalence and conditional proposition.
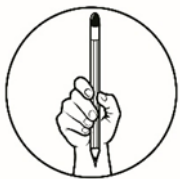
What I Know

Choose the letter of the best answer and write it on a separate sheet of paper.

1. What is the symbol used in referring to logically equivalent propositions?
    a. $\wedge$ c. $\Leftrightarrow$
    b. $\vee$ d. $\leftrightarrow$

For item numbers 2-7, verify $\sim[p \vee (\sim p \wedge q)] \Leftrightarrow (\sim p \wedge \sim q)$.

2. Which of the following supports the logical equivalence
$\sim[p \vee (\sim p \wedge q)] \Leftrightarrow (\sim p) \wedge \sim(\sim p \wedge q)$?
    a. De Morgan's Law   c. Idempotent Law
    b. Identity Law   d. Double Negation

3. Which of the following supports the logical equivalence
$(\sim p) \wedge \sim(\sim p \wedge q) \Leftrightarrow \sim p \wedge [\sim(\sim p) \vee (\sim q)]$?
    a. Double Negation   c. Identity Law
    b. Idempotent Law   d. De Morgan's Law

4. Which of the following supports the logical equivalence
   $\sim p \wedge [\sim(\sim p) \vee (\sim q) \Longleftrightarrow \sim p \wedge (p \vee \sim q)$?
   a. Double Negation
   b. Idempotent Law
   c. Identity Law
   d. De Morgan's Law

5. Which of the following supports the logical equivalence
   $\sim p \wedge (p \vee \sim q) \Longleftrightarrow (\sim p \wedge p) \vee (\sim p \wedge \sim q)$?

   a. Associative Law
   b. Distributive Law
   c. Commutative Law
   d. Switcheroo Law

6. Which of the following supports the logical equivalence
   $(\sim p \wedge p) \vee (\sim p \wedge \sim q) \Longleftrightarrow \phi \vee (\sim p \wedge \sim \wedge q)$?

   a. Switcheroo Law
   b. Absorption Law
   c. Domination Law
   d. Inverse Law

7. Which of the following supports the logical equivalence
   $\phi \vee (\sim p \wedge \sim \wedge q) \Longleftrightarrow \sim p \wedge \sim q$?

   a. Domination Law
   b. Identity Law
   c. Idempotent Law
   d. Double Negation

8. Which of the following logical equivalence is being depicted by
   $p \wedge (q \wedge r) \Longleftrightarrow (p \wedge q) \wedge r$?

   a. Identity Law
   b. Commutative Law
   c. Associative Law
   d. Distributive Law

9. Which of the following are logically equivalents?
   a. converse and contrapositive
   b. contrapositive and conditional
   c. conditional and inverse
   d. conditional and converse

For item numbers 10-15, refer to the conditional proposition, "If a quadrilateral is a square, then it has equal sides".

10. What type of conditionals is being depicted by the conditional statement, "If a quadrilateral is not a square, then it does not have equal sides."?
    a. converse
    b. contrapositive
    c. inverse
    d. equivalence

11. Which of the following represents item number 10?
    a. $\sim p \rightarrow \sim q$
    b. $p \rightarrow q$
    c. $\sim q \rightarrow \sim p$
    d. $q \rightarrow p$

12. What type of conditionals is being depicted by the conditional statement, "If a quadrilateral has equal sides, then it is a ssquare."?
    a. converse
    b. contrapositive
    c. inverse
    d. equivalence

13. Which of the following represents item number 12?
    a. $p \rightarrow q$
    b. $\sim p \rightarrow \sim q$
    c. $q \rightarrow p$
    d. $\sim q \rightarrow \sim p$

14. What type of conditionals is being depicted by the conditional statement, "If a quadrilateral does not have equal sides, then it is not a square."?
    a. converse
    b. contrapositive
    c. inverse
    d. equivalence

15. Which of the following represents item number 14?
    a. $p \rightarrow q$
    b. $\sim p \rightarrow \sim q$
    c. $q \rightarrow p$
    d. $\sim q \rightarrow \sim p$

Lesson 1

Logical Equivalence and Conditional Propositions

Do you easily believe in everything you hear? How do you know whether a statement is true or not? It is difficult to claim the truthfulness of one thing without a basis. That applies to everything. So, the next time you hear a rumor, before believing, one thing you must consider is to let it undergo a process. Because sometimes, there is no shortcut.

What's In

In the past lessons, you have learned the different logical operators, including negation ($\sim$), conjunction ($\wedge$), disjunction ($\vee$), conditional ($\rightarrow$), and biconditional ($\leftrightarrow$).

These logical operators connect propositions and make them compound ones. And truth values of said propositions can be determined through the use of a truth table.

Let us see whether you can still recall the truth values of logical operators when used in compound propositions.

Activity 1. Show Me What is True!

Fill in the missing cells of the table.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T			
T	F			
F	T			
F	F			

Third and fourth columns used conditional. All values for $p \rightarrow q$ "p implies q" are true except in the case where p is true and q is false. On the other hand, all values for $q \rightarrow p$ "q implies p" are true except in the case where q is true and p is false.

The last column involves disjunction $(p \rightarrow q) \vee (q \rightarrow p)$ "p implies q" or "q implies p". As you have noticed with the truth values you obtained in this column, all are true. And this we call a tautology τ, in which a proposition is always true.

Activity 2. Prove Me, I'm Wrong!

Complete the following table.

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$q \wedge \sim(p \rightarrow q)$
T	T			
T	F			
F	T			
F	F			

As recalled above, in the third column $p \rightarrow q$ "p implies q", all are true except in the case where p is true and q is false. Whereas, the fourth column $\sim(p \rightarrow q)$ "negation of p implies q" denies $p \rightarrow q$. Thus, making all that which are true in the third column, as false; and with that which is the only false in the former as the only true in the fourth column.

The fifth column involves the conjunction $q \wedge \sim(p \rightarrow q)$ "q and negation of p implies q". How did it differ from the last column in the first table? If you have observed, this time, all are false. This proposition that always results in false values is a contradiction ϕ.

Notes to the Teacher

Guide students in performing the activities. Explain further the reasons behind the answers in each activity.

What's New

Activity 3. Complete Me!

Decide for the truth values of the propositions in a truth table below.

p	q	$p \rightarrow q$	$\sim p$	$(\sim p) \vee q$	$(p \rightarrow q) \leftrightarrow [(\sim p) \vee q]$
T	T				
T	F				
F	T				
F	F				

Questions:
1. What truth values do you get for $p \rightarrow q$?
2. What truth values do you get for $\sim p$?
3. What truth values do you get for $(\sim p) \vee q$?
4. What have you noticed with every pair of corresponding truth values in the third and fifth columns?
5. What can you infer from the sixth column?

As mentioned in the other previous activities above, the conditional $p \rightarrow q$ results to all true values except in the case where p is true and q is false. For $\sim p$, values of p are simply negated. That is if p is true, then $\sim p$ is false, and vice-versa. On the other hand, for the disjunction $(\sim p) \vee q$, all are true except when both $\sim p$ and q are false. Comparing the third and fifth columns, you can easily notice that they have the same truth values. Moreover, it turns out that when propositions are perfectly alike, their biconditional will yield values that are all true. Hence, they result in a tautology, as observed in the last column.

What is It

Two propositions p and q are said to be **logically equivalent** if they have the same values in the truth table and if the biconditional $p \leftrightarrow q$ is a tautology. It is denoted by $p \Leftrightarrow q$ or $p \equiv q$.

The above activity shows that $(p \to q) \Leftrightarrow [(\sim p) \lor q]$. This logical equation is called Switcheroo Law.

Aside from this, below is a table of various logical equations.

Table of Logical Equivalences

Let *p, q,* and *r* be propositions.

Identity Law	$(p \land \tau) \Leftrightarrow p$	$(p \lor \phi) \Leftrightarrow p$
Domination Law	$(p \lor \tau) \Leftrightarrow \tau$	$(p \land \phi) \Leftrightarrow \phi$
Idempotent Law	$(p \lor p) \Leftrightarrow p$	$(p \land p) \Leftrightarrow p$
Inverse Law	$[p \lor (\sim p)] \Leftrightarrow \tau$	$[p \land (\sim p)] \Leftrightarrow \phi$
Double Negation	$\sim(\sim p) \Leftrightarrow p$	
Associative Law	$p \lor (q \lor r) \Leftrightarrow (p \lor q) \lor r$	$p \land (q \land r) \Leftrightarrow (p \land q) \land r$
Commutative Law	$p \lor q \Leftrightarrow q \lor p$	$p \land q \Leftrightarrow q \land p$
Distributive Law	$p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$
De Morgan's Law	$\sim(p \lor q) \Leftrightarrow (\sim p) \land (\sim q)$	$\sim(p \land q) \Leftrightarrow (\sim p) \lor (\sim q)$
Absorption Law	$p \lor (p \land q) \Leftrightarrow p$	$p \land (p \lor q) \Leftrightarrow p$
Switcheroo Law	$p \to q \Leftrightarrow \sim p \lor q$	

Example 1: Using logical equivalences, verify that $\sim(p \to q) \Leftrightarrow [p \land (\sim q)]$.

Proposition	Reason
$\sim(p \to q)$	Given
$\Leftrightarrow \sim[(\sim p) \lor q]$	Switcheroo Law
$\Leftrightarrow \sim(\sim p) \land (\sim q)$	De Morgan's Law
$\Leftrightarrow p \land (\sim q)$	Double Negation

Moreover, the **conditional proposition** $p \to q$ can be illustrated in three other conditional statements. They are the following:
1. **Converse**: $q \to p$
2. **Contrapositive**: $(\sim q) \to (\sim p)$
3. **Inverse**: $(\sim p) \to (\sim q)$

A conditional and its contrapositive are logically equivalent. In the same manner, based on a given conditional proposition, its converse and inverse are logically equivalent.

Example 2: Given the following conditional, "If the weather is fine, then I will go to the beach."; state its converse, contrapositive and inverse.

1. Converse $q \rightarrow p$: If I go to the beach, then the weather is fine.
2. Contrapositive $(\sim q) \rightarrow (\sim p)$: If I do not go to the beach, then the weather is not fine.
3. Inverse $(\sim p) \rightarrow (\sim q)$: If the weather is not fine, then I will not go to the beach.

What's More

Activity 1.1

Tell whether the propositions $[(p \rightarrow q) \wedge \sim q]$ and $\sim(p \vee q)$ are logically equivalent by filling in the correct logical equivalence for each statement.

Proposition	Reason
$[(p \rightarrow q) \wedge \sim q]$	Given
$\Leftrightarrow (\sim p \vee q) \wedge \sim q$	
$\Leftrightarrow \sim q \wedge (\sim p \vee q)$	
$\Leftrightarrow (\sim q \wedge \sim p) \vee (\sim q \wedge q)$	
$\Leftrightarrow (\sim q \wedge \sim p) \vee \phi$	
$\Leftrightarrow (\sim q \wedge \sim p)$	
$\Leftrightarrow (\sim p \wedge \sim q)$	
$\Leftrightarrow \sim(p \vee q)$	

Activity 1.2

Using the conditional proposition "If I eat foods high in carbohydrates, then I will gain weight.", state the following:

1. Converse : _____
2. Contrapositive: _____
3. Inverse : _____

What I Have Learned

Fill in the blanks with the correct words or phrases to complete the following statements.

1. Logically equivalent propositions are _____
2. Converse is a conditional statement _____
3. Contrapositive is a conditional statement _____
4. Inverse is a conditional statement _____
5. The pairs of conditional statements which are logically equivalents are

   _____

6. Name and state the laws supporting logical equivalences between propositions. _____
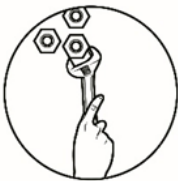
What I Can Do

This lesson helps you understand propositions which are logically equivalent. You also have learned the three other forms of conditional propositions. In this connection, construct a real-life conditional proposition and state its converse, contrapositive, and inverse. State which pairs are logically equivalent and explain what make them so.

Try scoring your essay using the rubric below.

Criteria	4	3	2	1
Statements **(score x 3)**	*All four conditional statements are correct.*	*Three conditional statements are correct.*	*Two conditional statements are correct.*	*One conditional statement is correct.*
Reasoning **(score x 2)**	*Very clear and very accurate justification*	*Clear and accurate justification*	*Clear justification*	*Vague justification*

Assessment

Multiple Choice. Choose the letter of the best answer and write it on a separate sheet of paper.

1. What do you call two propositions with the same truth values?
       a. logically equivalent   c. contradictions
       b. tautologies   d. fallacies

For item numbers 2-7, verify $\sim(q \to p) \lor (p \land q) \Leftrightarrow q$.

2. Which of the following supports the logical equivalence
   $\sim(q \to p) \lor (p \land q) \Leftrightarrow \sim(\sim q \lor p) \lor (p \land q)$?
       a. Absorption Law   c. De Morgan's Law
       b. Commutative Law   d. Switcheroo Law

3. Which of the following supports the logical equivalence
     $\sim(\sim q \lor p) \lor (p \land q) \Leftrightarrow (q \land \sim p) \lor (p \land q)$?
       a. Absorption Law and Double Negation
       b. Commutative Law and Double Negation
       c. De Morgan's Law and Double Negation
       d. Switcheroo Law and Double Negation

4. Which of the following supports the logical equivalence

   $(q \land \sim p) \lor (p \land q) \Leftrightarrow (q \land \sim p) \lor (q \land p)$?

       a. Associative Law   c. Distributive Law
       b. Commutative Law   d. Identity Law

5. Which of the following supports the logical equivalence
   $(q \land \sim p) \lor (q \land p) \Leftrightarrow q \land (\sim p \lor p)$?

       a. Distributive Law   c. Identity Law
       b. Idempotent Law   d. Switcheroo Law

6. Which of the following supports the logical equivalence
     $q \land (\sim p \lor p) \Leftrightarrow q \land \tau$?
       a. Absorption Law   c. Identity Law
       b. Domination Law   d. Inverse Law

7. Which of the following supports the logical equivalence $q \land \tau \Leftrightarrow q$?
       a. Absorption Law   c. Identity Law
       b. Domination Law   d. Inverse Law

8. Which of the following logical equivalences is being depicted by
   $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$?
   a. Associative Law   c. Distributive Law
   b. Commutative Law   d. Identity Law

9. Which of the following are logically equivalents?
   a. conditional and converse   c. converse and contrapositive
   b. contrapositive and inverse   d. converse and inverse

For item numbers 10-15, refer to the conditional proposition, "If Matmat is a responsible student, then he will do his assignments".

10. What type of conditionals is being depicted by the conditional statement, "If Matmat does his assignments, then he is a responsible student"?
    a. converse   c. inverse
    b. contrapositive   d. equivalence

11. Which of the following represents item number 10?
   a. $q \rightarrow p$ c. $\sim q \rightarrow \sim p$
    b. $p \rightarrow q$ d. $\sim p \rightarrow \sim q$

12. What type of conditionals is being depicted by the conditional statement, "If Matmat does not do his assignments, then he is not a responsible student"?
    a. converse   c. inverse
    b. contrapositive   d. equivalence

13. Which of the following represents item number 12?
    a. $p \rightarrow q$ c. $q \rightarrow p$
    b. $\sim p \rightarrow \sim q$ d. $\sim q \rightarrow \sim p$

14. What type of conditionals is being depicted by the conditional statement, "If Matmat is not a responsible student, then he will not do his assignments"?
    a. converse   c. inverse
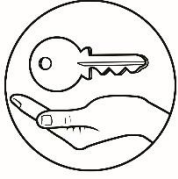   b. contrapositive   d. equivalence

15. Which of the following represents item number 14?
    a. $p \rightarrow q$ c. $q \rightarrow p$
    b. $\sim p \rightarrow \sim q$ d. $\sim q \rightarrow \sim p$
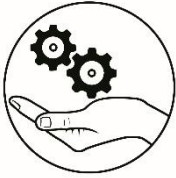
Additional Activities

A. Verify the logical equivalence $(p \land q) \to r \Leftrightarrow [(p \to r) \lor (q \to r)]$.

B. Think of a conditional statement. Cite its converse, contrapositive, and inverse.

Conditional: _____

Converse: _____

Contrapositive: _____

Inverse: _____

Answer Key

What I Know

1. c
2. a
3. d
4. a
5. b
6. d
7. b
8. c
9. b
10. c
11. a
12. a
13. c
14. b
15. d

What's More

Activity 1.1
1. Switcheroo Law
2. Commutative Law
3. Associative Law
4. Inverse Law
5. Identity Law
6. Commutative Law
7. De Morgan's law

Activity 1.2
1. If I gain weight, then I eat foods high in carbohydrates.
2. If I do not gain weight, then I do not eat foods high in carbohydrates.
3. If I do not eat foods high in carbohydrates, then I do not gain weight.

Assessment

1. a
2. d
3. c
4. b
5. a
6. d
7. c
8. c
9. d
10. a
11. a
12. b
13. d
14. c
15. b

References

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*DepED Material: General Mathematics Learner's Material